RAA 2014 Vol. **14** No. **8**, 979–991 doi: 10.1088/1674–4527/14/8/008
http://www.raa-journal.org *http://www.iop.org/journals/raa*

***R**esearch in*
***A**stronomy and*
***A**strophysics*

The influences of convective overshooting and semiconvection on the chemical evolution of massive stars *

Cai-Yun Ding[1,2,3] and Yan Li[1,2]

[1] Yunnan Observatories, Chinese Academy of Sciences, Kunming 650011, China;
dcy84310@ynao.ac.cn; ly@ynao.ac.cn
[2] Key Laboratory for the Structure and Evolution of Celestial Objects, Chinese Academy of Sciences, Kunming 650011, China
[3] University of Chinese Academy of Sciences, Beijing 100049, China

Received 2013 December 11; accepted 2014 February 11

Abstract In massive stars, convection in the interior is different from that of intermediate and small mass stars. In the main-sequence phase of small mass stars, there is a convective core and a radiative envelope, between which are the radiative intermediate layers with uneven chemical abundances. Semiconvection would occur in the intermediate layers between the convective core and the homogeneous envelope in massive stars. We treat core convective overshooting and semiconvection together as a process. We found that when decreasing overshooting, the semiconvection is more pronounced. In these two processes, we introduce one diffusive parameter D, which is different from other authors who have introduced different parameters for these two zones. The influences of the turbulent diffusion process on chemical evolution and other quantities of the stellar structure are shown in the present paper.

Key words: stars: evolution — stars: abundances — convection — diffusion

1 INTRODUCTION

Stellar evolution is driven by changes in chemical composition, which are caused by nuclear reactions. The extent to which the chemical elements are mixed has many influences, including the structural evolution of stars and the age of clusters, chemical evolution, and the surface abundances of stars. Nonstandard models of mixing in massive stars spring up analyzing observational data indicating departures from the results of standard models.

Overshooting affects the internal structure of stars by modifying the chemical abundance profiles. During the central burning phases, core overshooting brings elements from the outer layers to the inner layers, resulting in more massive convective cores. The stars spend more time burning their central hydrogen and generate higher luminosities. Mowlavi & Forestini (1994) found that overshooting does not modify the evolutionary features of the stars when applied to the boundaries of all convective zones. The common method to model the process involved in convective overshooting was by adding an extending zone to the convective zone ($0.1 \sim 0.2\,H_{\rm P}$ outside the edge of the convective zone), which agreed with observational results. Since the diffusion method was introduced

∗ Supported by the National Natural Science Foundation of China.

to simulate chemical mixing in stellar models, this method is popularly used to simulate the process of chemical mixing in the overshooting region.

Semiconvection was first proposed by Schwarzschild & Härm (1958) to obtain a coherent solution for the structure of massive stars. The developments of semiconvective zones were discovered to take place during the main sequence (MS) phase of massive stars and during the helium burning phase in low-mass stars (Schwarzschild & Härm 1965), and Paczyński (1970) also found the phenomenon to be present during the same evolutionary phase in intermediate-mass stars. The impact of semiconvection on the structural evolution of massive stars has been analyzed by several authors (Stothers 1965, 1970; Chiosi & Summa 1970; Eggleton 1972; Sreenivasan & Ziebarth 1974).

Semiconvection and core overshooting modify the structural evolution of massive stars after central hydrogen exhaustion. Stothers & Chin (1975) found that semiconvection is almost nonexistent for the 10 $M_{\odot}$ model and is of minor importance for the 15 $M_{\odot}$ model. But for the 30 $M_{\odot}$ model, semiconvection covers most of the intermediate zone and develops into full convection if the initial hydrogen and metal abundances are both high.

In the studies by Stothers (1966) and Hayashi & Cameron (1962), which treated the semiconvective zone as leading to the condition for convective neutrality and which took electron scattering as the sole source of opacity, and in studies by Iben (1966), which treated the semiconvective zone as fully convective and which took electron scattering plus bound-free absorption as the source of opacity, the stars would start and end the helium burning phase as a blue supergiant. Stothers & Chin (1975) treated the semiconvective zone as leading to the convective neutrality condition and took electron scattering plus bound-free absorption as the source of opacity, finding that the star starts the helium burning phase as a red supergiant.

Prior authors have treated the convective overshooting and semiconvection as two independent processes and used different methods to model the chemical mixing. The different methods to treat semiconvection and convective overshooting each have their own shortcomings. The diffusion parameters in those models are free. More free parameters mean less reliability. In the present paper, we use one element diffusion coefficient to control the extent of chemical mixing in the core convective overshooting region and the semiconvective zone. In addition, our diffusive parameters are calculated using the results of the turbulent convection model (TCM), which are not completely artificial.

There are six sections in the present paper. In Section 2, convective mixing in the overshooting regions and semiconvection zones are introduced. Section 3 describes the input physics and model assumptions in our stellar models. Then, we draw the chemical profiles with different parameter values in Section 4 and compare the differences in evolution from our stellar models with these different parameter values in Section 5. We summarize our main conclusions in Section 6.

2 CONVECTIVE MIXING IN THE OVERSHOOTING REGIONS AND SEMICONVECTION ZONES

The basic scenario of stellar convection is physically simple. A fluid with a temperature stratification such that cold fluid is located on top of hot fluid will be unstable. Schwarzschild described buoyancy driven instability for a chemically homogeneous fluid in 1905. When the local temperature gradient is larger than the adiabatic one (i.e. $\nabla_{\rm rad} > \nabla_{\rm ad}$, where $\nabla_{\rm rad}$ and $\nabla_{\rm ad}$ are the radiative and the adiabatic temperature gradients respectively), the hot fluid will adiabatically expand until its density becomes smaller than the environment, and then buoyancy occurs. For a chemically stratified fluid, the Ledoux criterion should be used to determine the convectively unstable zone.

There are many effects that can produce convective instability. Increasing $\nabla_{\rm rad}$ is one important means for producing convective instability. It is well known that the radiative temperature gradient

$\nabla_{\rm r}$ is defined as

$$\nabla_{\rm r} = \frac{3}{16\pi acG}\frac{\overline{\kappa}L_{\rm r}P}{m_{\rm r}T^4}\,, \tag{1}$$

where $L_{\rm r}$ is the luminosity at the radius r, P the pressure, T the temperature, $m_{\rm r}$ the mass inside r and $\overline{\kappa}$ the mean opacity. From Equation (1), it is evident that large luminosity and large opacity may lead to $\nabla_{\rm rad} > \nabla_{\rm ad}$. On the other hand, partial ionizations of some abundant elements, such as hydrogen and helium, can significantly lower $\nabla_{\rm ad}$, which helps to achieve the condition that $\nabla_{\rm rad} > \nabla_{\rm ad}$.

In massive MS stars, the nuclear energy production rate ε is large in the stellar core, leading to a large enough $L_{\rm r}$ to make $\nabla_{\rm rad} > \nabla_{\rm ad}$ there. As a result, convection develops in the central zone and a convective core will be formed that is much larger than the nuclear burning zone. On the other hand, a few convective shells are formed in the stellar envelope, because the opacity can be fairly large in the ionization zones of hydrogen and helium. Since these convective shells are quite thin, they do not significantly affect the structure and evolution of massive stars.

The major effects of convection on stellar structure and evolution are to transfer heat and to mix materials in the convective zone. For massive MS stars, convective motions are almost adiabatic in the stellar core, so that the temperature gradient is nearly adiabatic there. On the other hand, the timescale for the convective motions is much shorter than the typical timescale of the evolution. As a result, mixing in the convective core is assumed to be instant and complete. Therefore, the treatment of convection is simple for the convective core. However, the question of how to dispose elements to be mixed in the overshooting region and in the semiconvective zone leads to a large number of possible solutions.

Instantaneous mixing has been commonly adopted in the overshooting region by many authors. For example, Hirschi et al. (2004) assumed instantaneous mixing in an extended region that is 0.1 $H_{\rm P}$ thick from the Schwarzschild boundary, where $H_{\rm P}$ is the local pressure scale height calculated at the boundary of the convective core.

Another way to treat element mixing in the overshooting region, as well as in the semiconvection zone, is the so-called "exponentially decreasing mixing" (e.g. Herwig et al. 1997), which assumes that the convective mixing is a diffusion process. The diffusion equation for an element is

$$\frac{\partial X}{\partial t} = \frac{\partial}{\partial m_{\rm r}}\left[(4\pi r^2\rho)^2 D\frac{\partial X}{\partial m_{\rm r}}\right], \tag{2}$$

where X is the mass abundance of the considered chemical elements, D the element diffusion coefficient and ρ the density. There are many simulation methods that explore how to model the value of the element diffusion coefficient D.

Freytag et al. (1996) approximated the diffusion coefficient $D_{\rm ov}$ in the overshooting region beyond the convective core as an exponentially decreasing function

$$D_{\rm ov} = D_0{\rm e}^{\frac{-2z}{H_v}}\,, \tag{3}$$

where D_0 is a diffusion parameter near the Schwarzschild boundary of the convective core, and H_v is a typical distance that the convective cells can overshoot beyond the Schwarzschild boundary. It should be noticed that neither of the parameters in Equation (3) (i.e. D_0 and H_v), can be determined from the theory and they have to be assumed for various applications. Analogous to the molecular diffusion process, El Eid et al. (2009) suggested that in the overshooting region

$$D_{\rm ov} = \frac{1}{3}vl\,, \tag{4}$$

where v is the average velocity of convective elements, and l is the typical length of the convective mixing. Again, v and l should be modeled elsewhere in order to use the diffusion coefficient defined above.

Semi-convective mixing has been studied by many authors as follows (see, for example, Robertson & Faulkner 1972). The elements are just partially mixed in the semiconvective zone in order for the radiative temperature gradient to be equal to the adiabatic temperature gradient ($\nabla_{\rm rad} = \nabla_{\rm ad}$). This approach is certainly an extreme case for convective mixing in the semiconvection zone. However, based on Kato's linear analysis (Kato 1966), Langer et al. (1983) suggested a diffusion coefficient for the incomplete mixing in the semiconvection zone to be

$$D_{\rm sc} = \frac{\alpha}{6}\frac{\overline{\kappa}}{c_{\rm p}\rho}\frac{\nabla - \nabla_{\rm ad}}{\nabla_{\rm rad} - \nabla}\,, \tag{5}$$

where $c_{\rm p}$ is the specific heat at constant pressure, ∇ is the actual temperature gradient and α is a free parameter. In addition, Woosley et al. (2002) suggested that in the semiconvection zone

$$D_{\rm sc} = 0.1 D_{\rm r}\,, \tag{6}$$

where

$$D_{\rm r} = \frac{acT^3}{\overline{\kappa}\rho^2 c_{\rm p}}\,. \tag{7}$$

It should be noticed that convective overshooting and semiconvection are usually treated as two independent processes and modeled separately in the literature. However, they are physically related with each other. For massive MS stars, the convective overshooting exists all the time, modifying the chemical gradient profile when the convective core shrinks during the MS evolution. On the other hand, semiconvection appears by the end of the MS evolution in the outer part of the region with the chemical gradient, which will be called the chemical gradient zone hereafter. Consequently, the inner convective overshooting determines the outer development of semiconvection in the chemical gradient zone. More importantly, the convective mixing is not primarily caused by regular up and down motions but rather by the resultant turbulence. It is, therefore, desirable to use a turbulence model in the treatment of convective overshooting and semiconvection, as well as the resulting convective mixing.

We follow Li & Yang's method (Li & Yang 2007) to treat turbulence in both the overshooting region and the semiconvection zone. Solving the turbulent diffusion equations, we can obtain the radial and total kinetic energy of turbulence, as well as other quantities related to turbulence. The element diffusion coefficient D is defined according to the following equation

$$D = C_x \frac{\overline{u_{\rm r}' u_{\rm r}'}}{\sqrt{k}}\, l\,, \tag{8}$$

where $\overline{u_{\rm r}' u_{\rm r}'}$ is the radial kinetic energy and k the total kinetic energy of turbulence, $l = \alpha H_{\rm P}$ can be obtained from evolutionary models, α is the mixing-length parameter and C_x is a free parameter which describes the degree of convective mixing. It may be noticed that the mixing-length parameter α and the turbulent diffusion coefficient C_x are two important parameters that influence the chemical profiles. α chiefly affects the overshooting distance while C_x decides the shape of the chemical profile. Just as their names imply, α determines how far the eddies are effectively mixed and C_x measures the extent of mixing for the elements. In the present paper, we adopt 0.7 and 1.0 for α, and 10^{-9}, 5×10^{-9} and 10^{-8} for C_x, to analyze the influence of the parameters on chemical and structural evolution of massive stars.

3 INPUT PHYSICS AND MODEL ASSUMPTIONS

Our stellar evolution models are computed based upon the following assumptions. The stellar evolution code h04.f was originally written by Paczyński and Kozlowski, and updated by Sienkiewicz (2004)[1] and many other authors. Nuclear reaction rates are taken from Bahcall et al. (1995); Harris et al. (1983). The OPAL equation of state (Rogers et al. 1996) is used. Opacity is read from OPAL opacity tables (Rogers & Iglesias 1995; Iglesias & Rogers 1996) for $\lg T > 3.95$ and from opacity tables of Alexander & Ferguson (1994) when $\lg T < 3.95$.

In order to implement the convective mixing scheme introduced in Section 2 into our evolution code, we make some further simplifications as follows. Stellar models are computed using the Schwarzschild criterion to determine the boundary of the convective core as well as those of the convection zones in the stellar envelope. The standard mixing-length theory is used to obtain the actual temperature gradient. As soon as a stellar model is obtained, we solve for the TCM using the same method as Ding & Li (2014) in order to obtain the diffusion coefficient D according to Equation (8). The mixing in the convective core is assumed to be complete, while the diffusion Equation (2) is solved to determine the chemical profiles in the overshooting region and semiconvection zone. Such simplifications are acceptable if the convective mixing is moderate in the overshooting region as well as in the semiconvection zone.

We have adopted many parameters in the TCM, which have been discussed in detail by Zhang & Li (2012) and Ding & Li (2014). In the present paper, we choose the following values: for the diffusion parameters $C_s = C_{t1} = C_{e1} = 0.05$; for the dissipation parameters $C_T = 3.00$ and $C_e = 1.25$; for the anisotropic parameter $C_k = 2.5$. As discussed by Ding & Li (2014), there are two methods to treat the averaged entropy gradient. The Ledoux method assumes

$$\frac{\partial \overline{s}}{\partial r} = -\frac{c_{\rm P}}{H_{\rm P}}(\nabla - \nabla_{\rm ad} - \nabla_\mu)\,, \tag{9}$$

where the chemical gradient ∇_μ is defined as

$$\nabla_\mu = -\Big(\frac{\partial \ln \rho}{\partial \ln T}\Big)^{-1}_{\mu,p}\Big(\frac{\partial \ln \rho}{\partial \ln \mu}\Big)_{T,p}\frac{d\ln \mu}{d \ln P}\,. \tag{10}$$

On the other hand, the Schwarzschild method assumes

$$\frac{\partial \overline{s}}{\partial r} = -\frac{c_{\rm P}}{H_{\rm P}}(\nabla - \nabla_{\rm ad})\,. \tag{11}$$

We will compare their effects on the chemical evolution of the considered star.

The mass of the star we have considered here is 30 $M_\odot$. Since the overshooting from the convective core and semiconvection only happen only during the MS, we have only computed its evolution during the MS stage. The initial metal abundance is chosen to be 0.02 and the initial hydrogen abundance is 0.7. As the mass of the star is not large, we ignore the mass loss through stellar wind. This approximation will not result in a large effect on the chemical and structural evolution of the considered star. Stellar models usually consist of more than 3000 mesh points and evolve about 300 time steps, from zero age to the end of the MS stage.

4 THE CHEMICAL PROFILES WITH DIFFERENT PARAMETER VALUES

4.1 Comparison between Results with Different Turbulent Diffusion Coefficients C_x

We first choose the turbulent diffusion coefficient C_x to be 0, 10^{-9}, 5×10^{-9} and 10^{-8}, in order to see its effect on the chemical profiles of the star. It is evident in Figure 1 that the chemical gradient

[1] *ftp.camk.edu.pl/camk/rs/04/readme.04*

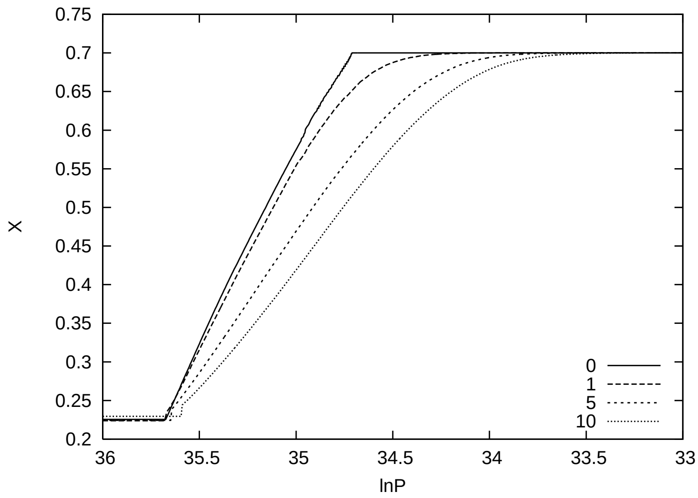

Fig. 1 The hydrogen abundance profiles of 30 $M_{\odot}$ models are shown with different turbulent diffusion coefficients. Values on the x-axis are the natural logarithm of the pressure. The Ledoux method is adopted to define the convective zone and calculate the turbulent quantities; the value of the mixing length parameter α is 1.0 and the turbulent diffusion parameters are 0.05. The solid line denoted by '0' means no convective mixing, the long dashed line denoted by '1' means the turbulent diffusion coefficient C_x is 10^{-9}, the short dashed line denoted by '5' means the turbulent diffusion coefficient C_x is 5×10^{-9}, and the dotted line denoted by '10' means the turbulent diffusion coefficient C_x is 10^{-8}.

becomes smaller when the turbulent diffusion coefficient C_x is larger. It is worth noticing that the hydrogen profile will be smoother with convective mixing than those with either the standard method (only mixing within the Schwarzschild boundary) or the traditional overshooting method (complete mixing including an overshooting region). This is a direct result of our convective mixing scheme, which treats convective overshooting and semiconvection in the same way. It can be seen that the complete mixing zone beyond the Schwarzschild boundary is very much limited, but a considerable amount of hydrogen is gradually transported from the whole chemical gradient zone into the convective core. When the convective mixing is done, the change in the hydrogen profile is more obvious around the top of the chemical gradient zone than at the surface of the convective core. This result indicates that the influence of semiconvection on the element profiles lies mainly in the outer part of the chemical gradient region. When estimating equivalent overshooting distance measured by the local pressure height at the surface of the convective core, we can measure the width of the two hydrogen profiles with and without the convective mixing around the middle part of the chemical gradient zone. It can be found that the width of the convective core overshooting is about $0.2\,H_{\rm P}$ when adopting $C_x = 5 \times 10^{-9}$.

To get an intuitive view of how many materials are mixed from the chemical gradient zone into the convective core, we plot the hydrogen profile against the stellar mass that is interior to the radius of r in Figure 2. It can be seen that the total mass of the core for a star with convective mixing in the overshooting region is only 5% larger than the original case without convective mixing when adopting $C_x = 5 \times 10^{-9}$, which amounts to about a 10% increase of the mass of the convective core. This effect evidently influences the evolution of the star. When the turbulent diffusion coefficient C_x is enlarged from 10^{-9} to 10^{-8}, the convective core obviously grows, so that the evolution of the star may be significantly influenced by the mixing outside of the traditional convective boundary.

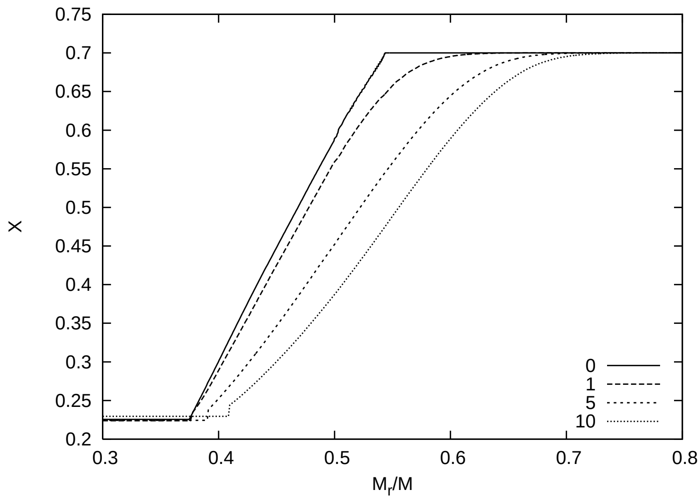

Fig. 2 The hydrogen abundance profiles of 30 $M_{\odot}$ models with different turbulent diffusion coefficients are shown. The different lines denote the same quantities as in Fig. 1. What is different from Fig. 1 is that values on the x-axis are the mass fraction inside the considered point.

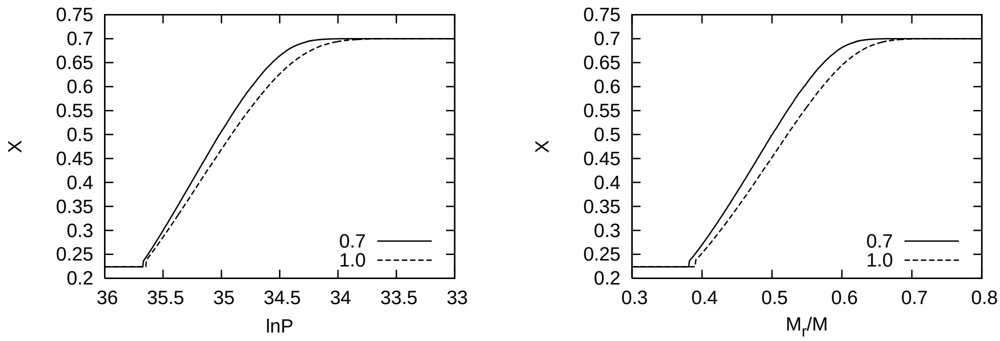

Fig. 3 The hydrogen abundance profiles of 30 $M_{\odot}$ models with different α are shown. The values on the x-axis are the natural logarithm of the pressure (*left*) and mass fraction inside the considered point (*right*). The Ledoux method is adopted to define the convective zone and calculate the turbulent quantities. The turbulent diffusion parameters are 0.05, and the turbulent diffusion coefficient C_x is 5×10^{-9}. The solid line denoted by '0.7' means α is 0.7, and the dashed line denoted by '1' means α is 1.0.

4.2 Comparison between Results with Different α

In Figure 3, the influence of the mixing length parameter α on the profile of hydrogen has been investigated. It can be seen that increasing α can lead to a slightly more extended hydrogen profile and a smaller chemical gradient. As already pointed out by Ding & Li (2014), increasing α leads to an increase in the turbulent velocity near the surface of the convective core, which results in an increase in the overshooting distance, as shown in Figure 3. In addition, it should be noticed that such an influence is more remarkable when using the mass fraction for the abscissa (seen in the right

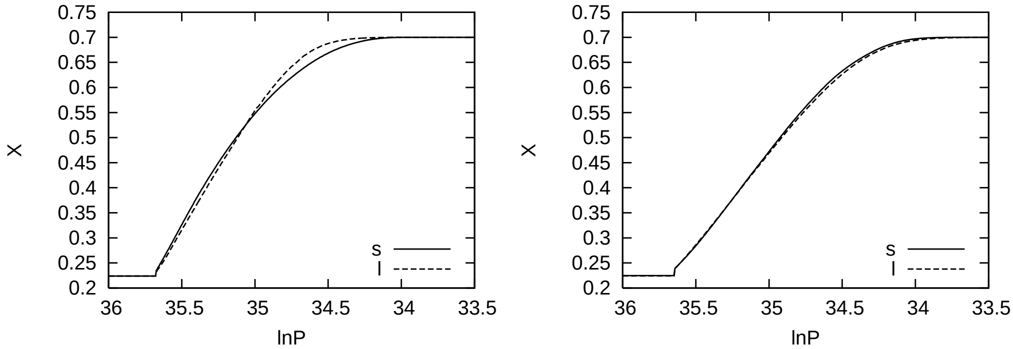

Fig. 4 The hydrogen abundance profiles of $30\,M_\odot$ models with the Schwarzschild method and the Ledoux method are shown. The values on the x-axis are the natural logarithm of the pressure. The mixing length parameter α is 1.0, the turbulent diffusion parameters are 0.05 and the turbulent diffusion coefficient C_x is 10^{-9} (*left*) and 5×10^{-9} (*right*). The solid line denoted by 's' means the Schwarzschild method is adopted to define the convective zone and calculate the turbulent quantities; the dashed line denoted by 'l' means the Ledoux method is adopted to define the convective zone and calculate the turbulent quantities.

panel in Fig. 3). Compared to the effect of the convective mixing parameter C_x, the effect of the mixing length parameter α seems more moderate but its influence on the evolution of the star should not be less significant.

4.3 Comparison between Results using the Schwarzschild Method and the Ledoux Method

In Figure 4, the influences of different convection processing methods on the chemical evolution of the star have been investigated. It can be seen that both the Ledoux method and the Schwarzschild method result in similar hydrogen profiles, except that the Schwarzschild method leads to slightly more significant mixing in the outer part of the chemical gradient zone than does the Ledoux method. It has been pointed out by Ding & Li (2014) that adopting the Ledoux method may increase the turbulent velocity near the surface of the convective core but decrease it in the semiconvection zone. This property explains the results obtained above. Therefore, the Ledoux method may enhance the overshooting beyond the convective core but weaken convective motions in the semiconvection zone.

On the other hand, it has been found that the convective mixing parameter has a considerable influence on the results with different treatments of convection. As shown in Figure 4, the difference between the hydrogen profile obtained by adopting the Ledoux method and that by adopting the Schwarzschild method is weakened if the turbulent diffusion coefficient C_x is increased. Since the Ledoux method is closer to the physical conditions in the chemical gradient zone, it is better to take 5×10^{-9} for the turbulent diffusion coefficient C_x.

4.4 Chemical Evolution of the Star

The influences of convective mixing in the overshooting region and semiconvection zone on the chemical evolution of the considered star are shown in Figure 5. Without the convective mixing in the overshooting region and semiconvection zone, the hydrogen profile in the chemical gradient zone will be almost a straight line, extending from the value of the hydrogen content of the convective core to that of the homogeneous envelope and gradually growing longer, as seen in the solid line in

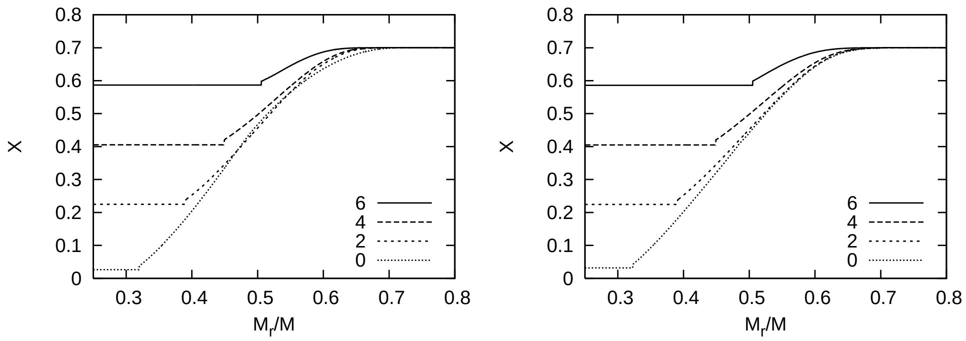

Fig. 5 The hydrogen abundance profiles are shown with 30 $M_\odot$ models adopting the Schwarzschild method and the Ledoux method at different evolutionary stages. Values on the x-axis are the mass fraction inside the considered points. The turbulent diffusion parameters are 0.05, the mixing length parameter α is 1.0 and the turbulent diffusion coefficient C_x is 5×10^{-9}; the Schwarzschild method (*left*) and the Ledoux method (*right*) are adopted to define the convective zone and to calculate the turbulent quantities. There are four lines which represent four different evolutionary stages: the solid line denoted by '6' means core hydrogen content is 0.59, the dashed line denoted by '4' means core hydrogen content is nearly 0.4, the short dashed line denoted by '2' means core hydrogen content is 0.22, and the dotted line denoted by '0' means core hydrogen content is 0.03.

Figure 1. With proper convective mixing considered in the overshooting region and semiconvection zone, the extent of the hydrogen profile significantly changes with stellar evolution. As the star evolves during the MS phase, the hydrogen profile becomes flatter in the chemical gradient zone, extending farther and more smoothly into the stellar envelope.

Hydrogen profiles adopting either the Ledoux method or the Schwarzschild method are also shown in Figure 5. Compared with those adopting the Schwarzschild method, it can be seen that the extent of chemical gradient zone is slightly shorter when adopting the Ledoux method to calculate the turbulent quantities and element mixing. These results are in agreement with properties of turbulent velocity profiles discussed by Ding & Li (2014). As the star evolves, the turbulent velocities in the semiconvective zone are larger when adopting the Schwarzschild method, which implies that the degree of element mixing would be stronger. Therefore, the hydrogen profile would extend farther outwards. By comparing the turbulent velocity profiles that adopt the Ledoux method and the Schwarzschild method, it can be found that the Ledoux method restrains the turbulent velocity in the semiconvective zone. This illustrates that the hydrogen profiles are less extended when adopting the Ledoux method than when adopting the Schwarzschild method.

5 EVOLUTION WITH DIFFERENT PARAMETER VALUES

5.1 HR Diagram of the Star Adopting the Ledoux or Schwarzschild Method

In Figure 6 we show the HR diagram of the considered star, adopting either the Ledoux method or the Schwarzschild method, as well as the model without convective mixing in the chemical gradient zone. It can be seen that the evolutionary tracks on the HR diagram for the stars that adopt a different method to treat turbulence show only a very small difference during the MS phase, but they significantly differ from the track that does not include convective mixing in the chemical gradient zone. It can be noticed that the luminosity of the star adopting the Ledoux method is slightly larger than that adopting the Schwarzschild method because, as seen in Figure 4, the overshooting distance of the convective core for the star adopting the Ledoux method is slightly larger than that of the star

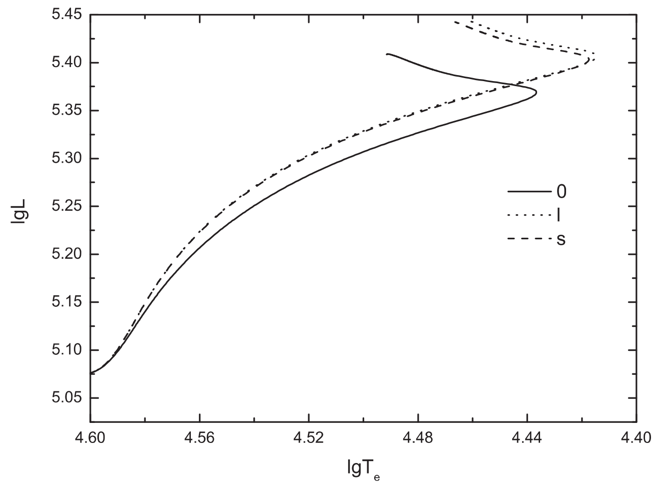

Fig. 6 H-R diagrams are shown with the MS of 30 $M_\odot$ models adopting the Schwarzschild method and the Ledoux method. In this diagram, the value of the mixing length parameter α is 1.0, the turbulent diffusion parameters are 0.05 and the turbulent diffusion coefficient C_x is 5×10^{-9}. There are three curves, whose differences lie in the convective mixing method adopted: the solid line means no convective mixing occurs, the dashed line means the Schwarzschild method is used in convective mixing and the dotted line means the Ledoux method is used in convective mixing.

adopting the Schwarzschild method. From the discussions of Ding & Li (2014), it can be seen that the turbulent velocity is slightly larger around the boundary of the convective core when adopting the Ledoux method than that adopting the Schwarzschild method. As more fuel is transported into the convective core for the star adopting the Ledoux method, its luminosity is slightly larger during the MS phase compared with the star adopting the Schwarzschild method.

The results of Figure 6 show that the difference between the HR diagram for stars either adopting the Ledoux method or the Schwarzschild method is negligible. From a physical perspective, the Ledoux method is more realistic with regard to describing conditions in the convective overshooting and semiconvection than the Schwarzschild method. Therefore, the Ledoux method is adopted in the following analysis.

5.2 HR Diagram with Different Mixing Length Parameter α

Figure 7 shows the HR diagram during the MS phase of the considered stars adopting different values of the mixing length parameter α. It can be seen that increasing the mixing length parameter α can lead to a larger luminosity for the star in the MS phase, which indicates that the influence of the mixing length parameter α on the HR diagram is important. This happens because, as seen in Figure 3, a larger value of α leads to a larger overshooting distance beyond the convective core, which can transport more hydrogen into the nuclear burning core of the star.

5.3 HR Diagram with Different Turbulent Diffusion Coefficient C_x

Figure 8 shows the HR diagram during the MS phase of stars adopting different values of the turbulent diffusion coefficient C_x. It can be seen that, when increasing the turbulent diffusion coefficient C_x, the luminosity of the model in the MS phase is larger. When the turbulent diffusion coefficient

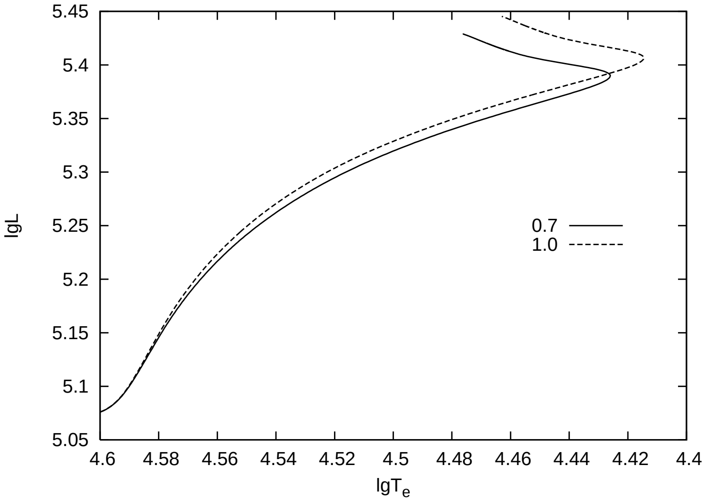

Fig. 7 The main sequence is shown on the H-R diagram of $30\,M_{\odot}$ models with different α. In this diagram, the Ledoux method is adopted to define the convective zone and calculate the turbulent quantities; the turbulent diffusion parameters are 0.05 and the turbulent diffusion coefficient C_x is 5×10^{-9}. There are two curves, whose differences lie in the mixing length parameter α adopted: the solid line means the mixing length parameter α is 0.7, and the dashed line means the mixing length parameter α is 1.0.

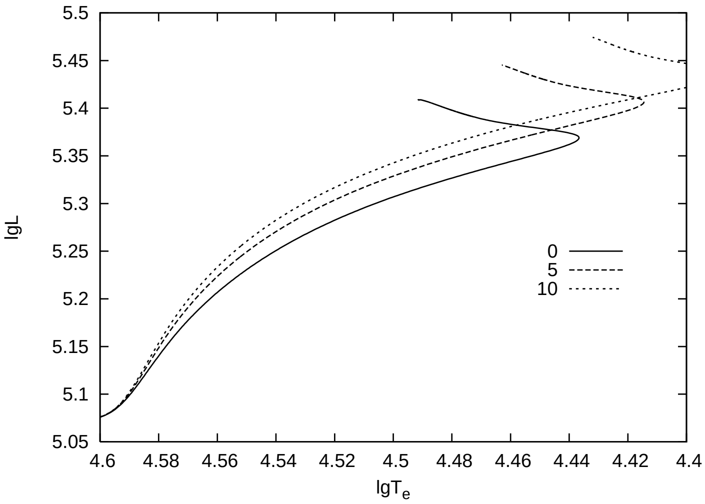

Fig. 8 The main sequence is shown on the H-R diagrams of $30\,M_{\odot}$ models with different C_x. In this diagram, the Ledoux method is adopted to define the convective zone and calculate the turbulent quantities; the value of the mixing length parameter α is 1.0 and the turbulent diffusion parameters are 0.05. There are three curves, whose differences lie in the turbulent diffusion coefficient C_x adopted: the solid line means the turbulent diffusion coefficient C_x is 0.0, the dashed line means the turbulent diffusion coefficient C_x is 5×10^{-9}, and the dotted line means the turbulent diffusion coefficient C_x is 10^{-8}.

C_x increases from 5×10^{-9} to 10^{-8}, the diversity in the luminosity of the models is obvious, which indicates the importance of the influence of the convective mixing parameter on the luminosity of the star in the MS phase.

By comparing Figures 6, 7 and 8, we may find that the influences of the mixing length parameter α and the turbulent diffusion coefficient C_x are important for the luminosity of stars in the MS phase, but the different methods of treating convective overshooting and semiconvection only have a trivial influence on the HR diagram of the stars that are considered.

6 SUMMARY AND FUTURE WORK

In the present paper, we have analyzed the characteristics of core convection in massive stars. The influences of the mixing length parameter α, different convective processing methods, the turbulent diffusion coefficient C_x on the element abundance profiles, element evolutions and H-R diagram have been analyzed. We have summarized the results as follows:

(1) The influence of the mixing length parameter α on the hydrogen profiles is reflected in the shift to the right of the hydrogen abundance profile, which demonstrates the decisive role of α on the overshooting width. This is meaningful in that the turbulent velocities are larger near the core when increasing α, which means that the hydrogen abundance would be smaller at the same mass point. A wider overshooting region means that there will be a wider MS band and larger luminosity in the MS phase.

(2) By adopting the Ledoux method instead of the Schwarzschild method to calculate the turbulent quantities and process convective mixing, the hydrogen abundances are found to be smaller in the inner part of the ∇_μ zone and larger in the outer part. This is meaningful in that the turbulent velocities are larger near the convective core and smaller in the outer part of the ∇_μ zone, which shows the restrictive effect of the Ledoux method on the element mixing and the occurrence of semiconvection. In addition, observations suggest the overshooting distance to be $0.15 \sim 0.2\, H_\mathrm{P}$, which supports the turbulent diffusion coefficient C_x being 5×10^{-9}.

By adopting either the Ledoux method or the Schwarzschild method to calculate the turbulent quantities and model convective mixing, the element evolution profiles have a similar trend. With element mixing in the overshooting region, the element evolutionary profiles are found to have an extended asymmetrical hydrogen profile as evolution proceeds, which is different from results achieved by using mixing length theory. The difference in the results from the Ledoux method and those using the Schwarzschild method is the degree of extension in the asymmetrical hydrogen profile. By adopting the Schwarzschild method, the asymmetrical hydrogen profiles extend further outward, which is consistent with the turbulent quantity profiles in a prior paper (Ding & Li 2014).

(3) By increasing the turbulent diffusion coefficient C_x, the convective core slightly enlarges and the intermediate asymmetrical hydrogen profile extends further outward, which illustrates the influence of C_x on the element profiles. A smooth element abundance profile can influence the range of a confined G mode's transport, which would influence characteristics of the G mode's distribution and the interval of oscillation frequencies, meaning the stellar structure would be affected.

From the above analysis it is found that when the Ledoux method is adopted to calculate turbulent quantities and model convective mixing, 5×10^{-9} is adopted for the turbulent diffusion coefficient C_x and α is 1.0, the core overshooting width is nearly $0.2\, H_\mathrm{P}$. From figures in a prior paper (Ding & Li 2014) it is found that when the parameter values used above are adopted, the semiconvection is nearly restrained until hydrogen is exhausted in the core.

Our results show opposite behavior in terms of changing directions for the degree of core convective overshooting and semiconvection when adopting the Ledoux method. Other than methods used by other authors, the models we adopt have only one free parameter which adjusts the degree of convective overshooting and semiconvection. In theoretical studies, more free parameters return less reliable results. This, to a certain extent, accounts for the validity of the structure of the turbulent convective velocity. These values of turbulent velocities should be important in understanding the process of semiconvection and in future research about convection and evolution of stars.

Acknowledgements We are very grateful for help and the shared experiences from many colleagues (Zhang Qian-Sheng, Lai Xiang-Jun, Su Jie, Wu Tao, Guo Jun-Jun, Chen Yan-Hui et al.). This work is co-sponsored by the National Natural Science Foundation of China (Grant Nos. 11333006 and 10973035) and by the Chinese Academy of Sciences (Grant No. KJCX2-YW-T24).

References

Alexander, D. R., & Ferguson, J. W. 1994, ApJ, 437, 879
Bahcall, J. N., Pinsonneault, M. H., & Wasserburg, G. J. 1995, Reviews of Modern Physics, 67, 781
Chiosi, C., & Summa, C. 1970, Ap&SS, 8, 478
Ding, C. Y., & Li, Y. 2014, MNRAS, 438, 1137
Eggleton, P. P. 1972, MNRAS, 156, 361
El Eid, M. F., The, L.-S., & Meyer, B. S. 2009, Space Sci. Rev., 147, 1
Freytag, B., Ludwig, H.-G., & Steffen, M. 1996, A&A, 313, 497
Harris, M. J., Fowler, W. A., Caughlan, G. R., & Zimmerman, B. A. 1983, ARA&A, 21, 165
Hayashi, C., & Cameron, R. C. 1962, ApJ, 136, 166
Herwig, F., Bloecker, T., Schoenberner, D., & El Eid, M. 1997, A&A, 324, L81
Hirschi, R., Maeder, A., & Meynet, G. 2004, in IAU Symposium 215, Stellar Rotation, eds. A. Maeder, & P. Eenens, 510
Iben, I., Jr. 1966, ApJ, 143, 516
Iglesias, C. A., & Rogers, F. J. 1996, ApJ, 464, 943
Kato, S. 1966, PASJ, 18, 374
Langer, N., Fricke, K. J., & Sugimoto, D. 1983, A&A, 126, 207
Li, Y., & Yang, J. Y. 2007, MNRAS, 375, 388
Mowlavi, N., & Forestini, M. 1994, A&A, 282, 843
Paczyński, B. 1970, Acta Astronomica, 20, 195
Robertson, J. W., & Faulkner, D. J. 1972, ApJ, 171, 309
Rogers, F. J., & Iglesias, C. A. 1995, in Astronomical Society of the Pacific Conference Series, 78, Astrophysical Applications of Powerful New Databases, eds. S. J. Adelman, & W. L. Wiese, 31
Rogers, F. J., Swenson, F. J., & Iglesias, C. A. 1996, ApJ, 456, 902
Schwarzschild, M., & Härm, R. 1958, ApJ, 128, 348
Schwarzschild, M., & Härm, R. 1965, ApJ, 142, 855
Sreenivasan, S. R., & Ziebarth, K. E. 1974, Ap&SS, 30, 57
Stothers, R. 1965, ApJ, 141, 671
Stothers, R. 1966, ApJ, 144, 959
Stothers, R. 1970, MNRAS, 151, 65
Stothers, R., & Chin, C.-W. 1975, ApJ, 198, 407
Woosley, S. E., Heger, A., & Weaver, T. A. 2002, Reviews of Modern Physics, 74, 1015
Zhang, Q. S., & Li, Y. 2012, ApJ, 746, 50