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LETTERS

New estimates of scale heights and spiral structures for non-edge-on spiral galaxies *

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Abstract On the basis of Poisson's equation for the logarithmic perturbation of matter density, we provide improved estimates of scale heights and spiral structures for non-edge-on spiral galaxies by subtracting the surface brightness distributions from observed images. As examples, the non-edge-on spiral galaxies PGC 24996, which is face-on, and M31, which is inclined, are studied. The scale height, pitch angle and inclination angle of M31, our nearest neighbor, that are presented in this work, agree well with previous research.

Key words: galaxy: disk — galaxies: fundamental parameters — galaxies: spiral — galaxies: structure

1 INTRODUCTION

van der Kruit & Searle (1981a,b, 1982a,b) proposed an effective method to obtain the scale heights for edge-on spiral galaxies based on measuring the light distributions of three dimensional galactic disks; however, this method is invalid for measuring the thickness of the disk for non-edge-on spiral galaxies. Peng et al. (1979) studied an exponential distribution of mass density for a three-dimensional galactic disk

$$\rho(r,\phi,z) = \frac{\alpha}{2}\sigma(r,\phi)\exp(-\alpha \mid z \mid)\,, \tag{1}$$

where $\alpha = 2/H$ is the thickness factor; here H represents the equivalent thickness of the galactic disk, and $H_s = 0.5H = 1/\alpha$ is the scale height. $\sigma(r,\phi)$ is the surface density of matter in the disk, which contains decompositions of a surface density and a perturbation density. By investigating the asymptotic expression of Poisson's equation for the logarithmic perturbation of the matter density (Peng et al. 1979), Peng (1988, hereafter P88) presented a mathematical method for determining the scale heights of non-edge-on spiral galaxies,

$$H = \frac{2r_0}{\sqrt{m^2+\Lambda^2}}\,, \tag{2}$$

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where m is the number of galactic spiral arms and Λ is the winding parameter of the spiral arms. Here r_0 is the forbidden radius, which denotes that spiral arms could not appear in the region $r \leq r_0$, where the density wave could not exist (Peng et al. 1979; Peng 1988).

Our Galaxy has been the subject of many researches, but the forbidden radius and the number of spiral arms have not been well determined (Hou et al. 2009). By collecting data from published literatures, e.g., the average value $r_0 = 3.7$ kpc of four forbidden radii of the spiral arms of Scutum-Crux, Sagittarius-Carina, Perseus and Norma-Outer (Nakanishi & Sofue 2006) and using $r_0 = 3.6 - 4.5$ kpc, models with 2–4 spiral arms have been presented by Hou et al. (2009) to outline the spiral structure of the Milky Way. As an example, we adopt $r_0 = 4.0$ kpc and the pitch angle $\mu = 11.6°$ (Vallée 1995) to test the scale height of our Galaxy by using equation (2) of the P88 method. Therefore the Galactic scale height would be $H_\mathrm{s} = 0.40$ kpc, which is in agreement with the values $0.30 - 0.35$ kpc given by previous researches (Gilmore & Reid 1983; Pritchet 1983; Gould et al. 1997). Based on P88, Peng's group investigated large numbers of scale heights and spiral structures of non-edge-on spiral galaxies (e.g., Ma et al. 1997a,b, 1998, 1999; Ma 2001a,b, 2002, 2003; Zhao et al. 2004; Hu 2006; Hu et al. 2006a,b,c, 2007, 2013a,b).

Nevertheless, in the P88 method, the key parameter of forbidden radius r_0 is difficult to exactly determine when the galactic central region is blurred by light from the bulge. In this work, we give improved estimates of the scale heights and spiral structures by subtracting the surface brightness distributions from observed images of non-edge-on spiral galaxies. As examples, the face-on case of PGC 24996 and the inclined case of M31 are studied. We note that the scale height, pitch angle and inclination angle of M31 presented in this study are in good agreement with previously published investigations.

2 METHOD

From P88, we learn that if the values of m, Λ and r_0 are known, one can obtain the scale heights of spiral galaxies from Equation (2). The number of spiral arms m can be determined directly from images of the galaxy. For face-on spiral galaxies, the winding parameter Λ can be derived by fitting equiangular logarithmic spirals to the spiral arms in the galactic disk

$$r = r_0 \exp\left[\frac{m}{\Lambda}(\phi - \phi_0)\right] , \tag{3}$$

where (r, ϕ) are polar coordinates and (r_0, ϕ_0) denote the most inward limit of the spiral arm, which is the forbidden radius.

In general, the forbidden radius r_0 could be determined by directly measuring the innermost point of a spiral arm in an observed image. However, within the central disk, the spiral arms are usually contaminated by light from the bulge component. Based on personal judgment about the spiral structure, one can only make a rough estimate of the innermost limit of a spiral arm. It is hence quite a subjective process to confirm the exact value for r_0 near the galactic center.

In most cases, disk galaxies are comprised of two components: bulge+disk decompositions (de Vancouleurs 1959; Simien & de Vaucouleurs 1986; Hu et al. 2006b). The brightness from the bulge, which can be described by an $r^{1/4}$ exponential law,

$$I_\mathrm{b}(r) = I_{\mathrm{b}_0} \exp[-(r/r_\mathrm{b})^{1/4}] , \tag{4}$$

and the disk brightness generally follows exponential law,

$$I_\mathrm{d}(r) = I_{\mathrm{d}_0} \exp(-r/r_\mathrm{d}) , \tag{5}$$

where I_{b_0} and I_{d_0} are the central surface brightness of the bulge and disk, respectively. r_b is the scale length of the bulge and r_d is the scale length of the disk.

Therefore, if a bulge+disk decomposition is subtracted from the image of a spiral galaxy, then a residual pattern would emerge that would only show the spiral structures without being blurred by the light of the bulge. Thus, without subjectivity and being free of contamination, we can then determine the exact value of r_0 by measuring the innermost location of a spiral arm.

3 RESULTS

In this study, we derive the forbidden radius, the equiangular logarithmic spirals and the winding parameter of the nearly face-on spiral galaxy PGC 24996 by directly fitting the pattern of the spiral arms in the residual image. Table 1 lists the parameters of the bulge+disk decomposition model for the face-on spiral galaxy PGC 24996. The scale height H_s, forbidden radius r_0 and winding parameter of the spiral arms are shown in Table 2. In Table 2 Column (7), $\Delta F/F$ is the total flux in the residual image compared to that of the original image of the galaxy, where ΔF is the sum of the pixel value (pixel flux) in the residual image, and F is the sum over the original observed image. Examining the sum of values of the pixels in the residual image can reveal whether the best-fitting to the surface brightness of the galaxy is satisfactory or not. The flux ratio $\Delta F/F$ will be small if the fit is good.

Figure 1 displays four panels showing the nearly face-on galaxy PGC 24996. In this work, the flux ratio $\Delta F/F = 7.6\%$.

Nevertheless, for non-face-on (between face-on and edge-on) spiral galaxies, in particular for inclined disk galaxies, it is not easy to obtain the bulge+disk decomposition model due to the disk inclination angle IA, which can add extra brightness from the edge of the disk. Another subtraction method is required.

Here, we give a simple treatment for obtaining the residual image of an inclined spiral galaxy. Firstly, we can get an approximate inclination angle IA_0 proposed by Aaronson et al. (1980), which is corrected from the Hubble (1926) formula

$$\mathrm{IA} = \arccos\sqrt{[(d_{25}/D_{25})^2 - q_0^2]/(1-q_0^2)}, \qquad q_0 = 0.2\,,$$
$$\mathrm{IA}_0 = \arccos\sqrt{1.042\left(\frac{d_{25}}{D_{25}}\right)^2 - 0.042} + 0.052\,, \tag{6}$$

where d_{25} and D_{25} are profiles of the apparent minor and major axes respectively, which come from the Third Reference Catalogue of Bright Galaxies (de Vancouleurs et al. 1991, hereafter RC3). Secondly, by using the disk inclination angle, we measure the average flux of the pixels in the elliptical zone versus the disk's major axis, then a residual pattern can be obtained after these elliptically

Table 1 Parameters of the Bulge+Disk Decomposition Model for the Face-on Spiral Galaxy PGC 24996

PGC (1)	band (2)	I_{b_0} (mag arcsec^{-2}) (3)	r_{b} (arcsec) (4)	I_{d_0} (mag arcsec^{-2}) (5)	r_{d} (arcsec) (6)
24996	i	24.80	48.78	29.36	2.41

Table 2 The Scale Height H_s, Forbidden Radius r_0 and Winding Parameter of PGC 24996

PGC (1)	Arm (2)	r_0 (kpc) (3)	Λ (4)	H_s (kpc) (5)	$H = 2H_s$ (kpc) (6)	$\Delta F/F$ (7)
24996	A-arm	1.23	7.86	0.15 ± 0.04	0.30 ± 0.08	7.6%
	B-arm	1.20	7.46	0.16 ± 0.05	0.32 ± 0.10	

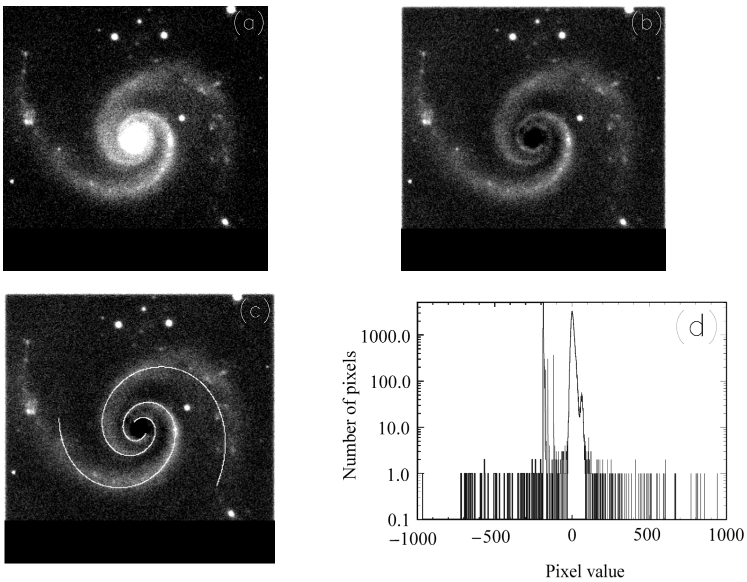

Fig. 1 (a) the observed image of PGC 24996 in the i band; (b) the residual image after a bulge+disk decomposition has been subtracted from the observed image; (c) the fitting pattern of the spiral arms; (d) flux values of the pixels in the residual image after the bulge+disk decomposition has been subtracted from the image of the spiral galaxy.

averaged flux data are subtracted from the observed image. Furthermore, we can adjust IA around IA_0, until a good spiral pattern appears in the residual-image. Thirdly, based on Equation (3) and corrected by the disk inclination angle IA, we can obtain the polar coordinates (ρ, θ) in the inclined plane

$$\rho(\theta, \mathrm{IA}) = \rho_0 \frac{f(\theta_0, \mathrm{IA})}{f(\theta, \mathrm{IA})} \exp\left[\frac{m}{\Lambda} B(\theta, \mathrm{IA})\right] , \tag{7}$$

where

$$\tan\theta = \tan\phi \cos \mathrm{IA} , \qquad \rho = \frac{r}{\sqrt{1 + \sin^2\theta \tan^2 \mathrm{IA}}} , \tag{8}$$

$$f(\theta, \mathrm{IA}) = \sqrt{\sin^2\theta + \cos^2\theta \cdot \cos^2 \mathrm{IA}} , \tag{9}$$

$$B(\theta, \mathrm{IA}) = \arctan\frac{\tan\theta}{\cos \mathrm{IA}} - \arctan\frac{\tan\theta_0}{\cos \mathrm{IA}} , \tag{10}$$

therefore, based on Equation (7), an approximate value of the winding parameter Λ_0 can be calculated by measuring the points along the spiral arm; then we can adjust the Λ value around Λ_0, until a good fit to the arm in the residual image is derived. Finally, after the forbidden radius r_0 is determined, the disk thickness $H = 2H_\mathrm{s} = 2/\alpha$ can be obtained from Equation (2).

Figure 2 shows four panels for the rather inclined spiral galaxy M31. The image is taken from the NASA/IPAC Extragalactic Database (NED). Panel (a) is the original observed image of M31; we can see that M31's spiral structures are difficult to determine in the observed image, which are are blurred by light from the bulge in the center of the galaxy. Panel (b) shows the residuals after the above mentioned elliptically averaged flux data have been subtracted from the original image. In practice, we have subtracted two sets of elliptically-averaged flux data to make M31's spiral

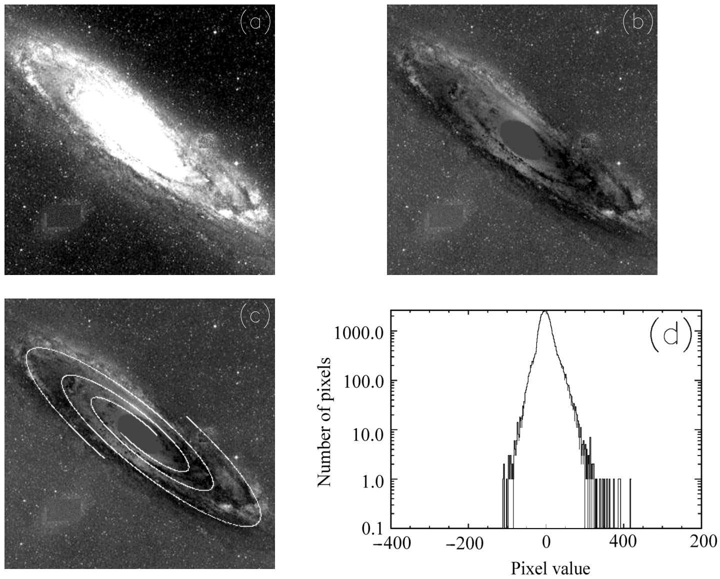

Fig. 2 Same as Fig. 1, but for M31. The image is taken from the NASA/IPAC Extragalactic Database (NED).

structures clear: one is the model for the disk and the other is for the bulge. We find that the bulge's ellipticity is about $e_{\rm b} = 0.54$, and the disk's ellipticity is about $e_{\rm d} = 0.23$. Panel (c) shows the results of the fitting for M31's spiral arms. Panel (d) gives a histogram for estimating the distribution of values of the pixels in the residual image. The sum of values can be used to judge whether the bulge's and disk's ellipticities ($e_{\rm b}$ and $e_{\rm d}$) of M31 are satisfactory or not. The results for M31 are listed in Table 3. Here we show that the flux ratio $\Delta F/F = 2.1\%$ (Table 3 Col. (9)) is very small.

4 CONCLUSIONS AND DISCUSSION

From Figures 1 and 2 and Tables 2 and 3, we can conclude that the two spiral structures of PGC 24996 and M31 are quite symmetrical. As shown in Tables 1 and 2, we base this study of PGC 24996 on an i-band image, which allows us see the central region of the galaxy more clearly. The flux ratio $\Delta F/F = 7.6\%$ shows that the fit of the bulge+disk decomposition model is good. The parameters of PGC 24996 derived from the i-band residual image are in agreement with Hu et al. (2007). As shown in Table 3 Column (6), the pitch angle of M31 is about 7.0°, which is in agreement with Ma et al. (1997a, $\mu = 7.7°$). The disk inclination angle IA= $76.8°$ listed in Column (2) is in good agreement with previous researches (IA=77°: Kent 1989 and Tremaine 1995; IA=75°: Hamble et al. 1995; and IA=77.5°: Ma et al. 1997a). It should be noted that the thickness of the disk in M31 obtained by Ma et al. (1997a, $H = 1.01$ kpc) is larger than ours ($H = 0.67 - 0.69$ kpc, Table 3 Col. (8)). This is due to the contamination from brightness of the bulge. The residual image only

Table 3 The Scale Height $H_{\rm s}$, Inclination Angle IA and Pitch Angle of M31

Name (1)	IA (°) (2)	Arm (kpc) (3)	r_0 (4)	Λ (5)	μ (°) (6)	$H_{\rm s}$ (kpc) (7)	$H = 2H_{\rm s}$ (kpc) (8)	$\Delta F/F$ (9)
M31	76.8	A-arm	5.52	16.28	7.0	0.34 ± 0.07	0.67 ± 0.15	2.1%
		B-arm	5.70	16.41	6.9	0.35 ± 0.10	0.69 ± 0.20	

displays the spiral arm structure without blurring of light from the bulge, so that the spiral arms can be traced to smaller radii; therefore, the innermost location of the spiral arm obtained in this study is closer to the galactic center, namely, the r_0 is smaller. From Equation (2), we can deduce that the smaller r_0 is, the thinner the galactic disk will be. Our result of $H = 0.67 - 0.69$ kpc is in good agreement with the conclusion of $H = 0.8$ kpc presented by de Vaucouleurs (1958).

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