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A study of short-period variation in solar activity *

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Abstract We introduce two methods to detect short-period variation in solar activity. These are called amplitude of low frequency fluctuation (ALFF) and fractional amplitude of low frequency fluctuation (FALFF). We find a positive correlation between short-period variation and 11-year variation of solar activity using these two methods. Through ALFF, we find that solar activity over a short period becomes intensive when the 11-year solar activity is intensive. The ALFF value of the short period activity varies with the peak in sunspot number as a quadratic function. Through FALFF we find that the ratio of short-period spectral intensity to intensity over the whole period of solar activity will increase when the 11-year period of solar activity is intensive. The short-period FALFF value varies with the peak in sunspot number according to a cubic function. Using ALFF, we obtain a yearly series of solar activity that varies over a short period of 1–5 yr from 1860 to 2003, which shows an obvious periodicity of about 22 yr, 33 yr, 11 yr and a century. These short period variations show good correlations with long term variations in solar activity.

Key words: Sun: activity — Sun: sunspots — methods: data analysis

1 INTRODUCTION

Solar activity, a decisive factor in global weather and climate, is closely connected to the survival of human beings (Friis-Christensen & Svensmark 1997; Friis-Christensen & Lassen 1991; Zhao & Han 2005). The variation of sunspot numbers has some obvious periods, such as the decade-scale cycle (Schwabe-Wolf cycle), which has a period of approximately 11 yr, the 22-year magnetic cycle, and a roughly 33-year period (Zhao & Han 2005). There are also some other long periods, such as the Gleisberg cycle, which has a century long timescale (Ochadlick et al. 1993; Zhao & Han 2005). However, solar activity over shorter periods of less than 11 yr (Krivova & Solanki 2002) is also important. In fact, the length of all these periods changes over time. Researchers have exploited many methods to study the sunspot cycle, and have accumulated progressively more data about variations in magnitude and amplitude of the solar cycle through a series of theoretical and observational approaches. There are some classical methods, such as power spectral analysis (Silverman & Shapiro 1983), and autoregressive (AR) spectral analysis (Ulrych & Bishop 1975), etc. A number of new methods have recently been introduced, for instance, wavelet analysis, which is used to study variations in the solar cycle (Han & Han 2002; Krivova & Solanki 2002; Le & Wang 2003; Ochadlick et al. 1993; Frick et al. 1997). At the same time, scientists have paid considerable attention to prediction of the maximum and minimum epochs of sunspot activity (Gholipour et al. 2005; Han 2000; Jiang et al. 2007).

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There has been some research on the short-period variation in solar activity. Blackman and Tukey studied the periodicities of solar activity by the spectral analysis method and found a periodicity of 5.7 yr, with a significance level just below 95% (Blackman & Tukey 1958). Rao removed the dominant 11-year period from the solar data and found more short periodicities with a high level of significance (more than 95%), and also found some relations between short-period solar activity and geophysics and meteorology (Rao 1973). More relations between short-period solar activity and geophysics have been studied (Lockyer & Lockyer 1902; Gnevyshev 1967; Bagiya et al. 2009). In this paper, in order to analyze how a sunspot's short-period amplitude and the proportion of the total amplitude taken up by a sunspot over the whole frequency domain changes over time, we introduce two methods, which are named the amplitude of low frequency fluctuation algorithm (ALFF) (Zang et al. 2007) and the fractional amplitude of low frequency fluctuation algorithm (FALFF) (Zou et al. 2008). We hope to investigate the relationship between the ALFF and FALFF values and short-period variation in solar activity.

Section 2 gives the method of ALFF and FALFF. Section 3 describes the sunspot number data. In Section 4, we give the ALFF and FALFF analysis. We present the discussions and conclusions in Section 5.

2 METHOD

2.1 Amplitude of Low Frequency Fluctuation (ALFF)

The ALFF algorithm is based on Fourier spectral analysis. The difference between the two methods is that the ALFF algorithm calculates the mean amplitude of a frequency domain from f_1 to f_2 as the characteristic value of this signal sequence instead of determining the character of a signal sequence by analyzing the maximum position of frequency. The bigger the characteristic value is, the more spectral intensity of solar activity this frequency domain will include, and the more intense solar activity in the period from $1/f_2$ to $1/f_1$ becomes. The ALFF algorithm is divided into two steps:

(1) From Fast Fourier Transform of the observed time series, we could get the amplitude distribution versus frequency. The definition of amplitude is

$$P(f) = \mathrm{abs}[\mathrm{FFT}(x)]\,, \tag{1}$$

where x is the signal sequence, and $P(f)$ is the amplitude value of the frequency f.

(2) A mean square root is calculated from f_1 to f_2 in the frequency domain

$$\mathrm{ALFF} = \frac{1}{N-1}\mathrm{sqrt}\left[\sum_{f_i=f_1}^{f_2} P^2(f_i)\right], \tag{2}$$

where N is the total number of frequencies from f_1 to f_2. The value returned by ALFF is the mean square root from f_1 to f_2. The bigger ALFF is, the larger the amplitude will be in the frequency domain from f_1 to f_2.

2.2 Fractional Amplitude of Low Frequency Fluctuation (FALFF)

The FALFF algorithm is an improved method based on the ALFF algorithm. It exchanges the mean square root from the frequency domain with the sum of fractional amplitude. The fractional amplitude is defined as

$$B(f_i) = \frac{P(f_i)}{\mathrm{sqrt}\left[\sum_{f_i=f_{\mathrm{start}}}^{f_{\mathrm{end}}} P^2(f_i)\right]}\,. \tag{3}$$

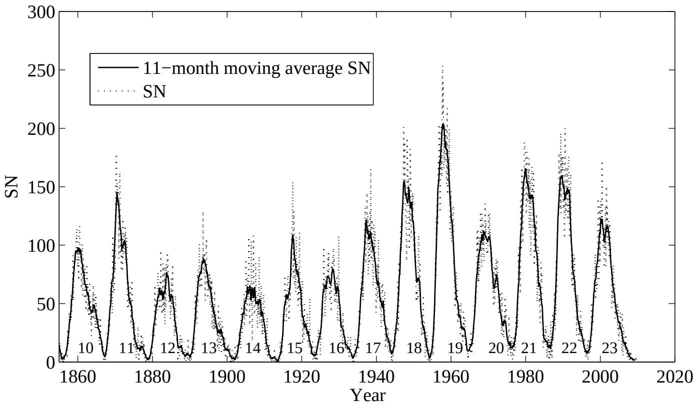

Fig. 1 SN variation, which is based on monthly data from January 1855 to October 2009, from the 10th cycle to the 23rd cycle.

The value of $B(f_i)$ indicates a ratio of the amplitude $P(f_i)$ accounting for the whole frequency domain, and $P(f_i)$ is defined by Equation (1). Then, we define the value returned by FALFF as

$$\mathrm{FALFF} = \mathrm{sqrt}\left[\sum_{f_i=f_1}^{f_2} B^2(f_i)\right]. \tag{4}$$

FALFF returns a fraction of the amplitude that occurs from frequency f_1 to f_2. The bigger FALFF is, the larger the ratio of the amplitude in this $[f_1,\ f_2]$ frequency band is compared to the whole frequency domain.

3 DATA

The relative sunspot numbers (SN) in 1-month intervals are obtained from solar data provided by the National Geophysical Data Center (NGDC), National Oceanic and Atmospheric Administration (NOAA) (*www.ngdc.noaa.gov/stp/solar/ssndata.html*), for the period from January 1855 to October 2009. The record of SN lasting 154 yr, containing 14 cycles, and starting from the 10th cycle to 23rd cycle, is shown in Figure 1, which exhibits prominent 11-year Schwabe cycles. In Figure 1, large gaps can be seen between two peaks of adjacent cycles. After the 11-month moving average is computed, the minimum of the peak is about 56.24 (in the 14th cycle) while the maximum peak is about 203.6 (in the 19th cycle). The range is about 147.36.

4 DATA ANALYSIS

4.1 Data Preprocessing

In order to make the variation in short-period activity more obvious, we subtract the trend for the decade-scale cycle using two steps.

First, in order to determine the starting point of each solar cycle, we compare the position of the minimum between the 11 point moving average series with the observed series, and then divide the series into 14 cycles.

Second, we subtract the trend from the decade-scale cycle. Each solar activity cycle changes asymmetrically. In each cycle, the time required to increase from minima to maxima is shorter than

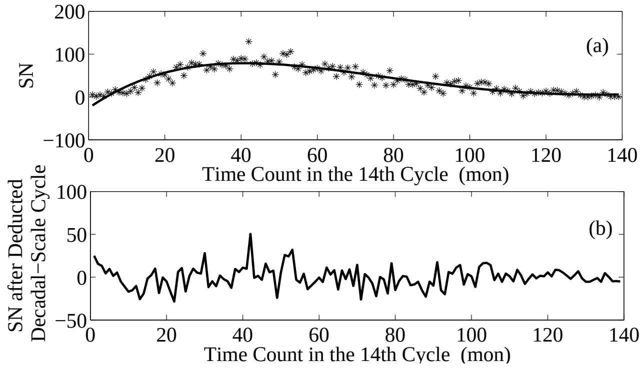

Fig. 2 Comparison between the observed SN and the result after subtracting the trend from the decade-scale cycle in the 14th cycle. (a) Variations in SN in the 14th cycle. The asterisks are the observed SN. The solid line is the quadratic polynomial fitting. (b) Variations in SN after subtracting the trend from the decade-scale cycle in the 14th cycle.

that required to decrease from maximum to minimum, and the maximum of the difference is about 65 mon, with an average of 35.5 mon. Figure 2(a), a scatterplot of monthly SN of the 14th cycle, clearly illustrates the pronounced asymmetry. If we use a sine or cosine function to subtract parts of the trend, we will possibly introduce an error.

In order to subtract the parts of the trend to make the short-period changes more apparent, we apply a quartic, polynomial regression to fit the trend representing the decade-scale cycle in each cycle; See the solid line in Figure 2(a). For example, we can see that the short-period change in the 14th solar activity cycle is much more obvious in Figure 2(b).

4.2 ALFF Analysis

We apply the ALFF algorithm for the 14 cycles of SN after subtracting the trend for the decade-scale cycle. Because we focus on the analysis of the relationship between the amplitudes for a short period of 1 to 5 yr and the variation in intensity for solar activity in the decade-scale cycle, we choose the range of period to be in the frequency domain [1,5] (unit: yr).

We firstly apply a Fast Fourier Transform to each solar cycle to obtain the distribution of amplitude $P(f)$ for each cycle through Equation (1). Then, we calculate the ALFF using Equation (2). We take the 14th solar cycle, for which the trend of the decade-scale cycle had been subtracted (for example) here, and give the $P(1/f)$ ($1/f$ means the period) distribution and illustrate the frequency band for the calculation of ALFF, shown as Figure 3. The frequency band for the calculation with FALFF is the same. Values for all 14 SN cycles using ALFF are calculated, and shown in Table 1.

Table 1 ALFF Values and Peak Values of SN

Solar Cycle	10th	11th	12th	13th	14th	15th	16th
SN Peak	96.950	144.600	75.630	88.650	63.180	108.700	79.520
ALFF	116.728	171.345	96.188	84.946	65.380	148.088	105.793
Solar Cycle	17th	18th	19th	20th	21st	22nd	23rd
SN Peak	121.200	154.800	202.900	111.300	165.300	159.400	122.600
ALFF	153.139	202.449	218.955	128.278	142.404	180.909	112.529

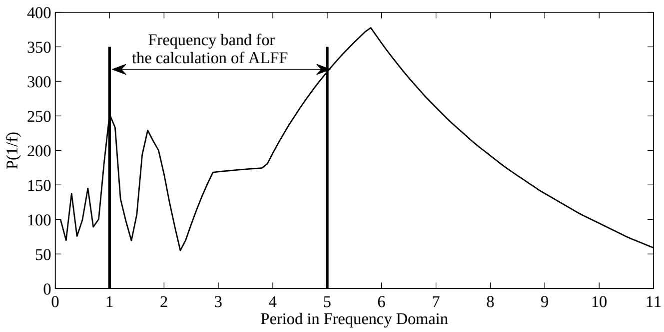

Fig. 3 $P(1/f)$ distribution in the 14th cycle, when the trend in the decade-scale cycle has been subtracted. The frequency band for the calculation of ALFF is marked by the double-headed arrow.

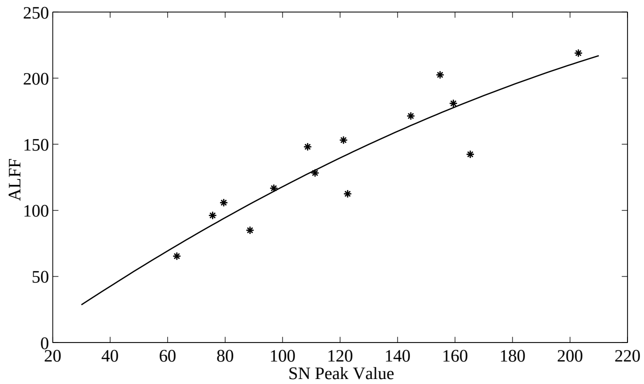

Fig. 4 Correlation between the ALFF and SN peak value. The asterisks are the real ALFF and SN values. The solid line is the quadratic polynomial regression line.

Figure 4 shows the result of fitting curves of ALFF values against SN peaks. The solid line is the regression line. Accordingly, the quadratic polynomial regression equation is

$$\mathrm{ALFF} = -0.0021x^2 + 1.5476x - 16.0104\,, \tag{5}$$

where x is the value of peaks in SN.

4.3 FALFF Analysis

We apply a Fast Fourier Transform to each set of solar cycle data, for which the trend of the decade-scale cycle has been subtracted, and then we obtain the distribution of fractional amplitude $B(f_i)$ through Equation (3). We calculate values with the FALFF method through Equation (4) and see how values from the FALFF method change compared to peak values for SN as shown in Table 2.

Figure 5 gives the scatterplot of FALFF values against SN peaks. Considering the shape of the graph, we try to apply cubic polynomial regression to fit the relationship between FALFF and SN peak,

$$\mathrm{FALFF} = 3.7 \times 10^{-7}x^3 - 1.6 \times 10^{-4}x^2 + 0.0228x - 0.567\,, \tag{6}$$

where x is the peak value of SN.

Table 2 FALFF Values and SN Peak Values

Solar Cycle	10th	11th	12th	13th	14th	15th	16th
SN Peak	96.950	144.600	75.630	88.650	63.180	108.700	79.520
FALFF	0.4525	0.4865	0.4512	0.3941	0.2656	0.4860	0.4751
Solar Cycle	17th	18th	19th	20th	21st	22nd	23rd
SN Peak	121.200	154.800	202.900	111.300	165.300	159.400	122.600
FALFF	0.4561	0.5030	0.4911	0.4810	0.3962	0.4973	0.4123

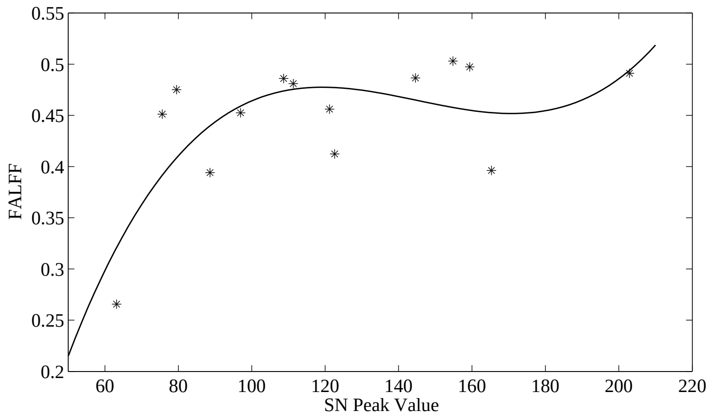

Fig. 5 Relationship between the peak values of FALFF and SN. The asterisks are the real FALFF and SN values, and the solid line is the fitting curve.

4.4 Correlation Analysis

We can see that there is a good correspondence between ALFF values of short periods of 1–5 yr and peak values of SN (Fig. 6(a)). The ALFF-SN correlation coefficient is 0.9021 for the 10th–23rd periods, with significance above the 99% confidence level.

There is also a relationship between the change in peaks of FALFF and SN, shown as Figure 6(b). The correlation coefficient is about 0.4841, with significance above the 90% confidence level.

In other words, when the 11-year solar activity is intensive, solar activity in a period of 1–5 yr also becomes intensive, and the total spectral intensity of solar activity will be concentrated in periods of 1–5 yr.

4.5 Variation of Solar Activity in a Short Period

If we consider each year of the sunspot variations as the center of a time window, five years before and after which the sunspot values form a solar cycle, and calculate the ALFF, we can obtain a series of yearly variations of 1–5 yr ALFF values from 1850 to 2003, which represents variations in the short-period of solar activity, as shown in Figure 7(a).

Figure 7(b) is the Fourier transform of the ALFF variation. We can see a variation in period from Figure 7(b). The short period solar activity over 1–5 yr shows an obvious 22-year magnetic cycle. There are also cycles at roughly 33-year (39.7-year), 11-year (11.35-year) and century timescales which can be seen in variation of the ALFF values.

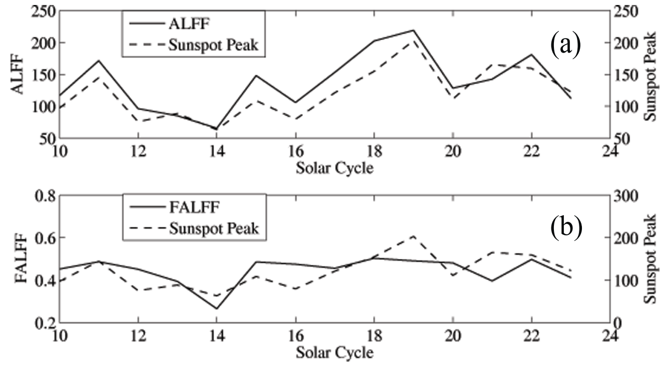

Fig. 6 Variation in ALFF and FALFF compared with the variation in peak values of SN. (a) ALFF (scale on left); (b) FALFF (scale on left). The solid lines are ALFF and FALFF respectively. The dashed lines are the peaks in variation of SN.

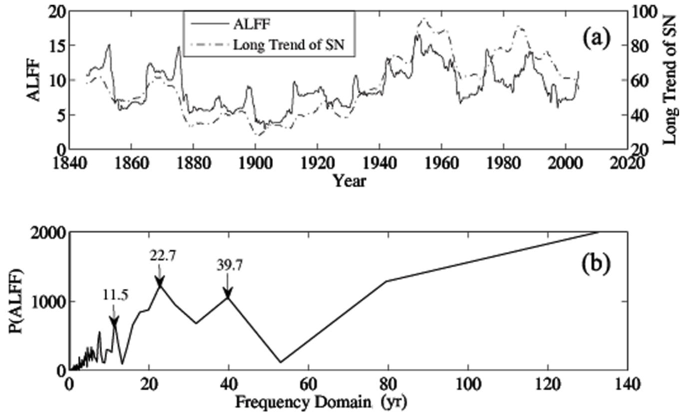

Fig. 7 (a) The 1–5 yr ALFF variation from 1850 to 2003, compared with the long term variation in solar activity. (b) Fourier transform of the variation in ALFF.

Figure 7(a) is a comparison of variation in the 1–5 yr ALFF values and variation in the long term trend of solar activity. We can also see an obvious long term trend that follows variation in ALFF from Figure 7(a). This short-period variation has a good correlation with variation in the long term trend of total solar activity, which is represented by an 11-year moving average series of sunspot number, as analyzed by Zhao et al. (2004). The correlation coefficient is about 0.7759, with significance above the 99.9% confidence level. From this correlation, we can detect that the variation in short-period solar activity may have a good correlation with the long term variation of the total solar activity.

5 DISCUSSION AND CONCLUSIONS

As demonstrated by ALFF and FALFF analysis, by incorporating correlation analysis between ALFF, FALFF and variation in peak SN, the main conclusions can be summarized as follows:

(1) The ALFF of short-period activity of 1–5 yr varies with peaks in SN as a quadratic function. The more intensive solar activity becomes, the larger the amplitude in the short-period cycle of 1–5 yr will be, and vice versa.
(2) The FALFF of short-period activity of 1–5 yr varies with peaks in SN as a cubic function. The more intensive solar activity becomes, the larger the ratios representing spectral intensity in the short-period activity of 1–5 yr will be compared to the whole frequency domain.

We also obtain variation in solar activity over a short period of 1–5 yr, which shows an obvious periodicity and has a good correlation with the long term variation of the total solar activity. There is a close correlation between ALFF and FALFF of the short period activity of 1–5 yr and peaks in SN.

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References

Bagiya, M. S., Joshi, H. P., Iyer, K. N., et al. 2009, in Annales Geophysicae, 27, 1047
Blackman, J. W., & Tukey, J. W. 1958, The Measurement of Power Spectra (New York: Dover Publications Inc)
Frick, P., Galyagin, D., Hoyt, D. V., et al. 1997, A&A, 328, 670
Friis-Christensen, E., & Lassen, K. 1991, Science, 254, 698
Friis-Christensen, E., & Svensmark, H. 1997, Advances in Space Research, 20, 913
Gholipour, A., Lucas, C., Araabi, B. N., & Shafiee, M. 2005, Journal of Atmospheric and Solar-Terrestrial Physics, 67, 595
Gnevyshev, M. N. 1967, Sol. Phys., 1, 107
Han, Y. B. 2000, Chinese Science Bulletin, 45, 1287
Han, Y. B., & Han, Y. G. 2002, Chinese Science Bulletin, 47, 609
Jiang, J., Chatterjee, P., & Choudhuri, A. R. 2007, MNRAS, 381, 1527
Krivova, N. A., & Solanki, S. K. 2002, A&A, 394, 701
Le, G.-M., & Wang, J.-L. 2003, ChJAA (Chin. J. Astron. Astrophys.), 3, 391
Lockyer, N., & Lockyer, W. J. S. 1902, in Proceedings of the Royal Society of London, 70, On Some Phenomena Which Suggest a Short Period of Solar and Meterological Changes, 500
Ochadlick, A. R., Jr., Kritikos, H. N., & Giegengack, R. 1993, Geophys. Res. Lett., 20, 1471
Rao, K. R. 1973, Sol. Phys., 29, 47
Silverman, S. M., & Shapiro, R. 1983, J. Geophys. Res., 88, 6310
Ulrych, T. J., & Bishop, T. N. 1975, Reviews of Geophysics and Space Physics, 13, 183
Zang, Y. F., He, Y., Zhu, C. Z., et al. 2007, Brain and Development, 29, 83
Zhao, J., & Han, Y. B. 2005, Earth, Moon, and Planets, 97, 69
Zhao, J., Han, Y.-B., & Li, Z.-A. 2004, ChJAA (Chin. J. Astron. Astrophys.), 4, 189
Zou, Q. H., Zhu, C. Z., Yang, Y. H., et al. 2008, Journal of Neuroscience Methods, 172, 137