The peculiar velocity and temperature profile of galaxy clusters

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Abstract Dynamical parameters like average velocity dispersion and temperature profile of galaxy clusters are determined using the theory of quasi-equilibrium thermodynamics. The calculated results of velocity dispersion show a good agreement between theory and simulations with the results of velocity dispersion from Abdullah et al. An adaptive mesh refinement grid-based hybrid code has been used to carry out the simulations. Our results indicate that the average velocity dispersion profile of 20 Abell galaxy clusters falls in the range of $500 - 1000 \, \text{km s}^{-1}$ and their temperature profile is of the order of $10^7$ to $10^8 \, \text{K}$ calculated on the basis of kinetic theory. The data in the plot show a significant contribution from gravitating particles clustering together in the vicinity of the cluster center and beyond a certain region this velocity dies out and becomes dominated by the Hubble flow due to which all the galaxy clusters in an expanding universe participate in Hubble expansion.

Key words: galaxies: clusters — gravitation — simulations

1 INTRODUCTION

In recent years, developments in the study of large scale structure formation in an expanding universe have become one of the exciting problems in cosmology. The auto-correlation functions, rapid computer simulations, theoretical analysis and observational evidence have added considerable knowledge to this field. Earlier studies on the applicability of statistical mechanics and thermodynamics to the cosmological many-body problem have yielded many interesting results and have also generated many related questions (Saslaw & Hamilton 1984; Saslaw & Fang 1996; Ahmad et al. 2002; Iqbal et al. 2006, 2011, 2012). The timescale for cluster formation is much longer than the timescale for a galaxy to cross the cluster. Although a cluster is still evolving, the galaxies in the cluster form a quasi-equilibrium distribution. Such distributions and evolution are governed by gravitational instability (Saslaw 2000). One of the interesting unknowns in the study of large-scale structure formation in an expanding universe is peculiar motions which can be studied by using galaxies or a cluster of galaxies. These motions provide an important tool for probing the peculiar velocity field. The peculiar velocity of galaxy clusters which represents their departures from the global cosmic expansion provides significant information about the related details of the cosmological many-body problem and the existence of a dark matter (DM) component. The use of the virial theorem $\langle 2T \rangle + \langle W \rangle = 0$ relating the time averages of kinetic and potential energies is even studied for the effects of configuration on the peculiar motion of galaxy clusters. Some studies have already been carried out to study the spatial configuration of galaxy clusters on the basis of the virial theorem (Saslaw 1987, 2000; Baryshev et al. 2001). The first peculiar velocity of galaxy clusters was reported by Gudehus
In this work, we study the peculiar velocity of galaxy clusters by using the kinetic equation of state and assuming that the system is virialized in which the growth of peculiar motions of various gravitating particles leads to the study of cluster formation. We describe N-body simulations using the ENZO 2.1 hydrodynamical code (O'Shea et al. 2005) which is an adaptive mesh refinement (AMR) code. The peculiar velocity results generated from the simulations are analyzed in detail to compare with the results obtained from theoretical calculations based on the quasi-equilibrium thermodynamic approach. More confirmations in this context will demonstrate the overall match between results of velocity dispersion generated from kinetic theory and simulations using the results of the velocity dispersion profile of Abdullah et al. (2011).

It is also interesting to examine the temperature profile of galaxy clusters on the basis of the same theory with an assumption that galaxy clusters are constituents of hot gas particles which contribute to the overall temperature of a galaxy cluster. We also calibrate intracluster temperature for a group of Abell galaxy clusters. The overall goal of this work is to come up with a simple, fundamental approach for studying various dynamical parameters like peculiar motion and temperature for 20 Abell galaxy clusters in an expanding universe.

In Section 2 we study the basic theoretical formalism of peculiar velocities followed by N-body simulations. Section 3 describes the intracluster temperature. Finally in Section 4, we discuss the results.

2 PECULIAR MOTION OF GALAXY CLUSTERS

Peculiar motion studies show that there is significant gas motion in galaxy clusters with velocities up to \(500 - 1000 \text{ km s}^{-1}\). The description of the peculiar velocity field of a cluster is an important step towards understanding of the nature of the forces acting on a cluster of galaxies. This peculiar velocity field can be studied by using galaxy clusters (Bahcall et al. 1994; Lauer & Postman 1994; Bahcall & Oh 1996; Moscardini et al. 1996; Borgani et al. 1997, 2000; Watkins 1997; Dale et al. 1999; Abdullah et al. 2011). The observational studies (Rines & Diaferio 2006), N-body simulations (Wojtak et al. 2005; Mamon et al. 2004; Cuesta et al. 2008) and a combination of both (Mahajan et al. 2011) have shown that virialized clusters in gravitating systems are surrounded by infall zones from which most of the galaxies move into the relaxed cluster, as is evident from the earlier result of (Gunn & Gott 1972). In order to understand the simple formalism of a peculiar velocity, we make use of quasi-equilibrium thermodynamics in which quasi-equilibrium evolution takes place through a sequence of quasi-equilibrium states whose properties change with time (Saslaw & Hamilton 1984; Saslaw et al. 1990; Ahmad et al. 2002; Iqbal et al. 2006, 2012).

To determine the peculiar motion of galaxy clusters with a fixed number of galaxies in a spherical volume of cluster size \(R\), the average kinetic energy of peculiar motions of \(N\) galaxies in a cluster can reasonably be connected to the average kinetic temperature \(T\) of a cluster (Saslaw 1986) defined as

\[
K = \frac{3}{2} NT = \frac{1}{2} M V^2.
\]

Here \(M\) is the mass of a cluster containing \(N\) galaxies and \(V\) represents the average velocity dispersion of a galaxy cluster. The average peculiar velocity dispersion of a sample of galaxy clusters can be determined by approximating the dynamical timescale \(t_{\text{dyn}}\) and crossing timescale \(t_c\) of a relaxed cluster of galaxies. The condition \(t_{\text{dyn}} \approx t_c\) is valid for a system to be in a state of quasi-equilibrium where the system can be described as virialized and the required condition for a cluster to be in virial equilibrium is that the macroscopic evolution of a system in quasi-equilibrium is slow compared with the crossing time of the system, so that the condition of equilibrium is approximately
satisfied. The description of the velocity dispersion can be understood in terms of its dependence on
the respective timescales and a suitable choice for the time unit is a representation of the dynamical
scale for the cluster (Yang & Saslaw 2011)

\[ t_{\text{dyn}} = \frac{1}{\sqrt{G \rho}} = \sqrt{\frac{R^3}{GM}}, \]

(2)

where \( \rho \) is the density of a given cluster, \( R \) the size of a cluster and \( G \) the gravitational constant.

\[ V_{\text{pec}} = \sqrt{\frac{3GM}{\pi R}}. \]

(3)

This is the average velocity dispersion of a cluster.

Table 1 shows the velocity dispersion profile of a sample of 20 Abell galaxy clusters using
SDSS-DR7 at virial radius \( 1 - 2 \ h^{-1} \ Mpc \) (Abdullah et al. 2011). The calculations shown in Column
5 are results of the average peculiar velocity dispersion calculated from Equation (3). The two sets
of results shown in Columns 4 and 5 are in good agreement.

<table>
<thead>
<tr>
<th>Name</th>
<th>( R ) (1 - 2 ( h^{-1} ) Mpc)</th>
<th>Cluster Mass ( (10^{14} M_\odot) )</th>
<th>( V_{\text{pec}} ) (km s(^{-1})) (Abdullah et al. 2011)</th>
<th>( V_{\text{pec}} ) (km s(^{-1}))</th>
</tr>
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### 2.1 The Simulations

Here, we describe \( N \) body simulations for understanding the peculiar motions of galaxy clusters
in an expanding universe. The \( N \) body simulations have been carried out by using the ENZO 2.1
hydrodynamical code (O’Shea et al. 2005). ENZO 2.1 is an AMR grid-based hybrid (\( N \)-body plus
hydrodynamical) code. We have used a flat Lambda cold dark matter (LCDM) background cos-
mology with parameters defined by \( \Omega_m = 0.27, \Omega_L = 0.73, \Omega_b = 0.0459, n_s = 0.962, \)
\( H_0 = 70.2 \text{ km s}^{-1} \text{ Mpc}^{-1} \) and \( \sigma_8 = 0.817 \) derived from WMAP5 (Komatsu et al. 2009). The
simulations have been initialized at redshift \( z = 60 \) using the Eisenstein & Hu (1999) transfer func-
tion and evolved up to \( z = 0 \). An ideal equation of state was used for the gas with \( \gamma = \frac{5}{3} \). Radiative
cooling was assumed from Sarazin & White (1987) for a fully ionized gas with metallicity of 0.5 times the solar value.

The simulation was performed in a box of comoving volume \((64 h^{-1} \text{ Mpc})^3\) containing \(64^3\) particles, each having \(9.05 \times 10^{10} M_{\odot}\), with three levels of AMR. This gives the highest effective resolution of \(128 \text{kpc}\). The halo centers have been identified using the enzohop algorithm (Eisenstein & Hut 1998). Considering a particular center obtained from enzohop, we calculate the density within a spheroidal shell. The shell radius is increased until it reaches the virial radius. The virial radius is defined as that radius where the average density of the spheroidal shell equals 200 times the critical density. We calculate the peculiar velocity as the root mean square velocity of the galaxy clusters inside the virial radius. For estimating the cosmic variance, the analysis was carried out for 10 different independent realizations of the simulation.

For comparison, the variation of velocity dispersion with virial cluster radius for three different data sets shown in Figure 1 clearly indicates that the theory and the simulation results are in good agreement with Abdullah et al. (2011).

### 3 KINETIC CLUSTER TEMPERATURE

The kinetic temperature of a cluster of galaxies is defined in terms of the average kinetic energy of its constituent particles (galaxies) and can be described by a system in quasi-thermodynamic equilibrium. It has been known for years that baryons in a DM dominated potential will heat up to a temperature that is determined by the properties of the DM mass profile (Rees & Ostriker 1977; Cavaliere & Fusco-Femiano 1978, 1981; Sarazin 1986; Cavaliere et al. 2009). In hydrostatic equilibrium, the total mass within a given radius is proportional to the local temperature at that radius. Thus it is imperative to measure the cluster temperature at large radii to a good precision. The largest samples to date used to measure temperature profiles have been with ASCA (Markevitch et al. 1998), BeppoSAX (De Grandi & Molendi 2002), XMM-Newton (Pointecouteau et al. 2005) and Chandra (Vikhlinin et al. 2005) data. The deep observational study, which used the Swift X-ray telescope that is able to measure the intercluster medium profiles in the region external to the virialized radii, has shown the best results compared with previous measurements (Moretti et al. 2011).
Intracluster gas, the hottest plasma in thermal equilibrium, has been heated to a temperature of tens of millions of degrees which causes the emission from this gas to be in the X-ray region of the electromagnetic spectrum (Böhringer & Werner 2010). Here our velocity dispersion profiles allow us to estimate the temperature of a galaxy cluster with an assumption that a cluster of galaxies is homologous and intracluster gas is in global equilibrium with the DM. In this case, where this gas behaves as an ideal gas, the temperature of the hot gas is described by the virial condition

$$\frac{3}{2} N k_B T_{cl} = \frac{1}{2} \sum_{i=1}^{N} m_i V_i^2$$

(4)

where \(m_i\) are masses of constituent particles and \(k_B\) is the Boltzmann constant. The calculation of cluster gas temperature at a location is given by the local velocity distribution. Here this velocity is of the order of the average velocity dispersion of the cluster. Considering this velocity as the order of magnitude of a mean velocity \((V_{\text{mean}})\) of the hot gas molecules, the above equation can be written as

$$\frac{3}{2} N k_B T_{cl} = \frac{1}{2} m V_{\text{mean}}^2$$

(5)

where \(m\) is the mean molecular weight which is the approximate atomic weight for a hydrogen atom. Here, we calculate the cluster temperature from the above equation using peculiar velocity values shown in Column 5 in Table 1. The values of \(m\) and \(V\) predict temperature in the range of millions of degrees, so we expect the cluster to be filled with an X-ray emitting plasma.

4 DISCUSSION

In this work, we present a quasi-equilibrium thermodynamic approach for studying the velocity dispersion profile and temperature profile of galaxy clusters in an expanding universe. The peculiar velocity field provides a basic understanding of the clustering rate of galaxies. The calculated velocity dispersion results obtained from Equation (3) fall in the range of 500 – 1000 km s\(^{-1}\) for 20 rich Abell galaxy clusters and therefore provide a good approximation such that the virialized systems of galaxy clusters can be well defined by the equation of state. An attempt has been made to carry out N-body simulations for the calculation of a peculiar velocity field by using the ENZO 2.1 hydrodynamical code. The simulation results and peculiar velocity results obtained by the quasi-equilibrium approach show a good agreement with Abdullah et al. (2011). The growth of clustering on the basis of the peculiar velocity field is illustrated in Figure 1. This is a plot of average velocity dispersion for each sample with a galaxy cluster versus cluster size. For smaller values of cluster size, the number density is large which corresponds to a higher rate of clustering and with the increase in size from the cluster center to the outer regions, the clustering rate weakens due to the global Hubble flow becoming dominant. The temperature profile of 20 Abell galaxy clusters using the kinetic equation is the same order of magnitude as hot X-ray gas in the clusters.

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References
