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A model of geometric delay in Space VLBI

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Abstract A model which includes the relativistic effect is derived that can be applied to space very long baseline interferometry (SVLBI) while taking observations of sources at infinite distance. In SVLBI, where one station is on a spacecraft, the length of the baseline and the orbiting station's maximum speed in an elliptical orbit around the Earth is much larger than the ground-based VLBI , which leads to a larger delay and higher delay rate. The delay models inside VLBI correlators are usually expressed as fifth-order polynomials during a limited time interval, which are evaluated by firmware in the correlator and track delays in the interferometer over the limited time interval. The higher SVLBI delay rate requires more accurate polynomial fitting and evaluation, as well as more frequent model updates.

Key words: astrometry — interferometric — analytical

1 INTRODUCTION

The technique of very long baseline interferometry (VLBI) is widely used in radio astronomical observations. In general, the angular resolution of VLBI is determined by the observing frequency and the length of the baseline. Limited by the size of the Earth, the angular resolution of ground VLBI (GVLBI) is not good enough for many compact sources. One technique to improve the resolution at a fixed frequency is to place a radio telescope in space, preferably in an elliptical orbit around the Earth. A radio telescope that is part of the satellite then observes in conjunction with ground-based radio telescopes, synthesizing an aperture whose effective resolution is that of a radio telescope which is much larger than the Earth. This technique is called space VLBI (SVLBI). SVLBI projects have been proposed or planned by several agencies, including Quasat, by the European Space Agency and NASA in 1980s (Schilizzi et al. 1984; Schilizzi 1988); VSOP, by the Institute for Space and Astronautical Science in Japan (Hirabayashi 1988), which was successfully launched on 1997 February 12 (Hirabayashi et al. 1997a,b); and, RADIOASTRON, by the Space Research Institute of the USSR Academy of Sciences (Kardashev & Slysh 1988), which was launched on 2011 July 18 (Alexandrov et al. 2012). The Chinese Space VLBI project (hereafter C-SVLBI), which was recently proposed by Shanghai Astronomical Observatory and National Space Science Center of China, plans to launch two antennas into Earth orbit as a first step (Hong et al. 2013; Shen et al. 2013).

The delay model is a representation of the apparent delay in the wavefront received at a radio telescope, which refers to its arrival time at the other telescope. Correlation in VLBI requires an accurate delay model for each instant of an observation. An accurate delay model corresponds to small residual delays and delay rates in correlation, which enable long coherent integrations in fringe fitting and translate into a capability for detecting fringes from weaker sources.

In GVLBI, the telescope is constructed on the Earth's surface, so its acceleration is about 34 mm s^{-2}. Considering that the maximum delay is about 21 ms, the effect of acceleration on the baseline in the delay interval is about 0.006 mm, which can be neglected in the GVLBI delay model (Petit & Luzum 2010). For SVLBI, the acceleration and the maximum delay are much larger. For example, if the spacecraft has an apogee height of 90 000 km and a perigee height of 1 000 km, then the delay is approximately 0.3 s and the maximum acceleration will be 8 m s^{-2}, yielding a difference of 0.36 m in the baseline of the delay interval. Thus, the effect of the spacecraft's acceleration should be included in the delay model for SVLBI.

Solutions for the delay and delay rate inside a VLBI correlator are produced at two-minute intervals (Benson 1995). A series of ten consecutive solutions are windowed into a quintic spline fitting algorithm and the resulting fifth-order polynomials are evaluated by firmware in the correlator and track the interferometer delays over two minute intervals that are used by the model (Wells et al. 1989; Benson 1995). For SVLBI, the delay rate is more difficult to address. The spacecraft's maximum speed in an elliptical orbit around the Earth is about 10 km s^{-1}, yielding a delay rate of 33 μs s^{-1}, which is much larger than the delay rate of GVLBI. With an acceleration close to 8 m s^{-2}, the SVLBI delay rate may require more accurate polynomial fitting or more frequent model updates to meet a certain accuracy.

In Section 2, a delay model including the effect of the spacecraft's acceleration for SVLBI is derived. Section 3 gives the simulation of the SVLBI model and the modification should be made when the model is applied in the SVLBI correlator. Section 4 gives the conclusion.

2 DELAY MODEL FOR SVLBI

2.1 Coordinate System Transformation

A reference system should be chosen so that the physical process under study can be described as simply as possible. Considering the effect of Earth's revolution in the solar system and the gravitational effect on signal propagation, the delay model should be described in the Barycentric Celestial Reference System (BCRS), whose time-coordinate is called TCB. Meanwhile, the positions and velocities of the VLBI stations (both the ground-based VLBI stations and the orbiting SVLBI stations) are given in the Geocentric Celestial Reference System (GCRS), whose time-coordinate is called TCG.

The coordinate transformation between BCRS and GCRS given by the XXIV IAU General Assembly is written as (Soffel et al. 2003):

$$t = \int \left[1 - \frac{U_{\rm ext}(\boldsymbol{X}_{\rm E})}{c^2} - \frac{{V_{\rm E}}^2}{2c^2} \right] dT - \frac{\boldsymbol{V}_{\rm E} \cdot \boldsymbol{X}}{c^2} + O(c^{-4})\,, \tag{1}$$

$$\boldsymbol{x} = \boldsymbol{R}_{\rm E} + \frac{1}{c^2} \left[U_{\rm ext} \boldsymbol{R}_{\rm E} + \frac{1}{2} (\boldsymbol{V}_{\rm E} \cdot \boldsymbol{R}_{\rm E}) \boldsymbol{V}_{\rm E} + (\boldsymbol{A}_{\rm E} \cdot \boldsymbol{R}_{\rm E}) \boldsymbol{R}_{\rm E} - \frac{1}{2} {R_{\rm E}}^2 \boldsymbol{A}_{\rm E} \right] + O(c^{-4})\,, \tag{2}$$

where $U_{\rm ext}(\boldsymbol{X}_{\rm E}) = \sum_{P \neq E} \frac{GM_P}{|\boldsymbol{X} - \boldsymbol{X_P}|}$ is the gravitational potential at the geocenter, neglecting the effects of Earth's mass. At a picosecond level, only the solar potential needs to be included (Petit & Luzum 2010). $\boldsymbol{V}_{\rm E}$ is the barycentric velocity of the geocenter. $\boldsymbol{X}_{\rm E}$ is the barycentric radius vector of the geocenter, $\boldsymbol{X}$ is the station's barycentric position vector, while $\boldsymbol{x}$ is the GCRS position vector. T refers to the barycentric coordinate time TCB, and t refers to TCG. c is the speed of light in a vacuum. $\boldsymbol{A}_{\rm E}$ is the acceleration of the geocenter and $\boldsymbol{R}_{\rm E} = \boldsymbol{X} - \boldsymbol{X}_{\rm E}$.

In particular, we assume that the proper time of the clock in the orbiting station has been transferred to TCG. The transformation of the proper time of the satellite clock to TCG will not be discussed in this paper; readers can refer to IERS Convention 2010, Chapter 10 for more details.

2.2 SVLBI Delay Model

In the barycentric frame, the delay equation is, to a sufficient level of approximation,

$$\Delta T = T_2 - T_1 = -\frac{\boldsymbol{K}}{c}\Big[\boldsymbol{X}_2(T_2) - \boldsymbol{X}_1(T_1)\Big] + \Delta T_{\rm grav}\,. \tag{3}$$

Here $\boldsymbol{K}$ is the unit vector from the barycenter to the source in the absence of gravitational or the aberrational bending; $\boldsymbol{X}_i$ is the barycentric radius vector of the $i^{\rm th}$ receiver at the TCB time T_i. $\Delta T_{\rm grav}$ is the general relativistic delay. The relationship between the receiving stations is shown in Figure 1.

As shown in Figure 1, the position vector of the orbiting station in BCRS can be approximated with coordinate velocity and acceleration as

$$\boldsymbol{X}_2(T_2) = \boldsymbol{X}_2(T_1) + \boldsymbol{V}_{\rm E}(T_2 - T_1) + \boldsymbol{V}_2(T_2 - T_1) + \tfrac{1}{2}\boldsymbol{a}_2(T_2 - T_1)^2\,. \tag{4}$$

where $\boldsymbol{V}_2$ is the orbital velocity of the space station, and $\boldsymbol{a}_2$ is its orbital acceleration. With Equation (4), Equation (3) can be rewritten as

$$\frac{\boldsymbol{K}}{2c}\boldsymbol{a}_2(T_2 - T_1)^2 + \left[1 + \frac{\boldsymbol{K}}{c}(\boldsymbol{V}_{\rm E} + \boldsymbol{V}_2)\right](T_2 - T_1) + \frac{\boldsymbol{K}}{c}\boldsymbol{B}_0 - \Delta T_{\rm grav} = 0\,, \tag{5}$$

with $\boldsymbol{B}_0 = \boldsymbol{X}_2(T_1) - \boldsymbol{X}_1(T_1)$. Equation (5) is a quadratic equation. According to Halley's method (Danby 1988), when a quadratic equation has the following form

$$\frac{1}{2}Ax^2 + Bx + C = 0\,, \tag{6}$$

its approximate solution is obtained by

$$x = -\frac{C}{B\left[1 - \frac{CA}{2B^2}\right]}\,. \tag{7}$$

This approximation gives third-order convergence when iterations are used to solve a quadratic equation (Sekido & Fukushima 2006). This is quite effective, especially when $A \cdot C \ll B^2$, as is the case here.

The solution of Equation (5) is obtained using Halley's method as

$$T_2 - T_1 = \frac{-\frac{\boldsymbol{K}}{c}\cdot\boldsymbol{B}_0 + \Delta T_{\rm grav}}{\left[1 + \frac{\boldsymbol{K}}{c}\cdot(\boldsymbol{V}_{\rm E} + \boldsymbol{V}_2)\right]\left\{1 - \frac{(\boldsymbol{K}\cdot\boldsymbol{a}_2)(\boldsymbol{K}\cdot\boldsymbol{B}_0)}{2c^2\left[1 + \frac{\boldsymbol{K}}{c}\cdot(\boldsymbol{V}_{\rm E} + \boldsymbol{V}_2)\right]^2}\right\}}\,. \tag{8}$$

The error that arises from approximation when Halley's method is applied is given by

$$\delta x \approx \frac{C^3\cdot A^2}{4B^5} = \frac{\left(\frac{\boldsymbol{K}}{c}\cdot\boldsymbol{B}_0\right)^3\left(\frac{\boldsymbol{K}}{c}\cdot\boldsymbol{a}_2\right)^2}{4\left[1 + \frac{\boldsymbol{K}}{c}\cdot(\boldsymbol{V}_{\rm E} + \boldsymbol{V}_2)\right]^5}\,. \tag{9}$$

The order of δx is $\left(\frac{\boldsymbol{K}}{c}\cdot\boldsymbol{a}_2\right)^2 \approx 10^{-16}$ s, which can be neglected.

When observations are taken with SVLBI, the time interval between the arrival of the signal at the two antennas is on the order of 0.1 s, so the coordinate transformation between BCRS and

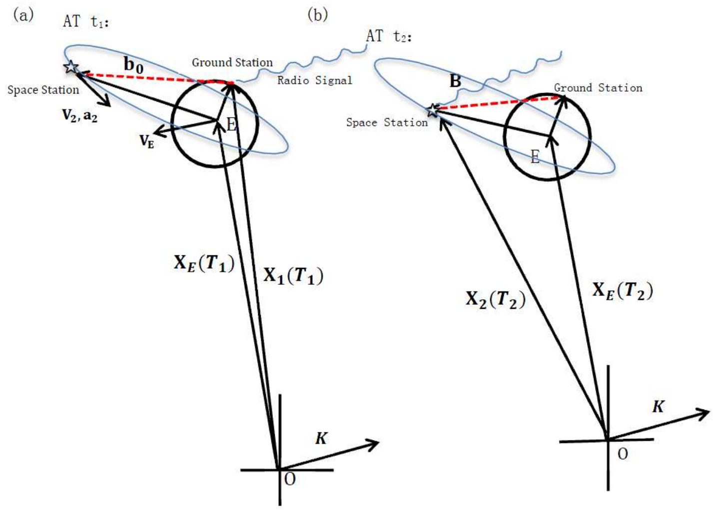

Fig. 1 Geometry of the receiving stations in SVLBI. (a) The orientation of the signal at the ground station. (b) The orbital movement of the antenna that is part of the satellite at the time the signal is received. The geocenter of the Earth has moved.

GCRS can be approximated up to the order of $(\frac{V_{\rm E}}{c})^2$ for 1 picosecond accuracy, where the coordinate velocity of the Earth and the external gravitational potential in the interval defined by Equation (1) and Equation (2) can be treated as constant

$$T_2 - T_1 = \left(1 + \frac{U_{\rm ext}}{c^2} + \frac{{V_{\rm E}}^2}{2c^2}\right)(t_2 - t_1) + \frac{\boldsymbol{V}_{\rm E}}{c^2}\cdot\Big[\boldsymbol{b}_0 + \boldsymbol{V}_2(t_2 - t_1)\Big]\,, \tag{10}$$

$$\boldsymbol{B}_0 = \boldsymbol{b}_0 - \frac{\boldsymbol{b}_0}{c^2}U_{\rm ext} + \frac{\boldsymbol{b}_0\cdot\boldsymbol{V}_{\rm E}}{2c^2}\boldsymbol{V}_{\rm E}\,. \tag{11}$$

Substituting Equation (10) and Equation (11) into Equation (8), the SVLBI delay model can be written as

$$\begin{aligned}\Delta t \;&=\; t_2 - t_1\\ &= \left\{-\frac{\boldsymbol{K}\cdot\boldsymbol{b}_0}{c}\left(1 - \frac{2U_{\rm ext}}{c^2} - \frac{\boldsymbol{V}_{\rm E}\cdot\boldsymbol{V}_2}{c^2} - \frac{V_{\rm E}^2}{2c^2}\right) - \frac{\boldsymbol{V}_{\rm E}\cdot\boldsymbol{b}_0}{c^2}\left(1 + \frac{\boldsymbol{K}\cdot\boldsymbol{V}_{\rm E}}{2c}\right)\right.\\ &\quad\left. + \frac{\boldsymbol{K}\cdot\boldsymbol{a}_2}{2c}\left[\frac{\boldsymbol{K}\cdot\left(\boldsymbol{b}_0 - \frac{\boldsymbol{b}_0}{c^2}U_{\rm ext} + \frac{\boldsymbol{b}_0\cdot\boldsymbol{V}_{\rm E}}{2c^2}\boldsymbol{V}_{\rm E}\right)}{c}\right]^2 + \Delta T_{\rm grav}\right\}\\ &\quad\Big/\left[1 + \frac{\boldsymbol{K}}{c}\cdot(\boldsymbol{V}_{\rm E}+\boldsymbol{V}_2)\right].\end{aligned} \tag{12}$$

The time derivative of Equation (12) is the delay rate.

3 MODEL CALCULATION AND FITTING IN THE VLBI CORRELATOR

In the VLBI correlator, the delay and delay rate are calculated at two-minute intervals. A series of ten consecutive solutions are windowed into a quintic spline-fitting algorithm (Benson 1995)

$$y = p_1 t^n + p_2 t^{n-1} + \cdots + p_n t + p_{n+1}\,, \qquad n = 5\,. \tag{13}$$

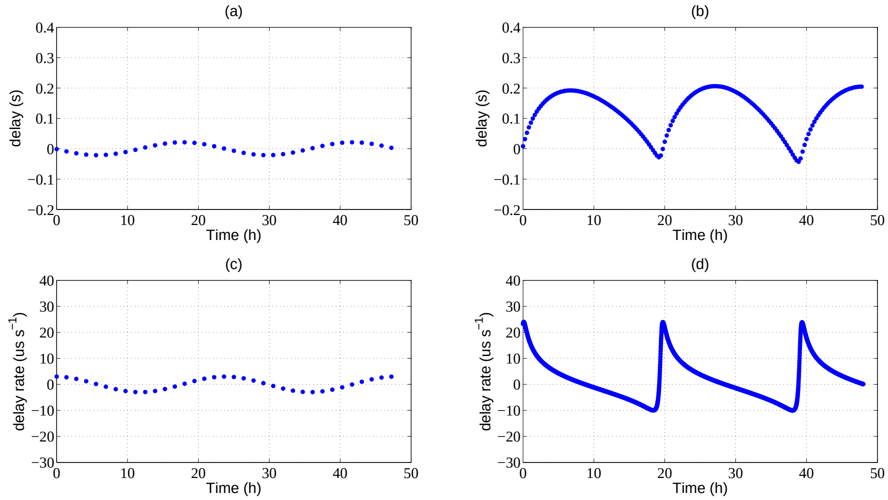

Fig. 2 The simulation delay and delay rate of GVLBI (G-G, *left*) and C-SVLBI (S-G, *right*).

The resulting fifth-order polynomials are evaluated by firmware in the correlator and track the interferometer delays over the intervening two-minute interval used by the model. The next two minutes will be evaluated using another fifth-order polynomial by fitting the next series of solutions. The precision of the fitting should meet the precision required by calculations from the model in the VLBI correlator, which has the following form (Shu & Zhang 2001)

$$\Delta\tau \leq \frac{N}{2B}, \qquad \Delta\dot{\tau} \leq \frac{1}{2T\cdot f}, \tag{14}$$

where B is the bandwidth of the base band converter, N is the number of delay channels, T is the integration time and f is the observing frequency.

For example, if N is 32 and B is 16 MHz, the integration time is 4 s, and the observing frequency of SVLBI can reach 50 GHz. The precision of calculations from the model is as follows

$$\Delta\tau \leq \frac{32}{2\cdot 16\text{MHz}} = 1\,\mu\text{s}, \qquad \Delta\dot{\tau} \leq \frac{1}{2\cdot 4\text{s}\cdot 50\text{ GHz}} = 2.5\text{ ps s}^{-1}. \tag{15}$$

In GVLBI, the maximum delay is about 21 ms, and the delay rate is less than 3 μs s^{-1}. For SVLBI, the maximum delay and fringe rate that must be accommodated are much larger. Take C-SVLBI as an example; the spacecraft has an apogee height of 60 000 km, and the maximum delay will be approximately 0.2 s. The maximum speed of the spacecraft around the perigee is about 8 km s^{-1}, corresponding to a delay rate of 26 μs s^{-1} as shown in Figure 2.

3.1 Simulation Results

In order to examine the effectiveness of the quintic spline fitting for SVLBI, we show the simulation results in this section.

The simulation conditions are shown as follows. The initial orbital elements of the C-SVLBI orbit are shown in Table 1. The simulation time is from 2004 September 8 at 04:00:00 UT to 2004 September 10 at 04:00:00 UT. The orbit of C-SVLBI is simulated with the Satellite Tool Kit (STK). We adopt the Shanghai 65-meter radio telescope as the ground station, which can observe in conjunction with the orbiting telescope. For GVLBI, in order to get the maximum delay, we consider

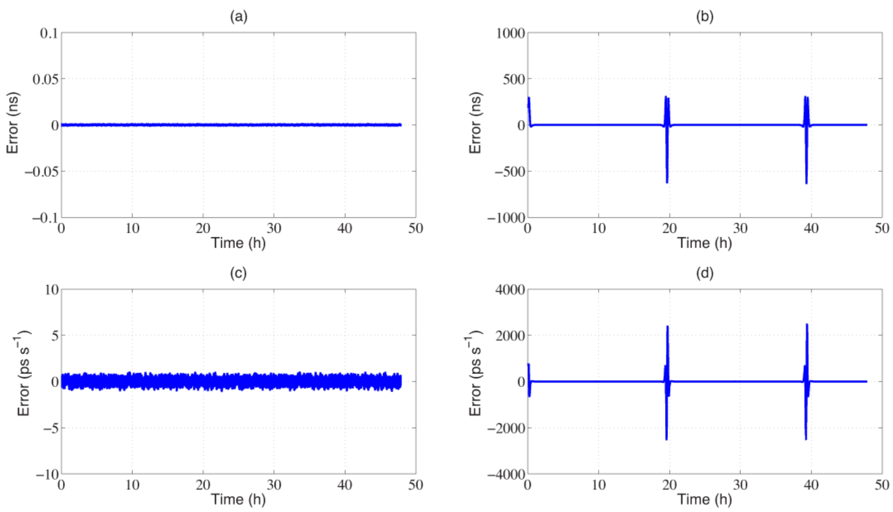

Fig. 3 The spline fitting error of delay and delay rate of GVLBI (*left*) and C-SVLBI (*right*).

Table 1 Initial Orbital Elements of the Orbiting Telescope

Orbit element	Value
Semimajor Axis	36 978.14 km
Eccentricity	0.79
Inclination	28.5°
Period	19.65 h
Perigee Altitude	1 200 km
Apogee Altitude	60 000 km

Notes: Two orbiting telescopes are planned to be launched as the first step of C-SVLBI, and the angle between the two orbital planes is about 120°.

Table 2 Positions of Ground-based Stations

	65-m telescope	GVLBI-1	GVLBI-2
Longitude	$121°11'59''$E	0°	90°/180°
Latitude	$31°05'57''$N	0°	0°
Altitude (m)	5	0	0

Notes: GVLBI-1 and GVLBI-2 refer to the two ground based telescopes.

two imaginary telescopes, which are located on the equator with a respective difference in longitude of 90° and 180°. The one with a difference of 180° has the maximum delay rate, as shown in Table 2.

We calculate the SVLBI delay and delay rate by using Equation (12) and its time derivative, and the GVLBI delay and delay rate by using the formulas given by IERS. The simulation results are shown in Figure 2. We use Equation (13) to fit the ten consecutive solutions, then we evaluate the intervening delay of two minutes and delay rate. We repeat these steps through the simulation. The fitting error is obtained by comparing the fitting result with the result from the model calculation. Compared with the GVLBI, the delay rate of SVLBI is more difficult to address. As shown in Figure 3, around the perigee, the error in the fifth-order fitting for the delay rate of the SVLBI is about 2 ns s^{-1}, which is much larger than the required precision calculated in Equation (15).

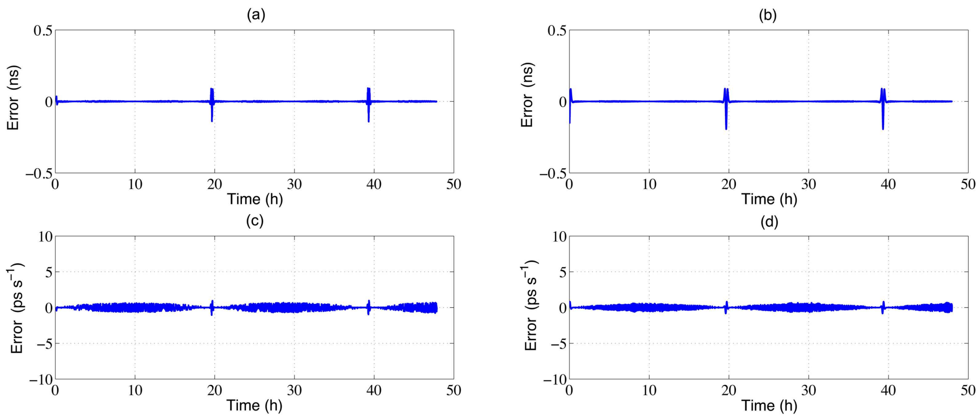

Fig. 4 The left panel shows the fitting result of a 2 min interval using an 8th-order polynomial. The right panel is the fitting error for a 30 s interval using a 5th-order polynomial.

As shown in Figure 4, if we use an eight-order polynomial to fit the delay and delay rate, the fitting result is much better. Another solution is to shorten the time interval used by calculations of the model to 1 min or less, say 30 s as shown in the right panel of Figure 4. Therefore, the SVLBI delay rate requires a more accurate polynomial fitting and evaluation, and more frequent model updates, especially around the perigee.

4 CONCLUSIONS

In this paper, a relativistic delay model for SVLBI is derived. The acceleration of the orbiting telescope is much larger than the GVLBI, and should be included in the delay model. The SVLBI delay rates are higher than GVLBI. The delay models used by VLBI correlators are expressed as polynomials in time. The higher delay rate requires more accurate polynomial fitting and evaluation, and more frequent model updates.

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References

Alexandrov, Y. A., Andreyanov, V. V., Babakin, N. G., et al. 2012, Solar System Research, 46, 458

Benson, J. M. 1995, in ASPC, 82, Very Long Baseline Interferometry and the VLBA, eds. J. A. Zensus, P. J. Diamond, & P. J. Napier, 117

Danby, J. M. A. 1988, Fundamentals of Celestial Mechanics, Richmond: Willman-Bell, c1992, 2nd ed.

Hirabayashi, H. 1988, IAU Symp. 129, The Impact of VLBI on Astrophysics and Geophysics, eds. M. J. Reid & M. Moran, 449 (Dordrecht: Kluwer)

Hirabayashi, H., Edwards, P., et al., 1997a, VSOP News No.55 (*http://www.vsop.isas.jaxa.jp/obs/Newsletters.html*) and (*http://www.vsop.isas.jaxa.jp/obs/news/www.55.html*)

Hong, X. Y., Shen, Z. Q., et al., 2013, The Second China-U.S. Workshop on Radio Astronomy Science and Technology (*http://rast2.csp.escience.cn/dct/page/65579*)

Hirabayashi, H., Edwards, P., et al., 1997b, VSOP News No.56 (*http://www.vsop.isas.jaxa.jp/obs/Newsletters.html*) and (*http://www.vsop.isas.jaxa.jp/obs/news/www.56.html*)

Kardashev, N. S., & Slysh, V. I. 1988, in The Impact of VLBI on Astrophysics and Geophysics, IAU Symposium, 129, eds. M. J. Reid, & J. M. Moran, 433 (Dordrecht: Kluwer)
Petit, G., & Luzum, B. 2010, IERS Conventions (2010) in IERS Technical Note, 36, 1
Schilizzi, R. T., Burke, B. F., Booth, R. S., et al. 1984, in VLBI and Compact Radio Sources, IAU Symposium, 110, eds. R. Fanti, K. I. Kellermann, & G. Setti, 407 (Dordrecht: Reidel)
Schilizzi, R. T. 1988, in The Impact of VLBI on Astrophysics and Geophysics, IAU Symposium, 129, eds. M. J. Reid & J. M. Moran, 441 (Dordrecht: Kluwer)
Sekido, M., & Fukushima, T. 2006, Journal of Geodesy, 80, 137
Shen, Z. Q., Zhang, Z. P., Wu, Z. B., et al., 2013, The Second China-U.S. Workshop on Radio Astronomy Science and Technology (*http://rast2.csp.escience.cn/dct/page/65579*)
Shu, F., & Zhang, X. 2001, Shanghai Observatory Annals, 22, 100
Soffel, M., Klioner, S. A., Petit, G., et al. 2003, AJ, 126, 2687
Wells, D., Benson, J., Broadwell, C., Horstkotte, J., & Romney, J. 1989, Software Architecture for the VLBA Correlator, Tech. rep., Technical Report, National Radio Astronomy Observatory