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Preliminary limits on deviation from the inverse-square law of gravity in the solar system: a power-law parameterization *

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Abstract New physics beyond the standard model of particles might cause a deviation from the inverse-square law of gravity. In some theories, it is parameterized by a power-law correction to the Newtonian gravitational force, which might originate from the simultaneous exchange of particles or modified and extended theories of gravity. Using the supplementary advances of the perihelia provided by INPOP10a (IMCCE, France) and EPM2011 (IAA RAS, Russia) ephemerides, we obtain preliminary limits on this correction. In our estimation, we take the Lense-Thirring effect due to the Sun's angular momentum into account. The parameters of the power-law correction and the uncertainty of the Sun's quadrupole moment are simultaneously estimated with the method of minimizing χ^2. From INPOP10a, we find $N = 0.605$ for the exponent of the power-law correction. However, from EPM2011, we find that, although it yields $N = 3.001$, the estimated uncertainty in the Sun's quadrupole moment is much larger than the value given by current observations. This might be caused by the intrinsic nonlinearity in the power-law correction, which makes the estimation very sensitive to the supplementary advances of the perihelia.

Key words: gravitation — ephemerides — celestial mechanics

1 INTRODUCTION

Although gravitation was the first known fundamental force in the Universe, it still cannot be included into a quantum framework, such as the standard model of strong, weak and electromagnetic interactions. It is undoubtedly a grand challenge to unify gravitation with the three others. Some candidate theories of quantum gravity predict there may be some possible deviation from the inverse-square law (ISL) of gravity. Therefore, searching for such deviation experimentally and observationally might shed light on new physics (see Adelberger et al. 2003, for a review).

Historically, the experimental tests of ISL were used to set limits on violations that took the form

$$F(r) = G\frac{m_1 m_2}{r^{2+\epsilon}}. \quad (1)$$

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Here G is the gravitational constant, m_i $(i = 1, 2)$ is the mass of the ith body and r is the distance between them. The parameter ϵ represents the deviation from ISL. Different theoretical scenarios might yield different values of ϵ (e.g. Reissner 1916; Weyl 1917; Nordström 1918; Mostepanenko & Sokolov 1987b; Ferrer & Grifols 1998; Ferrer & Nowakowski 1999; Randall & Sundrum 1999; Dobrescu & Mocioiu 2006; Navarro & van Acoleyen 2005, 2006a,b; Adelberger et al. 2007, 2009). From the perspective of Gauss's Law, the exponent 2 is a purely geometrical effect of three dimensional space, so this parameterization was not well-grounded in theory. Many theoretical models of modified gravity parameterize the deviation using the Newtonian gravitational potential with an additional Yukawa correction (Fischbach et al. 1986, 1992). That is,

$$V(r) = V_{\rm N}(r) + V_{\rm YK}(r), \tag{2}$$

where

$$V_{\rm N}(r) = \frac{Gm_1m_2}{r}, \tag{3}$$

$$V_{\rm YK}(r) = \frac{Gm_1m_2}{r}\alpha \exp\left(-\frac{r}{\lambda}\right). \tag{4}$$

Here α is a dimensionless strength parameter and λ is a length scale (see Fischbach & Talmadge 1999, for a review of constraints on α and λ). Recently, some works have been devoted to astronomical tests of the Yukawa correction (e.g. Iorio 2002, 2007b, 2008b; Deng et al. 2009; Lucchesi & Peron 2010; Lucchesi 2011; Deng & Xie 2013) and Li et al. (2014) found α is at the level of 10^{-11} and λ is about 0.2 astronomical units (au) with the motions of planets in the solar system's planets. With such a parameterization, Xie & Deng (2014) investigated the possibility of detecting a deviation from ISL in exoplanets using transit timing variations and found that these effects are still at least two orders of magnitude below the current capabilities of observation.

Other researchers have considered power-law modifications to the ISL which have the form of (Fischbach et al. 2001)

$$V(r) = V_{\rm N}(r) + V_{\rm PL}(r), \tag{5}$$

where $V_{\rm PL}(r)$ is the power-law correction to the Newtonian potential and it is

$$V_{\rm PL}(r) = \frac{Gm_1m_2}{r}\alpha_N\left(\frac{r_0}{r}\right)^{N-1}. \tag{6}$$

Here α_N is a dimensionless constant, N is the exponent of the power-law and r_0 corresponds to a new length scale associated with a non-Newtonian process. Terms with $N = 2$ and $N = 3$ may be generated by the simultaneous exchange of two massless scalars and two massless pseudoscalar particles, respectively (Feinberg & Sucher 1979; Drell & Huang 1953; Mostepanenko & Sokolov 1987a), while $N = 5$ may be generated by the simultaneous exchange of two massless axions (Ferrer & Grifols 1998) or a massless neutrino-antineutrino pair (Fischbach 1996). There are three trivial cases: (i) when $\alpha_N = 0$, $V_{\rm PL}$ vanishes; (ii) when $N = 0$, $V_{\rm PL}$ is a constant and it will not affect the equations of motion; and (iii) when $N = 1$, the gravitational potential $V(r)$ has the same structure as ISL but with a "new" gravitational constant $G' = G(1 + \alpha_N)$.

Equation (6) can be transformed to the MOdified Newtonian Dynamics (MOND) (Milgrom 1983c,a,b) by taking $N = 1 - 2n$ where $2n$ is the exponent of r in the interpolating function of MOND. MOND suggests that gravitation departs from ISL when dynamical accelerations are small and it can explain the asymptotically flat rotation curves of spiral galaxies and the Tully-Fisher law (for a recent review see Famaey & McGaugh 2012, and references therein). Iorio (2008a) found that the range $1 \leq n \leq 2$ $(-3 \leq N \leq -1)$ is neatly excluded at much more than the 3σ level with the solar system ephemeris EPM2004 (IAA RAS, Russia) (Pitjeva 2005). In recent years, it was also found that power-law corrections generated by modified and extended theories of gravity can simulate astrophysical dark matter (e.g. Capozziello et al. 2004, 2007; Capozziello & Francaviglia 2008). However, Iorio & Ruggiero (2008) showed that, with the parameters determined by the rotation curves of galaxies, the power-law correction is not compatible with the motions of planets in the solar system. When $\alpha_N = -1$ and $N = 1 - \beta$, Equation (6) reduces to the power-law correction, which was investigated by Iorio & Ruggiero (2008).

Inspired by the idea of tests of modified gravity using orbital motions of celestial bodies and artificial objects (e.g. Damour & Esposito-Farèse 1994; Iorio 2002, 2007b, 2008c; Deng et al. 2009; Deng 2011; Iorio 2012c; Iorio & Saridakis 2012; Deng & Xie 2013; Xie & Deng 2013, 2014; Deng & Xie 2014), we will try to find quantitative limits on the power-law correction by making use of the supplementary advances of the perihelia provided by INPOP10a (IMCCE, France) (Fienga et al. 2011) and EPM2011 (IAA RAS, Russia) (Pitjeva 2013) ephemerides. These two ephemerides were recently used in detecting gravitational effects and testing gravitational theories (e.g. Iorio & Saridakis 2012; Iorio 2013b; Xie & Deng 2013; Iorio 2014a,c; Li et al. 2014; Deng & Xie 2014; Liang & Xie 2014). Since INPOP10a and EPM2011 are significantly improved compared with EPM2004, we expect to obtain refined results.

In Section 2, we will calculate advances in the perihelia of planets in the solar system by treating the power-law correction as a small disturbance and then connect them with the data of ephemerides. In Section 3, the supplementary advances of the perihelia provided by INPOP10a and EPM2011 will be used to obtain the limits of their parameters when the Lense-Thirring effect due to the Sun's angular momentum and the uncertainty of the Sun's quadrupole moment are taken into account. Our conclusions and discussion will be presented in Section 4.

2 TWO-BODY PROBLEM WITH A POWER-LAW CORRECTION

We consider a gravitational two-body problem of massive particles with the power-law correction of Equation (6). The effective gravitational potential of this system can be written as

$$\bar{V}(r) = \bar{V}_{\rm N}(r) + \bar{V}_{\rm PL}(r), \tag{7}$$

where

$$\bar{V}_{\rm N}(r) = \frac{\mu}{r}, \tag{8}$$

$$\bar{V}_{\rm PL}(r) = \frac{\mu}{r}\alpha_N\left(\frac{r_0}{r}\right)^{N-1}. \tag{9}$$

Here $\mu \equiv G(m_1 + m_2)$. Such a power-law correction will introduce an *additional* advance of the periastron (Iorio & Ruggiero 2008). In order to investigate the secular evolution of the orbit of a planet in the solar system in the presence of this correction, we need to average $\bar{V}_{\rm PL}$ over one Keplerian period P of the planet, that is

$$\left\langle\bar{V}_{\rm PL}\right\rangle \equiv \frac{1}{P}\int_0^P \bar{V}_{\rm PL}{\rm d}t = \frac{\mu A_{N-1}}{P}\int_0^P r^{-N}{\rm d}t, \tag{10}$$

where $A_{N-1} = \alpha_N r_0^{N-1}$. Its derivative with respect to eccentricity e is

$$\frac{\partial}{\partial e}\left\langle\bar{V}_{\rm PL}\right\rangle = N\frac{\mu A_{N-1}a^{-N}}{2\pi}\int_0^{2\pi}\frac{\cos E - e}{(1 - e\cos E)^{N+1}}{\rm d}E, \tag{11}$$

where a is the semimajor axis and E is the eccentric anomaly. Therefore, the secular precession of the periastron caused by this power-law correction can be obtained as (Danby 1962; Adkins & McDonnell 2007; Iorio 2007a, 2012b,e)

$$\dot{\omega}_{\rm PL} = \frac{\sqrt{1-e^2}}{na^2e}\frac{\partial}{\partial e}\left\langle\bar{V}_{\rm PL}\right\rangle = \mu^{1/2}A_{N-1}Na^{-N-1/2}f(e,N), \tag{12}$$

where

$$f(e,N) \equiv \frac{\sqrt{1-e^2}}{2\pi e}\int_0^{2\pi}\frac{\cos E - e}{(1-e\cos E)^{N+1}}\mathrm{d}E. \tag{13}$$

This result can return the one given by Iorio & Ruggiero (2008) when $\alpha_N = -1$ and $N = 1-\beta$. In the case of planets in the solar system, it is closely connected to the supplementary advances of the perihelia $\dot{\omega}_{\rm sup}$ provided by modern ephemerides, such as INPOP10a (Fienga et al. 2010, 2011) and EPM2011 (Pitjeva 2013; Pitjeva & Pitjev 2013; Pitjev & Pitjeva 2013).

INPOP10a and EPM2011 were obtained by fitting the "standard model" of dynamics to observational data, where "standard model" means Newton's law of gravity and Einstein's general relativity (GR) (apart from the Lense-Thirring effect, see below for details). Therefore, the effects of the power-law correction were neither modeled in INPOP10a nor in EPM2011, and the parameters α_N, r_0 and N were not determined in these least-square fittings. In this sense, the results we obtain in the next section may not be considered as genuine "constraints" (they would be so if one solved for them in a covariance analysis by reanalyzing the data with modified software including these effects) but rather as preliminary indications of acceptable values to the best of the contemporary knowledge in the field of ephemerides, so that we call them "preliminary limits" (see Iorio 2014a, for a further discussion).

These $\dot{\omega}_{\rm sup}$ might represent possibly mismodeled or unmodeled parts of perihelion advances according to Newton's law and GR. They are almost all compatible with zero, so that they can be used to draw bounds on quantities parametrizing unmodeled "forces," like the power-law correction in this case. Nonetheless, the latest results by EPM2011 (Pitjeva & Pitjev 2013; Pitjev & Pitjeva 2013) returned non-zero values for Venus and Jupiter. Although the level of their statistical significance was not too high and further investigations are required, we still take them into account in this work. In the recent past, an extra non-zero effect on Saturn's perihelion was studied (Iorio 2009b). The ratios of the non-zero values of the supplementary precessions of Venus and Jupiter by EPM2011 (Pitjeva & Pitjev 2013; Pitjev & Pitjeva 2013) have recently been used to test a potential deviation from GR (Iorio 2014c).

In the construction of $\dot{\omega}_{\rm sup}$ (see Fienga et al. 2010, for details), the effects caused by the Sun's quadrupole mass moment $J_2^{\odot}$ are considered and isolated in the final results, but the perihelion shifts caused by the Lense-Thirring effect (Lense & Thirring 1918) due to the Sun's angular momentum $S_{\odot}$ are absent. Therefore, the entire relation between $\dot{\omega}_{\rm PL}$ and $\dot{\omega}_{\rm sup}$ is

$$\dot{\omega}_{\rm sup} = \dot{\omega}_{\rm PL} + \dot{\omega}_{\rm LT} + \dot{\omega}_{\delta J_2^{\odot}}. \tag{14}$$

Here, the Lense-Thirring term $\dot{\omega}_{\rm LT}$ is (Lense & Thirring 1918; Iorio 2001, 2009a; Renzetti 2013)

$$\dot{\omega}_{\rm LT} = -\frac{6GS_{\odot}\cos i}{c^2 a^3 (1-e^2)^{3/2}}, \tag{15}$$

where c is the speed of light, $S_{\odot} = 1.9\times10^{41}$ kg m^2 s^{-1} (Pijpers 2003) and i is the inclination of the planetary orbit with respect to the equator of the Sun. The uncertainty of $S_{\odot}$ is currently about 1% (Pijpers 2003). This effect of the Sun on planetary motions has been studied in several works (e.g. Iorio 2005b; Iorio et al. 2011; Iorio 2012a). Equation (15) only holds in a coordinate system whose z axis is aligned with the Sun's angular momentum. A general formula for an arbitrary orientation can be found in Iorio (2011, 2012d). This is useful in extrasolar planets and black holes, for which the orientation of the spin axis is generally unknown.

We add the third term in Equation (14) to include the uncertainty of the Sun's quadrupole moment $\delta J_2^{\odot}$ (Iorio 2005a), which is currently about $\pm10\%$ of $J_2^{\odot}$ (Damiani et al. 2011; Pireaux & Rozelot 2003; Rozelot et al. 2004; Rozelot & Damiani 2011; Rozelot & Fazel 2013). The Sun's quadrupole moment in INPOP10a is fitted to observations with $J_2^{\odot} = (2.40\pm0.25)\times10^{-7}$ (Fienga et al. 2011) and its value in EPM2011 is $J_2^{\odot} = (2.0 \pm 0.2) \times 10^{-7}$ (Pitjeva 2013). This uncertainty in $J_2^{\odot}$ can cause an extra precession for a planet, which is (Kozai 1959)

$$\dot{\omega}_{\delta J_2^{\odot}} = \frac{3}{2}\frac{\delta J_2^{\odot} R_{\odot}^2}{p^2} n\left(2 - \frac{5}{2}\sin^2 i\right), \tag{16}$$

where n is the Keplerian mean motion, $R_{\odot}$ is the Sun's radius and $p = a(1-e)$. The higher order multipoles , such as $J_4^{\odot}$, have a negligible impact on the perihelion precessions (see Renzetti 2013, for a recent calculation of the $J_4^{\odot}$ precessions). There are also post-Newtonian GR effects driven by $J_2^{\odot}$ (Iorio 2013a, 2014b). While they may have an impact in other systems, such as close extrasolar planets with highly eccentric orbits, they can be left aside in the present case (i.e. our Sun and its planets).

The effect of the cosmological constant Λ, which should be considered as somewhat "standard" in GR in view of the observed acceleration of the Universe (e.g. Riess et al. 1998; Perlmutter et al. 1999), has not been included in INPOP10a and EPM2011, so it should also appear in Equation (14). Its effects on the perihelion of planets were studied (e.g. Iorio 2008c; Arakida 2013; Liang & Xie 2014). However, Λ can be left out from the analysis of the present work since it mainly affects the outer planets but not the inner planets.

3 PRELIMINARY LIMITS ON PARAMETERS OF THE POWER-LAW CORRECTION

The INPOP10a (Fienga et al. 2011) ephemeris provides $\dot{\omega}_{\rm sup}$ for some planets in the solar system: Mercury, Venus, Earth-Moon Barycenter (EMB), Mars, Jupiter and Saturn. Similarly, EPM2011 (Pitjeva 2013) also gives those values for the planets from Mercury to Saturn. These numbers are taken from table 5 in Fienga et al. (2011) and tables 4 and 5 in Pitjeva & Pitjev (2013) and Pitjev & Pitjeva (2013) respectively (see Table 1 for details). It can be found that $\dot{\omega}_{\rm sup}$ of Mercury and Venus from EPM2011 are considerably larger than those of INPOP10a, while Venus and Jupiter have non-zero values of $\dot{\omega}_{\rm sup}$ in EPM2011.

In order to apply the method of minimizing χ^2 for estimating the parameters in a more convenient way, we rewrite $\dot{\omega}_{\rm PL}$ for a planet in the solar system as

$$\dot{\omega}_{\rm PL} = C_{\odot} A F(a, e, N), \tag{17}$$

where

$$C_{\odot} = \mu_{\odot}^{1/2} {\rm au}^{-3/2}, \tag{18}$$

$$A = A_{N-1} {\rm au}^{-N+1}, \tag{19}$$

$$F(a, e, N) = N \bar{a}^{-N-1/2} f(e, N). \tag{20}$$

Here $\bar{a} \equiv a/{\rm au}$. With the notation $X = \delta J_2^{\odot}/J_2^{\odot}$, we can also rewrite Equation (14) as

$$\dot{\omega}_{\rm sup} = C_{\odot} A F(a, e, N) - \frac{6GS_{\odot}\cos i}{c^2 a^3 (1-e^2)^{3/2}} + \frac{3}{2} X \frac{J_2^{\odot} R_{\odot}^2}{p^2} n\left(2 - \frac{5}{2}\sin^2 i\right). \tag{21}$$

This can be used to construct χ^2 as

$$\chi^2 = \sum_j \frac{C_{\odot}^2}{\sigma_j^2}\left[AF(a_j, e_j, N) + P_j X - Q_j\right]^2, \tag{22}$$

where

$$P_j = \frac{3}{2}\frac{J_2^{\odot} R_{\odot}^2}{C_{\odot} p_j^2} n_j \left(2 - \frac{5}{2}\sin^2 i_j\right), \tag{23}$$

$$Q_j = \frac{6GS_{\odot}\cos i_j}{c^2 C_{\odot} a_j^3 (1-e_j^2)^{3/2}} + C_{\odot}^{-1}\dot{\omega}_j^{\rm sup}. \tag{24}$$

Table 1 Supplementary Advances of the Perihelia $\dot{\omega}_{\rm sup}$ Given by INPOP10a and EPM2011

	$\dot{\omega}_{\rm sup}$ (mas cy^{-1})	
	INPOP10a [a]	EPM2011 [b]
Mercury	0.4 ± 0.6	-2.0 ± 3.0
Venus	0.2 ± 1.5	2.6 ± 1.6
EMB	-0.2 ± 0.9	–
Earth	–	0.19 ± 0.19
Mars	-0.04 ± 0.15	-0.020 ± 0.037
Jupiter	-41 ± 42	58.7 ± 28.3
Saturn	0.15 ± 0.65	-0.32 ± 0.47

[a] Taken from table 5 in Fienga et al. (2011). [b] Provided by table 4 in Pitjeva & Pitjev (2013) and table 5 in Pitjev & Pitjeva (2013).

Here j denotes each planet in Table 1. It can be easily checked that A, N and X are all dimensionless. They can relate the parameter ϵ in Equation (1) with

$$\epsilon = -AN\bar{r}^{-N+1}[\ln(r)]^{-1} + \mathcal{O}(\alpha_N^2)\,,$$

where $\bar{r} \equiv a/\mathrm{au}$.

For estimating the values of A, N and X, we need to solve the equations of $\partial\chi^2/\partial A = 0$, $\partial\chi^2/\partial X = 0$ and $\partial\chi^2/\partial N = 0$. Equation (22) shows the linear dependence of A and X and the nonlinear dependence of N. Therefore, we can *analytically* solve for the expressions of A and X according to the equations $\partial\chi^2/\partial A = 0$ and $\partial\chi^2/\partial X = 0$ first and then substitute them into $\partial\chi^2/\partial N = 0$ to *numerically* solve for N. From INPOP10a, we find $N = 0.605$, $A = 1.88 \times 10^{-12}$ and $X = 5.77\%$ (see Appendix A for details). This result is consistent with and refines the constraint obtained by Iorio (2008a).

However, from EPM2011, we find $N = 3.001$, $A = 4.746 \times 10^{-11}$ and $X = -1100\%$. This estimated X is three orders of magnitude larger than the limit of $\pm 10\%$ set by current observations (Damiani et al. 2011; Pireaux & Rozelot 2003; Rozelot et al. 2004; Rozelot & Damiani 2011; Rozelot & Fazel 2013) (see Appendix A for details). This might be explained by the nonlinear dependence of N in Equation (22). This nonlinearity makes the method of minimizing χ^2 very sensitive to the supplementary advances of the perihelia so that differences between those values of INPOP10a and EPM2011 can make them return radically different results.

4 CONCLUSIONS AND DISCUSSION

Using the supplementary advances of the perihelia provided by INPOP10a (Fienga et al. 2011) and EPM2011 (Pitjeva 2013) ephemerides, we estimate preliminary limits on the deviation from the ISL of gravity, which is parameterized by a power-law correction to the Newtonian gravitational force. Taking the uncertainty in the Sun's quadrupole moment into account and estimating it along with the parameters of the power-law correction, we find $N = 0.605$ for the exponent of the correction from INPOP10a with the method of minimizing χ^2. However, from EPM2011, we find that, although it yields $N = 3.001$, the estimated uncertainty in the Sun's quadrupole moment is much larger than the value of $\pm 10\%$ given by current observations (Damiani et al. 2011; Pireaux & Rozelot 2003; Rozelot et al. 2004; Rozelot & Damiani 2011; Rozelot & Fazel 2013). This might be explained by its intrinsic nonlinearity in the power-law correction.

With tremendous advances in techniques for deep space exploration in the solar system, ephemerides are going to be increasingly improved by high-precision datasets provided from tracking spacecraft and by sophisticated data analysis methodology (e.g. Fienga et al. 2013; Verma et al.

2013, 2014). The resulting limits on deviation of gravity from the ISL are expected to be better in the future.

It will also be necessary and important to do a similar analysis for these deviations with other local systems by using proper observable quantities (e.g. radial velocities, timing, eclipsing times and so on). These local systems can be extrasolar planets (e.g. Xie & Deng 2014), some wide compact binaries that host neutron stars and/or white dwarfs, or other binary systems, such as α Centauri AB (e.g. Iorio 2013b).

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Appendix A: MINIMIZING χ^2

Based on Equation (22) and using $\partial\chi^2/\partial A = 0$, we can find

$$\left[\sum_j \frac{1}{\sigma_j^2} F_j^2(N)\right] A + \left[\sum_j \frac{1}{\sigma_j^2} P_j F_j(N)\right] X = \sum_j \frac{1}{\sigma_j^2} F_j(N) Q_j. \tag{A.1}$$

From $\partial\chi^2/\partial X = 0$, we can have

$$\left[\sum_j \frac{1}{\sigma_j^2} P_j F_j(N)\right] A + \left[\sum_j \frac{1}{\sigma_j^2} P_j^2\right] X = \sum_j \frac{1}{\sigma_j^2} P_j Q_j, \tag{A.2}$$

where $F_j(N) \equiv F(a_j, e_j, N)$. With the above two equations, we can solve for A and X as

$$A \;=\; D^{-1} \sum_{j,k} \frac{1}{\sigma_j^2 \sigma_k^2} \left[P_j Q_j P_k F_k(N) - P_j^2 Q_k F_k(N)\right], \tag{A.3}$$

$$X \;=\; D^{-1} \sum_{j,k} \frac{1}{\sigma_j^2 \sigma_k^2} \left[P_j Q_k F_j(N) F_k(N) - P_j Q_j F_k^2(N)\right], \tag{A.4}$$

where

$$D = \sum_{j,k} \frac{1}{\sigma_j^2 \sigma_k^2} \left[P_j P_k F_j(N) F_k(N) - P_j^2 F_k^2(N)\right]. \tag{A.5}$$

Substituting Equations (A.3) and (A.4) into $\partial\chi^2/\partial N = 0$, which is

$$\left[\sum_j \frac{1}{\sigma_j^2} F_j(N) F_j'(N)\right] A + \left[\sum_j \frac{1}{\sigma_j^2} P_j F_j'(N)\right] X = \sum_j \frac{1}{\sigma_j^2} F_j'(N) Q_j, \tag{A.6}$$

we can obtain

$$\begin{aligned} h(N) \;\equiv\; & \sum_{l,j,k} \frac{1}{\sigma_l^2 \sigma_j^2 \sigma_k^2} F_l(N) F_l'(N) \left[P_j Q_j P_k F_k(N) - P_j^2 Q_k F_k(N)\right] \\ & + \sum_{l,j,k} \frac{1}{\sigma_l^2 \sigma_j^2 \sigma_k^2} P_l F_l'(N) \left[P_j Q_k F_j(N) F_k(N) - P_j Q_j F_k^2(N)\right] \\ & - \sum_{l,j,k} \frac{1}{\sigma_l^2 \sigma_j^2 \sigma_k^2} Q_l F_l'(N) \left[P_j P_k F_j(N) F_k(N) - P_j^2 F_k^2(N)\right] \\ =\; & 0, \end{aligned} \tag{A.7}$$

Table A.1 Summary of the Numerical Solutions of N, A and X

Ephemeris	N [a]	A (10^{-13})	X (%)	χ^2
INPOP10a	0.605	18.77	5.77	1.070
	2.830	−5.231	13.00	1.079
	3.005	−459.1	901.2	1.032
	3.186	2.416	−0.899	1.082
EPM2011	3.001	474.6	−1100	8.084

[a] N is numerically solved by Equation (A.7) in the domain $N \in [-10, 10]$.

where $F'_j(N) = \partial F_j(N)/\partial N$. The roots of Equation (A.7) are numerically found in the domain $N \in [-10, 10]$ by the method of bisection (Press et al. 1992). Their values based on INPOP10a and EPM2011 are listed in Table A.1. Here, as we discussed in Section 1, we discard two trivial cases: $N = 0$ and $N = 1$. Although $N = 3.005$ can give the minimal χ^2 based on INPOP10a, its resulting value of X is not physically reasonable. Therefore, the best values given by INPOP10a are $N = 0.605$, $A = 1.88 \times 10^{-12}$ and $X = 5.77\%$. However, from EPM2011, we find $N = 3.001$, $A = 4.746 \times 10^{-11}$ and $X = -1100\%$. This estimated X is three orders of magnitude larger than $\pm 10\%$ set by current observations (Damiani et al. 2011; Pireaux & Rozelot 2003; Rozelot et al. 2004; Rozelot & Damiani 2011; Rozelot & Fazel 2013).

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