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***A**stronomy and*
***A**strophysics*

Equilibria of a charged artificial satellite subject to gravitational and Lorentz torques

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Abstract The attitude dynamics of a rigid artificial satellite subject to a gravity gradient and Lorentz torques in a circular orbit are considered. Lorentz torque is developed on the basis of the electrodynamic effects of the Lorentz force acting on the charged satellite's surface. We assume that the satellite is moving in a Low Earth Orbit in the geomagnetic field, which is considered to be a dipole. Our model of torque due to the Lorentz force is developed for an artificial satellite with a general shape, and the nonlinear differential equations of Euler are used to describe its attitude orientation. All equilibrium positions are determined and conditions for their existence are obtained. The numerical results show that the charge q and radius ρ_0 of the center of charge for the satellite provide a certain type of semi-passive control for the attitude of the satellite. The technique for this kind of control would be to increase or decrease the electrostatic screening on the satellite. The results obtained confirm that the change in charge can affect the magnitude of the Lorentz torque, which can also affect control of the satellite. Moreover, the relationship between magnitude of the Lorentz torque and inclination of the orbit is investigated.

Key words: space vehicles — atmospheric effects — celestial mechanics — kinematics and dynamics

1 INTRODUCTION

An artificial satellite moving in Low Earth Orbit (LEO) or High Earth Orbit naturally tends to accumulate an electrostatic charge. Ambient plasma and the photoelectric effect can produce a Lorentz force on satellites in LEO. Interactions between the spacecraft and plasma are the main source for spacecraft charging. Due to plasma interactions, surface charging on the spacecraft is the major source of anomalies in its orbit (Garrett 1981; Purvis et al. 1984). In some cases, the accumulation of electrostatic charge can affect instruments and other devices onboard the satellite, which may ultimately lead to difficulties in operating the satellite. For example, the newly launched LARES satellite could be effected by electrostatic charging (Ciufolini et al. 2012). Similarly, the space shuttle has been investigated for effects due to charging (BiléN et al. 1995). Different research efforts have led to the development of technology for active mitigation of satellite charging through the control of charge. The effect of electrostatic charge may negatively impact the error budget of satellites that are designed for experiments about fundamental physics by damaging onboard electronic instruments or by interfering with scientific measurements. Damage to electronic instruments is rare but may be harmful in many ways. Interference in scientific measurements is very common due to spacecraft charging; see references Everitt et al. (2011), Worden & Everitt (2013), Nobili et al. (2009), Iorio (2009) and Iorio et al. (2004) and the references there in.

Saad & Ismail (2010) determined the orbital effects of the Lorentz force on the motion of an electrically charged artificial satellite moving in Earth's magnetic field. The influence of the geomagnetic field manifests itself predominantly by the Lorentz force. Then in 1990, Coprophilagy studied variations in the orbital elements due to the Lorentz force with variations in natural charge. Pollock et al. (2010) showed that the Lorentz force can be used to save substantial amounts of propellant in maneuvers that change inclination. Heilmann et al. (2012) showed that the effect of an electric dipole moment induced by the Earth's electric field at high altitude is very small compared to the electromagnetic effect. Peng & Gao (2012) showed that the Lorentz force can be implemented for J_2 invariant formation given that the deputy spacecraft has an electrostatic charge. Therefore, the Lorentz force is a possible means for charging and thus controlling spacecraft orbits without consuming propellant. Peck (2005) was the first to introduce a control scheme. The orbits of a spacecraft that are accelerated by the Lorentz force are termed Lorentz–augmented orbits, because the Lorentz force cannot completely replace traditional rocket propulsion. After Peck (2005), a series of papers (King et al. 2003; Natarajan & Schaub 2006; Streetman & Peck 2007; Yamamoto & Yamakawa 2008; Yamakawa et al. 2010) applied charge control techniques to utilize Lorentz forces to control the orbit of a satellite.

Abdel-Aziz (2007) studied the stability of an equilibrium position due to Lorentz torque in the case of a uniform magnetic field and cylindrical shape for an artificial satellite. Yamakawa et al. (2012) investigated the attitude motion of a charged satellite having the shape of a dumbbell pendulum due to Lorentz torque. Their study of the stability of equilibrium points only focused on the pitch within the equatorial plane.

In this paper, we focus on the attitude motion of an artificial satellite with a general shape moving in a circular orbit under gravity gradient torque and Lorentz torque. Euler equations will be used to describe the attitude dynamics of the satellite. Determination of the equilibrium orientation of a satellite under the action of gravitational and Lorentz torques is one of the basic problems addressed in this paper. Finally, we will analyze the equilibrium positions based on control of the center of charge for the satellite relative to its center of mass and amount of charge.

Before we move onto the next section to formulate the problem in question, we would like to point out that electromagnetic effects caused by a Lorentz force on satellites moving in the gravitational field of the Earth, which are the subject of this paper, should not be confused with purely gravitational effects, which are dubbed "gravitomagnetic" and arise from general relativity. They are widely discussed in literature (Mashhoon et al. 2001; Mashhoon 2007; and Iorio & Lichtenegger 2005). The name "gravitomagnetic" is due to a purely formal resemblance of the Lense-Thirring effects, occurring in stationary spacetimes generated by stationary mass-energy currents, such as a rotating planet, with the linear equations of electromagnetism by Maxwell and with the Lorentz force acting on electrically charged bodies moving in a magnetic field (Iorio et al. 2011; Iorio et al. 2002; Renzetti 2013; Mashhoon 2013).

Section 2 gives the formulation of the spacecraft and description of the coordinate system used. In Section 3, expression of the torque due to Lorentz force is derived. In Section 4, certain equilibrium positions are identified. In Section 5 numerical experiments are used to explain the effect of charge and inclination on ρ_0 and torque. Conclusions are given in Section 6.

2 FORMULATION OF THE PROBLEM

A rigid spacecraft is considered, whose center of mass moves in the Newtonian central gravitational field of the Earth in a circular orbit with radius r. We suppose that the spacecraft is equipped with an electrostatically charged protective shield, having an intrinsic magnetic moment. The rotational motion of the spacecraft about its center of mass will be analyzed, considering the influence of gravity gradient torque T_{G} and the torque T_{L} due to Lorentz forces. The torque T_{L} results from the interaction of the geomagnetic field with the charged screen that is part of the electrostatic shield.

The rotational motion of the satellite relative to its center of mass is investigated in the orbital coordinate system $C_{x_{\mathrm{o}}y_{\mathrm{o}}z_0}$(see Fig. 1) with $C_{x_{\mathrm{o}}}$ tangent to the orbit in the direction of motion; $C_{y_{\mathrm{o}}}$ lies along the normal to the orbital plane and $C_{z_{\mathrm{o}}}$ lies along the radius vector r of the point O_E relative to the center of the Earth. Our investigation is carried out assuming the rotation of the orbital coordinate system relative to the inertial system has angular velocity Ω. As an inertial coordinate system, the system O_{XYZ} is taken, whose axis $OZ(k)$ is directed along the axis of Earth's rotation, the axis $OX(i)$ is directed toward the ascending node of the orbit, and the plane coincides with the equatorial plane. In addition, we assume that the satellite's principal axes of inertia $C_{x_by_bz_b}$ are rigidly fixed to a satellite (i_b, j_b, k_b). The satellite's attitude may be described in several ways. In this paper the attitude will be described by the angle of yaw ψ, the angle of pitch θ and the angle of roll φ, between the satellite's $C_{x_by_bz_b}$ and the set of reference axes O_{XYZ}. The three angles are obtained by rotating the satellite's axes from an attitude coinciding with the reference axes to describe the attitude in the following way:

– Allow a rotation ψ about the z-axis;

– About the newly displaced y-axis, rotate through θ;

– Finally, allow a rotation φ about the final position of the x-axis.

Although the angles ψ, θ and φ are often referred to as Euler angles, they differ from classical Euler angles in that only rotation takes place about each axis, whereas in the classical Euler angular coordinates two rotations are made about the z-axis. The relationship between the orbital coordinate

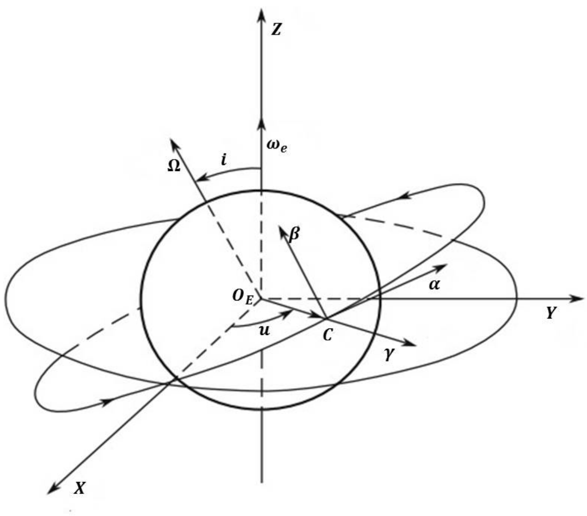

Fig. 1 Coordinates used in the derivation of the equations of motion.

system and reference system O_{XYZ} is determined below:

$$
\begin{aligned}
\hat{\boldsymbol{\imath}} &= -\sin u\boldsymbol{\alpha} + \cos u\boldsymbol{\gamma}\,,\\
\hat{\boldsymbol{\jmath}} &= \cos i \cos u\boldsymbol{\alpha} - \sin i\boldsymbol{\beta} + \cos i \sin u\boldsymbol{\gamma}\,,\\
\hat{\boldsymbol{\kappa}} &= \sin i \cos u\boldsymbol{\alpha} + \cos i\boldsymbol{\beta} + \sin i \sin u\boldsymbol{\gamma}\,,
\end{aligned}
\tag{1}
$$

where i is the orbital inclination, $u = \Omega t + u_0$ is the argument of latitude, Ω is the orbital angular velocity of the satellite's center of mass, u_0 is the initial latitude and $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are unit vectors along the axes of the orbital coordinate system. These vectors are the different directions of the tangent to the plane of the orbit, its radius and the normal of the orbit, respectively (Gerlach 1965).

The relationship between the reference frames $C_{x_b y_b z_b}$ and $C_{x_o y_o z_0}$ is given by the matrix A which is the matrix of unitary vectors $\alpha_i,\ \beta_i,\ \gamma_i\ (i = 1, 2, 3)$.

$$
A = \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix},
\tag{2}
$$

where

$$
\begin{aligned}
\alpha_1 &= \cos\theta\cos\psi\,,\\
\alpha_2 &= -\cos\phi\sin\psi + \sin\phi\,\sin\theta\cos\psi,\\
\alpha_3 &= \sin\phi\sin\psi\ + \cos\phi\sin\theta\ \cos\psi,\\
\beta_1 &= \cos\theta\sin\psi,\\
\beta_2 &= \cos\varphi\ \cos\psi\ + \sin\phi\sin\theta\sin\psi,\\
\beta_3 &= -\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi,\\
\gamma_1 &= -\sin\theta\,,\\
\gamma_2 &= \sin\phi\cos\theta\,,\\
\gamma_3 &= \cos\phi\cos\theta,
\end{aligned}
\tag{3}
$$

and

$$
\boldsymbol{\alpha} = \alpha_1 i_b + \alpha_2 j_b + \alpha_3 k_b,\quad \boldsymbol{\beta} = \beta_1 i_b + \beta_2 j_b + \beta_3 k_b,\quad \boldsymbol{\gamma} = \gamma_1 i_b + \gamma_2 j_b + \gamma_3 k_b.
\tag{4}
$$

3 TORQUE DUE TO THE LORENTZ FORCE

The geomagnetic field with magnetic induction $\boldsymbol{B}$ is approximated by a dipole. The spacecraft is supposed to be equipped with a charged surface (screen) with area S and electric charge $q = \int_S \sigma\, dS$ distributed over the surface with density σ. Therefore, we can write the torque of these forces relative to the spacecraft's center of mass as follows (Griffiths 1989)

$$
\boldsymbol{T}_{\mathrm{L}} = \int_S \sigma\boldsymbol{\rho} \times (\boldsymbol{V} \times \boldsymbol{B})dS,
\tag{5}
$$

where $\boldsymbol{\rho}$ is the radius vector of the screen's element dS relative to the spacecraft's center of mass and $\boldsymbol{V}$ is the velocity of the element dS relative to the geomagnetic field. As in Tikhonov et al. (2011), the torque $\boldsymbol{T}_{\mathrm{L}}$ can be written as

$$
\boldsymbol{T}_{\mathrm{L}} = (T_{Lx}, T_{Ly}, T_{Lz}) = q\boldsymbol{\rho}_0 \times A^{\mathrm{T}}(\boldsymbol{V}_{\mathrm{rel}} \times \boldsymbol{B}_{\mathrm{o}}),
\tag{6}
$$

$$
\boldsymbol{\rho}_0 = x_0 i_b + y_0 j_b + z_0 k_b = q^{-1} \int_S \sigma\,\boldsymbol{\rho}\, dS,
\tag{7}
$$

where $\boldsymbol{\rho}_0$ is the radius vector of the center of charge of the spacecraft relative to its center of mass and A^{T} is the transpose of the matrix of unitary vectors A. As in Gangestad et al. (2010), we use

$$\boldsymbol{V}_{\mathrm{rel}} = (V_{\mathrm{rel1}}, V_{\mathrm{rel2}}, V_{\mathrm{rel3}}) = \boldsymbol{V} - \boldsymbol{\omega}_e \times \boldsymbol{r} = \boldsymbol{r}(\Omega - \omega_E \cos i) \times \boldsymbol{\alpha} + R\omega_E \sin i \cos u\, \boldsymbol{\beta} \,, \tag{8}$$

where $\boldsymbol{V}_{\mathrm{rel}}$ is the velocity vector of the spacecraft's center of mass relative to the geomagnetic field, $\boldsymbol{V}$ is the initial velocity of the satellite, $\boldsymbol{\omega}_e = \omega_e \hat{k}$ is the angular velocity of the diurnal rotation of the geomagnetic field that moves with the Earth and $\boldsymbol{B}_{\mathrm{o}}$ is the magnetic field in the orbital coordinates. Substituting Equations (5)–(7) into Equation (8), we can write the final form of the components of torque due to the Lorentz force as:

$$T_{Lx} = q \left\{ \begin{array}{l} y_0 \left[\alpha_3 V_{\mathrm{rel2}} B_{\mathrm{o3}} - \beta_3 V_{\mathrm{rel1}} B_{\mathrm{o3}} + \gamma_3 (V_{\mathrm{rel1}} B_{\mathrm{o2}} - V_{\mathrm{rel2}} B_{\mathrm{o1}}) \right] \\ -z_0 (\alpha_2 V_{\mathrm{rel2}} B_{\mathrm{o3}} - \beta_2 V_{\mathrm{rel1}} B_{\mathrm{o3}} + \gamma_2 (V_{\mathrm{rel1}} B_{\mathrm{o2}} - V_{\mathrm{rel2}} B_{\mathrm{o1}}) \end{array} \right\}, \tag{9}$$

$$T_{Ly} = q \left\{ \begin{array}{l} z_0 \left[\alpha_1 V_{\mathrm{rel2}} B_{\mathrm{o3}} - \beta_1 V_{\mathrm{rel1}} B_{\mathrm{o3}} + \gamma_1 (V_{\mathrm{rel1}} B_{\mathrm{o2}} - V_{\mathrm{rel2}} B_{\mathrm{o1}}) \right] \\ -x_0 (\alpha_3 V_{\mathrm{rel2}} B_{\mathrm{o3}} - \beta_3 V_{\mathrm{rel1}} B_{\mathrm{o3}} + \gamma_3 (V_{\mathrm{rel1}} B_{\mathrm{o2}} - V_{\mathrm{rel2}} B_{\mathrm{o1}}) \end{array} \right\}, \tag{10}$$

$$T_{Lz} = q \left\{ \begin{array}{l} x_0 \left[\alpha_2 V_{\mathrm{rel2}} B_{\mathrm{o3}} - \beta_2 V_{\mathrm{rel1}} B_{\mathrm{o3}} + \gamma_2 (V_{\mathrm{rel1}} B_{\mathrm{o2}} - V_{\mathrm{rel2}} B_{\mathrm{o1}}) \right] \\ -y_0 (\alpha_1 V_{\mathrm{rel2}} B_{\mathrm{o3}} - \beta_1 V_{\mathrm{rel1}} B_{\mathrm{o3}} + \gamma_1 (V_{\mathrm{rel1}} B_{\mathrm{o2}} - V_{\mathrm{rel2}} B_{\mathrm{o1}}) \end{array} \right\}. \tag{11}$$

Like in Wertz (1978), we can write the components of the magnetic field in the orbital system directed to the tangent of the orbital plane, normal to the orbit and in the direction of the radius, respectively, as below:

$$\begin{aligned} B_{\mathrm{o1}} &= \frac{B_0}{2r^3} \sin\theta'_m \left[3\cos(2\nu - \alpha_m) + \cos\alpha_m \right], \\ B_{\mathrm{o2}} &= -\frac{B_0}{2r^3} \cos\theta'_m \,, \\ B_{\mathrm{o3}} &= \frac{B_0}{2r^3} \sin\theta'_m \left[3\sin(2\nu - \alpha_m) + \sin\alpha_m \right], \end{aligned} \tag{12}$$

where $B_0 = 7.943 \times 10^{15}$ is the intensity of the magnetic field, $\theta'_m = 168.6°$ is the co-elevation of the dipole, $\alpha_m = 109.3°$ is the east longitude of the dipole and ν is the true anomaly measured from the ascending node.

4 EQUILIBRIUM POSITIONS AND ANALYTICAL CONTROL LAW

The equations of motion for a rigid artificial satellite are usually written in terms of the Euler - Poisson variables $\boldsymbol{\omega}$, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$, and have the following form (Abdel-Aziz 2007).

$$\frac{d\,\boldsymbol{\omega}}{dt} I + \boldsymbol{\omega} \times \boldsymbol{\omega} I = \boldsymbol{T}_{\mathrm{G}} + \boldsymbol{T}_{\mathrm{L}} \,, \tag{13}$$

$$\frac{d\,\boldsymbol{\alpha}}{dt} + \boldsymbol{\alpha} \times \boldsymbol{\omega} = -\Omega\boldsymbol{\gamma} \,, \quad \frac{d\,\boldsymbol{\beta}}{dt} + \boldsymbol{\beta} \times \boldsymbol{\omega} = 0 \,, \quad \frac{d\,\boldsymbol{\gamma}}{dt} + \boldsymbol{\gamma} \times \boldsymbol{\omega} = \Omega\boldsymbol{\alpha}, \tag{14}$$

where $\boldsymbol{T}_{\mathrm{G}} = 3\Omega^2 \boldsymbol{\gamma} \times \boldsymbol{\gamma} I$ is the well known formula of the gravity gradient torque. I is the inertia matrix of the spacecraft, Ω is the orbital angular velocity and $\boldsymbol{\omega}$ is the angular velocity vector of the spacecraft. The components of $\boldsymbol{T}_{\mathrm{G}}$ can be written as

$$\begin{aligned} T_{Gx} &= 3\Omega^2 \gamma_2 \gamma_3 (C - B) \,, \\ T_{Gy} &= 3\Omega^2 \gamma_1 \gamma_3 (A - C) \,, \\ T_{Gz} &= 3\Omega^2 \gamma_1 \gamma_2 (B - A) \,, \end{aligned} \tag{15}$$

where A, B and C are moments of inertia of the spacecraft. According to Gerlach (1965), the angular velocity of the spacecraft in the inertial reference frame is $\boldsymbol{\omega} = (\omega_x, \omega_y, \omega_z)$, and in the orbital reference frame it is $\boldsymbol{\omega}_{\mathrm{o}} = (\omega_{\mathrm{o}x}, \omega_{\mathrm{o}y}, \omega_{\mathrm{o}z})$ as given below:

$$\begin{aligned}
\omega_x &= \dot{\phi} - \dot{\psi}\sin\theta,\\
\omega_y &= \dot{\theta}\,\cos\phi + \dot{\psi}\cos\theta\,\sin\phi,\\
\omega_z &= -\dot{\theta}\,\sin\phi + \dot{\psi}\cos\theta\,\cos\phi,
\end{aligned} \tag{16}$$

and

$$\begin{aligned}
\omega_{\mathrm{o}x} &= \dot{\phi} - \dot{\psi}\sin\theta - \Omega\,\sin\psi\cos\theta,\\
\omega_{\mathrm{o}y} &= \dot{\theta}\,\cos\phi + \dot{\psi}\cos\theta\,\sin\phi - \Omega(\cos\varphi\,\cos\psi\, + \sin\phi\sin\theta\sin\psi),\\
\omega_{\mathrm{o}z} &= -\dot{\theta}\,\sin\phi + \dot{\psi}\cos\theta\,\cos\phi - \Omega(-\sin\phi\cos\psi + \,\cos\phi\sin\theta\sin\psi).
\end{aligned} \tag{17}$$

It is well known that the orbital system rotates in space with a fixed orbital angular velocity Ω about the axis, which is perpendicular to the orbital plane. The relation between the angular velocity in the two systems is $\boldsymbol{\omega} = \boldsymbol{\omega}_{\mathrm{o}} - \Omega\boldsymbol{\beta}$.

At equilibrium positions, the right hand side of Equation (13) will be zero. Substituting Equations (9)–(11) and Equation (15) in Equation (13) and after some algebraic manipulation we get the following equilibrium positions:

Equilibrium 1.

$$\theta = 0,\ \phi = 0,\ \psi = \frac{\pi}{2},\ (\alpha_1, \alpha_2,\ \alpha_3) = (0, -1, 0),\ \ (\beta_1, \beta_2,\ \beta_3) = (1, 0, 0), \tag{18}$$

$$(\gamma_1,\ \gamma_2,\ \gamma_3) = (0, 0, 1), \tag{19}$$

$$x_0 = \frac{-V_{\mathrm{rel1}}B_{\mathrm{o3}}}{V_{\mathrm{rel1}}B_{\mathrm{o2}} - V_{\mathrm{rel2}}B_{\mathrm{o1}}}z_0, \qquad y_0 = \frac{-V_{\mathrm{rel2}}B_{\mathrm{o3}}}{V_{\mathrm{rel1}}B_{\mathrm{o2}} - V_{\mathrm{rel2}}B_{\mathrm{o1}}}z_0. \tag{20}$$

Equilibrium 2.

$$\theta = 0\,,\ \phi = \frac{\pi}{2}\,,\ \psi = 0, (\alpha_1\,,\ \alpha_2\,,\ \alpha_3) = (1, 0, 0)\,,\ (\beta_1\,,\ \beta_2\,,\ \beta_3) = (0, 0, -1)\,, \tag{21}$$

$$(\gamma_1, \gamma_2, \gamma_3) = (0, 0, 1), \tag{22}$$

$$x_0 = \frac{V_{\mathrm{rel2}}}{V_{\mathrm{rel1}}}z_0\,, \qquad y_0 = \frac{V_{\mathrm{rel1}}B_{\mathrm{o2}} - V_{\mathrm{rel2}}B_{\mathrm{o1}}}{V_{\mathrm{rel1}}B_{\mathrm{o3}}}z_0\,. \tag{23}$$

Equilibrium 3.

$$\theta = \frac{\pi}{2}\,,\ \phi = 0\,,\ \psi = 0, (\alpha_1, \alpha_2\,,\ \alpha_3) = (0, 0, 1)\,,\ (\beta_1, \beta_2\,,\ \beta_3) = (0, 1, 0)\,, \tag{24}$$

$$(\gamma_1, \gamma_2, \gamma_3) = (-1, 0, 0)\,, \tag{25}$$

$$x_0 = \frac{V_{\mathrm{rel2}}B_{\mathrm{o1}} - V_{\mathrm{rel1}}B_{\mathrm{o2}}}{V_{\mathrm{rel2}}B_{\mathrm{o3}}}z_0, \qquad y_0 = \frac{-V_{\mathrm{rel1}}}{V_{\mathrm{rel2}}}z_0. \tag{26}$$

Equilibrium 4.

$$\theta = 0\,,\ \phi = 0\,,\ \psi = 0\,,\ (\alpha_1, \alpha_2\,,\ \alpha_3) = (1, 0, 0)\,,\ (\beta_1, \beta_2\,,\ \beta_3) = (0, 1, 0)\,, \tag{27}$$

$$(\gamma_1, \gamma_2, \gamma_3) = (0, 0, 1), \tag{28}$$

$$x_0 = \frac{V_{\mathrm{rel1}} B_{\mathrm{o3}}}{V_{\mathrm{rel1}} B_{\mathrm{o2}} - V_{\mathrm{rel2}} B_{\mathrm{o1}}} z_0 \,, \qquad y_0 = \frac{V_{\mathrm{rel2}} B_{\mathrm{o3}}}{V_{\mathrm{rel1}} B_{\mathrm{o2}} - V_{\mathrm{rel2}} B_{\mathrm{o1}}} z_0 \,. \tag{29}$$

It can be seen that the four equilibrium positions depend on z_0, which can be used to control the equilibrium positions. We will study the relationship between the magnitude of the torque, the magnitude of the radius vector of the center of charge of the spacecraft relative to its center of mass, the amount of charge and the inclination of the orbit. This analysis will be done for two different values of z_0,

– $z_0 = k\, B_{\mathrm{o2}},\;\; k = -\frac{2r^3}{B_0}$ which approximately equals unity (1 meter).
– $z_0 = 4$.

5 NUMERICAL RESULTS

5.1 Equilibrium 1

In this equilibrium position, the attitude motion of the satellite is only in the ψ direction. The magnitude of the radius vector $\boldsymbol{\rho}_0$ is given by $\|\boldsymbol{\rho}_0\| = \sqrt{x_0^2 + y_0^2 + z_0^2}$. In case of Equilibrium 1, the values of x_0 and y_0 can be determined from Equation (20), which will give the magnitude of $\boldsymbol{\rho}_0$ as a function of u, i and z_0.

$$\rho_0(u,i,z_0) = \|\boldsymbol{\rho}_0\| = z_0 \sqrt{\begin{array}{l} 1{+}2.98{\times}10^{30}\left[\frac{-1.1\times10^{-3}+7.27\times10^{-5}\cos(i)}{D_{\mathrm{eq1}}}\right]^2 \\ +1.57{\times}10^{22}\left[\frac{\cos(u)\sin(i)}{D_{\mathrm{eq1}}}\right]^2 \end{array}} \,, \tag{30}$$

where

$$D_{\mathrm{eq1}} = 2.07{\times}10^{12}{-}1.37{\times}10^{11}\cos(i){+}2.83{\times}10^{11}\cos(u)\sin(i). \tag{31}$$

Similarly, the magnitude of torque $\boldsymbol{T}_{\mathrm{L}}$ can be determined from Equations (6) to (11).

$$\|\boldsymbol{T}_{\mathrm{L}}(q,u,i,r)\| = \frac{qz_0}{r^2 D_{\mathrm{eq1}}} \sqrt{\begin{array}{l} \cos^2 u \sin^2 i \Big[2.52\times10^{15} + 1.10\times10^{13}\cos^2 i \\ +2.84\times10^{14}\cos u \sin i + 1.95\times10^{13}\cos^2 u\, \sin^2 i \\ +\cos i(-3.33\times10^{14} - 1.88\times10^{13}\cos u\, \sin i)\Big] \end{array}} \,. \tag{32}$$

It can be seen from Equation (30) that $\|\boldsymbol{\rho}_0(u,i,z_0)\|$ is independent of r even though its components depend on it. Equation (32) gives the magnitude of the torque. $\|\boldsymbol{\rho}_0(u,i,z_0)\|$ is an almost periodic function of inclination i and latitude u with a maximum value of 1.4029 m and minimum value of 1.236 m for $z_0 = kB_{\mathrm{o2}} = 0.96$. Since the function is almost periodic, these optimum values occur at various values of i and u. For example the maximum occurs at $(i,u) =$ (23.63, 21.99) and (58.05, 53.41). Similarly the minimum occurs at $(i,u) = (39.00, 37.70)$ and (58.05, 56.55). To see the dependence of $\|\boldsymbol{\rho}_0(u,i,z_0)\|$ on the inclination i and latitude u, please refer to Figure 2. It can be seen from both Equation (30) and Figure 2 that z_0 can be used to control ρ. In a similar way, z_0 can be used to control torque, as can be seen in Equation (32). The relationship of torque with r and q is straightforward. It can be seen from Equation (32) that the torque is directly proportional to q and inversely proportional to r^2.

Figure 3 also shows that q can be used to control the torque if desired. It can also be seen from Figure 3 which is given for fixed values of q, u and r that torque has a maximum value of the order 10^{-13} for each value of inclination i.

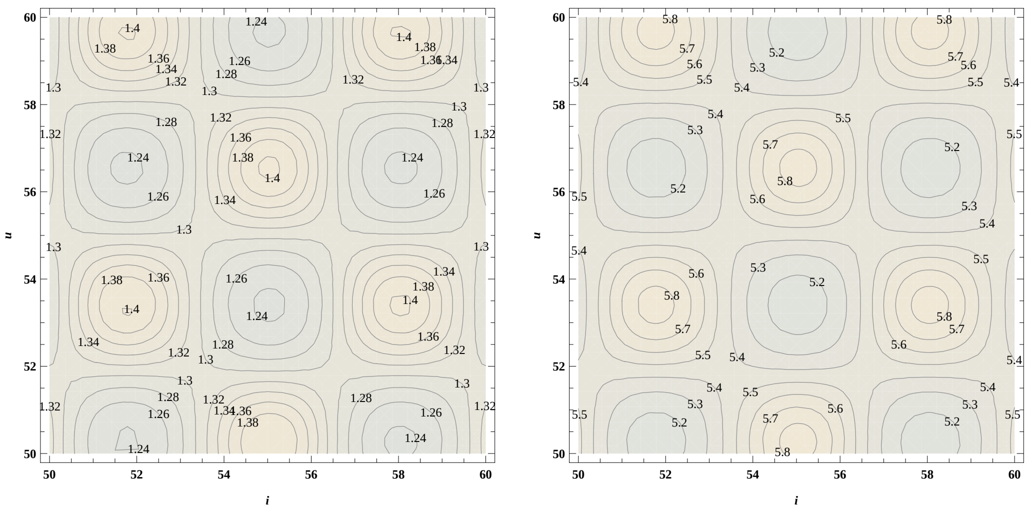

Fig. 2 (a) Contour plot of $\|\rho_0(u, i, z_0)\|$ with maximum and minimum values occurring more than once confirming its periodic behavior. z_0 is taken to be 0.96 in the case of Equilibrium 1. (b) Same as (a) except that z_0 is taken to be 4.

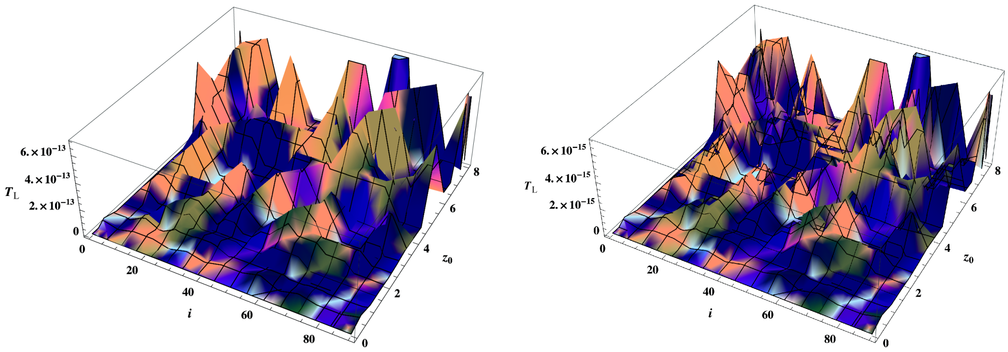

Fig. 3 $\|\boldsymbol{T}_{\rm L}(q, u, i, r)\|$ for fixed values of charge $q = 10\,000$ C (*left*), 100 C (*right*), $r = 6900$ km and $u = 40$ in the case of Equilibrium 1.

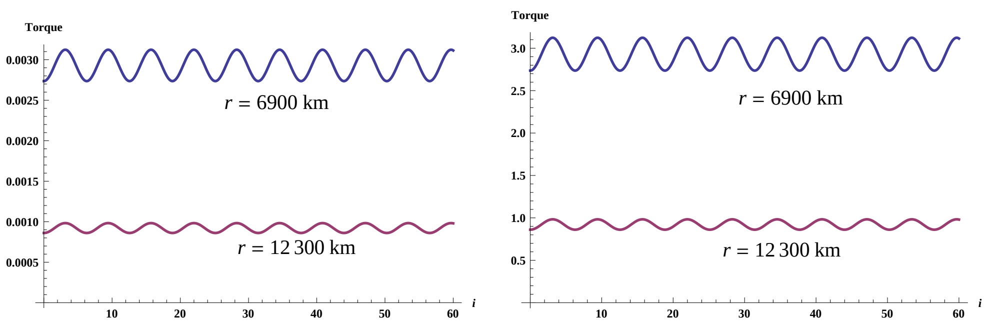

Fig. 4 $\|\boldsymbol{T}_{\rm L}(q, u, i, r)\|$ for a fixed value of charge $q = 0.01$ C (*left*), $q = 10$ C (*right*) and $z_0 = 1$ in the case of Equilibrium 2.

5.2 Equilibrium 2

In this equilibrium position, the attitude motion of the satellite is only in the roll direction. In this case, $\|\boldsymbol{\rho}_0(z_0)\|$ is only a linear function of z_0. It has a value of $2.47z_0$. Torque is only a function of inclination i, charge q and r.

$$\|\boldsymbol{T}_{\rm L}(q,i,r)\| = 1.27 \times 10^{16}\frac{z_0}{r^2}\left|q(0.0011 - 0.0000727\cos i)\right|. \tag{33}$$

In the same way as in Equilibrium 1, it is directly proportional to q and inversely proportional to r^2. However, unlike Equilibrium 1, torque in this case is a periodic function of the inclination i for fixed values of q and r^2. For fixed values of charge $q = 0.01$ C, or $q = 10$ C, $z_0 = 1$, and $r = 6900$ km or $r = 12\,300$ km the optimum values of torque change periodically. To see the periodic behavior of the torque and a comparison of the torque for two different values of r, see Figure 4. From the comparison for $r = 6900$ km and $r = 12\,300$ km we can see that the value of the Lorentz torque is higher in LEO. When charge is increased from 0.01 C to 10 C the magnitude of Lorentz torque significantly increases. This means that electrostatic charge can be used as some type of control if desired, as can be seen in Figure 4.

5.3 Equilibrium 3

In this case, $\|\boldsymbol{\rho}_0\|$ is a linear function of z_0. It has a value of $1.42138z_0$. The torque in this case is zero. The attitude motion of the satellite is in the pitch direction and the electrostatic charge of the screen surface is almost constant, which makes the components of the Lorentz torque zero.

5.4 Equilibrium 4

This position is a special case that can only happen when the orbital system coincides with the principal axis of inertia, which is rigidly fixed to the satellite. For Equilibrium 4, as described in Section 4, $\|\boldsymbol{\rho}_0\|$ and $\|\boldsymbol{T}_{\rm L}\|$ are determined in the same way as in the case of Equilibrium 1.

$$\begin{aligned}
\|\boldsymbol{\rho}_0(u,i,z_0)\| &= z_0\sqrt{1 + \frac{2.98(0.11 - 0.727\cos i)^2}{D_{\rm eq4}} + \frac{1.57(\cos u \sin i)^2}{D_{\rm eq4}}}, \\
D_{\rm eq4} &= (42.8 - 2.83\cos i - 1.37\cos u \sin i)^2 .
\end{aligned} \tag{34}$$

$$\begin{aligned}
\|\boldsymbol{T}_{\rm L}(q,u,i,z_0,r)\| &= \frac{qz_0 \times 10^{11}}{r^2} \\
&\times \sqrt{\begin{array}{l}\left[19 - 1.25(\cos i + \cos u \sin i)\right]^2 + \left[19 - 1.25(\cos i - \cos u \sin i)\right]^2 \\ +\left(3.6 - 47.6\cos i + 1.57\cos^2 i + 1.57\cos^2 u \sin^2 i\right)^2 \frac{1}{D_{\rm eq4}}\end{array}}.
\end{aligned} \tag{35}$$

It can be seen from Equation (34) that $\|\boldsymbol{\rho}_0(u,i,z_0)\|$ is independent of r even though its components depend on it. Equation (35) gives the magnitude of torque. $\|\boldsymbol{\rho}_0(u,i,z_0)\|$ is an almost periodic function of inclination i and latitude u with a maximum value of 1.057 m and minimum value of 1.04611 m for $z_0 = 0.96$. Since the function is almost periodic, these optimum values occur at various values of i and u. For example, the maximum occurs at $(i,u) = (61.33, 34.56)$.

Similarly the minimum occurs at $(i,u) = (58.05, 53.40)$. For some other occurrences of the optimum values, see Figure 5. It can be seen from both Equation (34) and Figure 5 that z_0 can be used to control ρ_0. In a similar way, z_0 can be used to control torque as can be seen in Equation (35). The relationship of torque with r and q is straightforward. It can be seen from Equation (35) that the torque for Equilibrium 4 is directly proportional to q and inversely proportional to r^2. Therefore, q

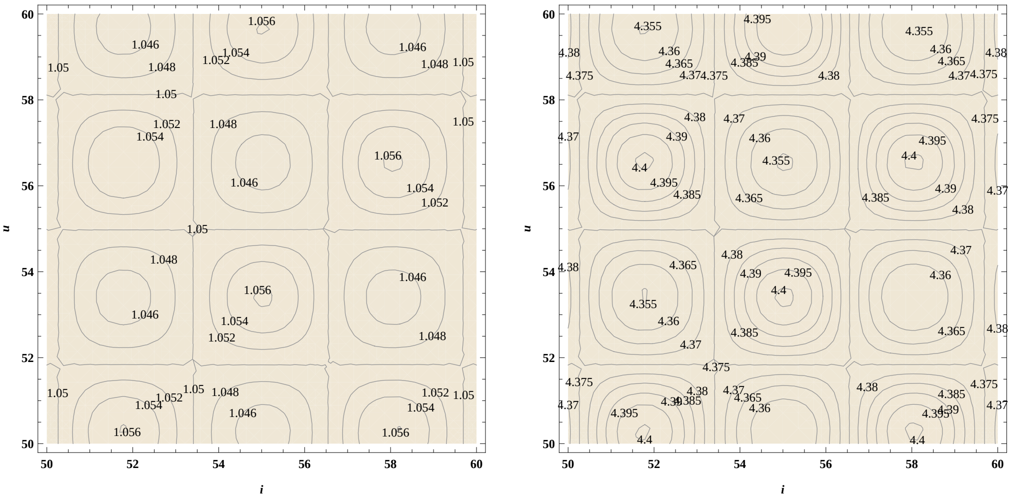

Fig. 5 Same as Fig. 2 except for Equilibrium 4.

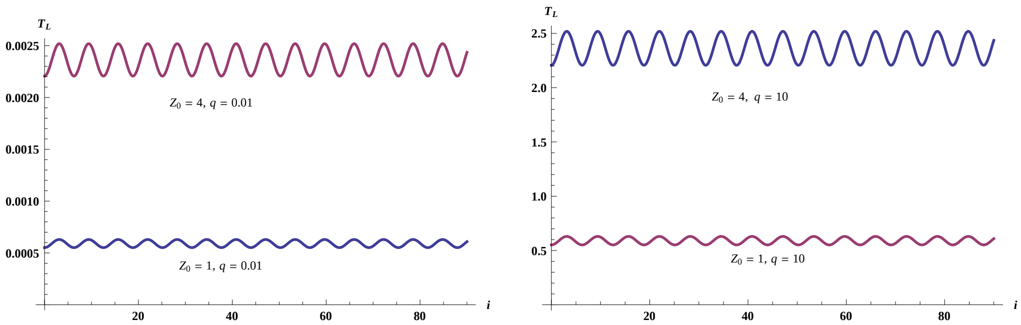

Fig. 6 $\|\boldsymbol{T}_{\mathrm{L}}(q, u, i, r)\|$ for fixed value of altitude ($r = 6900$ km), latitude ($u = 20$) and two different values of $z_0 = 1, z_0 = 4$, and $q = 0.01$ C, $q = 10$ C in the case of Equilibrium 4.

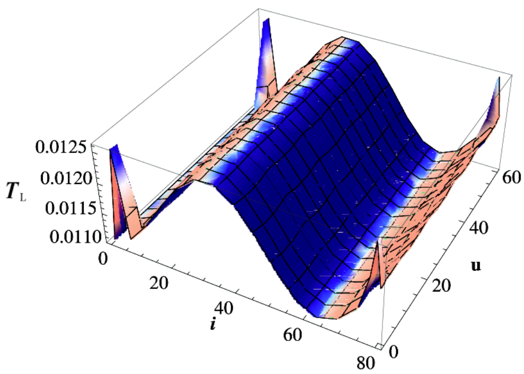

Fig. 7 $\|\boldsymbol{T}_{\mathrm{L}}(q, u, i, r)\|$ for fixed values of $q = 0.01$ C, $z_0 = 2$ and $r = 6900$ km in the case of Equilibrium 4.

and z_0 can be used to control torque if desired. To completely describe the torque, its representative graph is given in Figure 6. In the same way as in Equilibrium 2, when the charge is increased from 0.01 C to 10 C, the magnitude of Lorentz torque significantly increases. This means that the electrostatic charge can be used as some type of control if desired, which can be seen in Figure 6. It can also be seen from Figure 7, which is given for fixed values of $q = 0.01\,\mathrm{C}$, $z_0 = 2$ and $r = 6900$ km, that torque has a maximum value of the order 10^{-2} for each value of inclination i.

6 CONCLUSIONS

To control the attitude of a charged satellite with a general shape, we have proposed utilizing a Lorentz torque with a gravity gradient torque. The effect of the Lorentz torque on the attitude dynamics and the orientation of the equilibrium positions is discussed. The satellite is assumed to move in a circular orbit in the geomagnetic field. For this particular setup, we derived four equilibrium positions. The attitude motion for these equilibrium positions is analyzed in detail for different values of charge (q), magnitude of the radius vector of the center of charge for the satellite relative to its center of mass (ρ_0), inclination and latitude. The numerical results confirm that the Lorentz torque has a significant effect on the attitude orientation of the satellite for any inclination, especially in highly inclined orbits.

In the case of Equilibria 1, 2 and 4, it is shown that the value of charge q can control the magnitude of the Lorentz torque. We can choose the optimal torque to create a natural force which can be used to control the attitude of the satellite. In case of Equilibrium 1, a very high amount of charge is needed to generate a reasonable amount of torque. That is, a $1000\,\mathrm{C}$ charge is needed to generate a Lorentz torque of the order 10^{-13}. On the other hand, in the case of Equilibria 2 and 4, a charge of $0.01\,\mathrm{C}$ will generate a torque of the order 10^{-3}. This means that the use of charge as a control is a more realistic option in Equilibria 2 and 4. This also means that the Lorentz force can be used to control the satellite without consuming too much propellant. The installation of such a control on a satellite is dependent on the size of the surfaces of the satellite and how the screen is charged, which can be implemented by manufacturing a system of electrodes simulating the controlled electrostatic layer. Such a kind of control may be used instead of a magnetic control system because it is easy to constrain the mass of the satellite and decrease the cost.

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