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New upper limit on the cosmological constant from solar system dynamics *

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Abstract The cosmological constant Λ is the simplest model for explaining the dark energy which supposedly drives the observed accelerated expansion rate of the Universe. Together with the concept of cold dark matter, it satisfactorily accommodates a wealth of observations related to cosmology. Due to its assumed constancy throughout the Universe, Λ might also affect the dynamics of the planets in the solar system, although with extremely small effects. However, modern high-precision ephemerides provide a promising tool for constraining it. Using the supplementary advances in the perihelia provided by current INPOP10a and EPM2011 ephemerides, we obtain a new upper limit on Λ in the solar system when the Lense-Thirring effect due to the Sun's angular momentum and the uncertainty of the Sun's quadrupole moment are properly taken into account. These two factors were mostly absent in previous works dealing with Λ. We find that INPOP10a yields an upper limit of $\Lambda = (0.26 \pm 1.45) \times 10^{-43}\ \mathrm{m}^{-2}$ and EPM2011 gives $\Lambda = (-0.44 \pm 8.93) \times 10^{-43}\ \mathrm{m}^{-2}$. Such bounds are about 10 times less than previously estimated results.

Key words: cosmology: theory — celestial mechanics — ephemerides — relativity

1 INTRODUCTION

Observations of Type Ia supernovae (e.g. Riess et al. 1998; Perlmutter et al. 1999) indicate we live in a Universe that is undergoing an accelerated expansion. A possible way to explain this behavior is to introduce the presence of repulsive dark energy (for a recent review see Amendola & Tsujikawa 2010, and references therein). Among various candidates of dark energy, the simplest one is the cosmological constant Λ (see Weinberg 1989; Carroll et al. 1992; Carroll 2001; Peebles & Ratra 2003; Padmanabhan 2003, for reviews). Measurements of the cosmic microwave background radiation (e.g. Bennett et al. 2013; Planck Collaboration 2013) imply the ratio between the energy density of Λ and the critical density of the Universe has a value of $\Omega_\Lambda \approx 0.7$, which means $\Lambda \sim 10^{-52}\ \mathrm{m}^{-2}$. The concordance model (Peebles 1993) containing Λ and cold dark matter accurately coincides with the current observations of the Universe. However, as a constant intrinsically associated with spacetime, Λ exists everywhere, even at a scale much smaller than cosmological scales, although its influence will be extremely small, which suggests that Λ might also be constrained and detected *locally* with highly accurate measurements. Local bounds on cold dark matter have also been obtained

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from planetary motions (e.g. Anderson et al. 1995; Iorio 2006b; Khriplovich & Pitjeva 2006; Iorio 2010a,b, 2013; Pitjeva & Pitjev 2013), but this issue is beyond the scope of this work.

The effects of Λ in the solar system have been studied and it is found that these effects can cause *additional* precession in the orbit of a planet $\dot{\omega}_\Lambda$ (e.g. Kurmakaev 1972; Cardona & Tejeiro 1998; Rindler 2001; Kerr et al. 2003; Iorio 2006a; Jetzer & Sereno 2006; Kagramanova et al. 2006; Adkins et al. 2007; Iorio 2008; Arakida 2013; Xie & Deng 2013). Cardona & Tejeiro (1998) estimated its bound as $|\Lambda| \leq 10^{-39}$ m^{-2} by using measurements of Mercury's perihelion shift (Will 1993). Kagramanova et al. (2006) reduced this by two orders of magnitude yielding $|\Lambda| \leq 10^{-41}$ m^{-2} with the updated perihelion shift of Mercury (Nordtvedt 2000). Iorio (2006a) and Jetzer & Sereno (2006) found $|\Lambda| \lesssim 10^{-42}$ m^{-2} with ephemeris EPM2004 (Pitjeva 2005). Inspired by these works, we will try to find a new upper limit on Λ by making use of the supplementary advances in the perihelia provided by modern high-precision ephemerides: INPOP10a (IMCCE, France) (Fienga et al. 2011) and EPM2011 (IAA RAS, Russia) (Pitjeva 2013). It is worth mentioning that Λ has not been modeled in the software associated with these two ephemerides. The latest version of the INPOP ephemerides is INPOP13a (Verma et al. 2014), although no supplementary perihelion precessions have been determined with it. These two ephemerides were recently used in detecting gravitational effects and testing gravitational theories (e.g. Iorio & Saridakis 2012; Xie & Deng 2013; Iorio 2014a,b; Li et al. 2014; Deng & Xie 2013). Since INPOP10a and EPM2011 are significantly improved compared with previous compilations, we expect to obtain a tighter upper limit. In our investigation, we will include the effects caused by the Sun's angular momentum and the uncertainty of the Sun's quadrupole moment, which were absent in previous investigations.

In Section 2, we will connect predicted $\dot{\omega}_\Lambda$ with the data provided by ephemerides. In Section 3, the supplementary advances in the perihelia provided by INPOP10a and EPM2011 will be used to estimate an upper limit on Λ when the effects due to the Sun's angular momentum and the uncertainty in the Sun's quadrupole moment are taken into account. Conclusions and discussion will be presented in Section 4.

2 CONFRONTATION OF $\dot{\omega}_\Lambda$ WITH DATA

With the existence of Λ, the spacetime of a spherically symmetrical mass, such as the Sun, can be described by the Schwarzschild–de Sitter metric (Rindler 2001). This can cause an additional secular precession in the orbit of a planet, which is (see Kurmakaev 1972; Cardona & Tejeiro 1998; Rindler 2001; Kerr et al. 2003; Kagramanova et al. 2006, for details of derivation)

$$\dot{\omega}_\Lambda = \frac{\sqrt{1-e^2}}{2n}\Lambda\, c^2\,, \tag{1}$$

where e is the eccentricity, n is the Keplerian mean motion and c is the speed of light. It is undoubted that $\dot{\omega}_\Lambda$ is very small so that it is difficult to detect, as some previous works claimed. However, it is closely connected with the supplementary advances in the perihelia $\dot{\omega}_{\rm sup}$ provided by modern ephemerides, such as INPOP10a (Fienga et al. 2010, 2011) and EPM2011 (Pitjeva 2013; Pitjeva & Pitjev 2013; Pitjev & Pitjeva 2013).

INPOP10a and EPM2011 were obtained by fitting the "standard model" of dynamics to observational data, where "standard model" means Newton's law of gravity and Einstein's general relativity (GR) (apart from the Lense-Thirring effect, see below for details). Therefore, the effects of Λ were *neither* modeled in INPOP10a *nor* in EPM2011, and Λ was not determined in these least-squares fittings. In fact, it is *not* trivial to incorporate celestial mechanics with the background of the (accelerating) expanding Universe (see Kopeikin 2012; Kopeikin & Petrov 2013, and references therein). In this sense, the results we obtain in the next section may not be considered to be genuine "constraints" (they would be so if one solved for them in a covariance analysis by reanalyzing the data with modified software including these effects), but rather as preliminary indications of an acceptable upper limit computed by the best contemporary knowledge in the field of ephemerides (see Iorio 2014a, for a further discussion).

These $\dot{\omega}_{\rm sup}$ might represent possibly mismodeled and unmodeled parts of perihelion advances according to Newton's law of gravity and Einstein's GR. They are almost all compatible with zero so that they can be used to draw upper bounds on quantities parameterizing unmodeled effects like Λ in this case. Nonetheless, the latest results by EPM2011 (Pitjeva & Pitjev 2013; Pitjev & Pitjeva 2013) returned non-zero values for Venus and Jupiter. Although the level of their statistical significance is not too high and further investigations are required, we still take them into account in this work. In the recent past, a non-zero extra effect on Saturn's perihelion was studied (Iorio 2009). Also, about the non-zero values of the supplementary precessions of Venus and Jupiter by EPM2011 (Pitjeva & Pitjev 2013; Pitjev & Pitjeva 2013), their ratios have recently been used to test a potential deviation from GR (Iorio 2014b).

In the construction of $\dot{\omega}_{\rm sup}$ (see Fienga et al. 2010, for details), the effects caused by the Sun's quadrupole mass moment $J_2^\odot$ are considered and isolated in the final results, but the perihelion shifts caused by the Lense-Thirring effect (Lense & Thirring 1918) due to the Sun's angular momentum $S_\odot$ are absent. Therefore, the entire relation between $\dot{\omega}_\Lambda$ and $\dot{\omega}_{\rm sup}$ is

$$\dot{\omega}_{\rm sup} = \dot{\omega}_\Lambda + \dot{\omega}_{\rm LT} + \dot{\omega}_{\delta J_2^\odot} \,. \tag{2}$$

Here, the Lense-Thirring term $\dot{\omega}_{\rm LT}$ is

$$\dot{\omega}_{\rm LT} = -\frac{6GS_\odot \cos i}{c^2 a^3 (1-e^2)^{3/2}} \,, \tag{3}$$

where c is the speed of light, $S_\odot = 1.9 \times 10^{41}$ kg m^2 s^{-1} (Pijpers 2003), a is the semi-major axis and i is the inclination of the planetary orbit with respect to the equator of the Sun. The uncertainty in $S_\odot$ is currently about 1% (Pijpers 2003). This effect of the Sun on planetary motion has been studied in several works (e.g. Iorio 2005b; Iorio et al. 2011; Iorio 2012a). Equation (3) only holds in a coordinate system whose z axis is aligned with the Sun's angular momentum. A general formula for an arbitrary orientation can be found in Iorio (2011, 2012b). It is useful in extrasolar planets and black holes, for which the orientation of the spin axis is generally unknown.

We add the third term in Equation (2) to include the uncertainty in the Sun's quadrupole moment $\delta J_2^\odot$ (Iorio 2005a), which is currently about $\pm 10\%$ of $J_2^\odot$ (Damiani et al. 2011; Pireaux & Rozelot 2003; Rozelot et al. 2004; Rozelot & Damiani 2011; Rozelot & Fazel 2013). The Sun's quadrupole moment in INPOP10a is fitted to observations with $J_2^\odot = (2.40 \pm 0.25) \times 10^{-7}$ (Fienga et al. 2011) and its value in EPM2011 is $J_2^\odot = (2.0 \pm 0.2) \times 10^{-7}$ (Pitjeva & Pitjev 2013). This uncertainty in $J_2^\odot$ can cause an extra precession for a planet as described by Kozai (1959)

$$\dot{\omega}_{\delta J_2^\odot} = \frac{3}{2} \frac{\delta J_2^\odot R_\odot^2}{p^2} n \left(2 - \frac{5}{2} \sin^2 i \right) , \tag{4}$$

where $R_\odot$ is the Sun's radius and $p = a(1-e)$. The multipoles of higher order like $J_4^\odot$ have a negligible impact on the perihelion precessions (see Renzetti 2013, for a recent calculation of the $J_4^\odot$ precessions).

3 UPPER LIMITS ON Λ

The INPOP10a (Fienga et al. 2011) ephemeris provides $\dot{\omega}_{\rm sup}$ for some planets in the solar system: Mercury, Venus, Earth-Moon Barycenter (EMB), Mars, Jupiter and Saturn. Similarly, EPM2011 (Pitjeva 2013) also gives those values of the planets from Mercury to Saturn. These numbers are taken from table 5 in Fienga et al. (2011) and tables 4 and 5 in Pitjeva & Pitjev (2013) and Pitjev & Pitjeva (2013) respectively (see Table 1 for details). It can be found that values of $\dot{\omega}_{\rm sup}$ for Mercury

Table 1 Supplementary Advances in the Perihelia $\dot{\omega}_{\rm sup}$ Given for INPOP10a and EPM2011

	$\dot{\omega}_{\rm sup}$ (mas cy^{-1})	
	INPOP10a [a]	EPM2011 [b]
Mercury	0.4 ± 0.6	-2.0 ± 3.0
Venus	0.2 ± 1.5	2.6 ± 1.6
EMB	-0.2 ± 0.9	–
Earth	–	0.19 ± 0.19
Mars	-0.04 ± 0.15	-0.020 ± 0.037
Jupiter	-41 ± 42	58.7 ± 28.3
Saturn	0.15 ± 0.65	-0.32 ± 0.47

Notes: [a] Taken from table 5 in Fienga et al. (2011). [b] Provided by table 4 in Pitjeva & Pitjev (2013) and table 5 in Pitjev & Pitjeva (2013).

Table 2 Summary of Λ and $\delta J_2^{\odot}$ Estimated by $\dot{\omega}_{\rm sup}$

	Λ (10^{-43} m^{-2})	$\delta J_2^{\odot}/J_2^{\odot}$ (%)	Adopted Data / Ephemeris
Cardona & Tejeiro (1998)	$\|\Lambda\| \leq 10^4$	–	$\dot{\omega}_{\rm sup}^{\rm Mercury}$ = 100 mas cy^{-1} (Will 1993)
Kagramanova et al. (2006)	$\|\Lambda\| \leq 10^2$	–	$\dot{\omega}_{\rm sup}^{\rm Mercury}$ = 0.43 mas cy^{-1} (Nordtvedt 2000)
Iorio (2006a)/Jetzer & Sereno (2006)	$\|\Lambda\| \lesssim 10$	–	EPM2004 (Pitjeva 2005)
This work [a]	0.26 ± 1.45	5.6 ± 1.0	INPOP10a (Fienga et al. 2011)
	-0.44 ± 8.93	8.2 ± 4.8	EPM2011 (Pitjeva & Pitjev 2013)

Notes: [a] The results are obtained according to all the planets in Table 1.

Table 3 Summary of Λ (10^{-43} m^{-2}) Given for Individual Planets

INPOP10a	Λ [a]	Λ [b]	EPM2011	Λ [a]	Λ [c]
Mercury	988	0.50	Mercury	295	-903
Venus	70	31	Venus	336	288
EMB	-2.5	-10	Earth	24	15
Mars	0.25	-0.70	Mars	0.98	-0.18
Jupiter	-235	-235	Jupiter	337	337
Saturn	0.35	0.35	Saturn	-0.74	-0.74

Notes: [a] The uncertainty in $J_2^{\odot}$ is not included in the estimation. [b] $\delta J_2^{\odot}/J_2^{\odot}$ is taken into account and the value is fixed at 5.6% (see Table 2). [c] $\delta J_2^{\odot}/J_2^{\odot}$ is included and it is taken as 8.2% (see Table 2).

and Venus from EPM2011 are considerably larger than those from INPOP10a, while Venus and Jupiter have non-zero values of $\dot{\omega}_{\rm sup}$ in EPM2011.

By using the method of weighted least squares, we simultaneously estimate an upper limit on Λ and $\delta J_2^{\odot}/J_2^{\odot}$ with *all* the planets in Table 1. We find that (i) INPOP10a yields an upper limit of $\Lambda = (0.26 \pm 1.45) \times 10^{-43}$ m^{-2} and $\delta J_2^{\odot}/J_2^{\odot} = (5.6 \pm 1.0)\%$; and (ii) EPM2011 gives $\Lambda = (-0.44 \pm 8.93) \times 10^{-43}$ m^{-2} and $\delta J_2^{\odot}/J_2^{\odot} = (8.2 \pm 4.8)\%$. These results are summarized in Table 2. Although EPM2011 seems to favor a negative Λ, it might be positive in the range of uncertainties. For comparison, Table 3 lists estimated values of Λ given for individual planets in Table 1. It contains two cases: (1) where $\delta J_2^{\odot}/J_2^{\odot}$ is ignored; and (2) where $\delta J_2^{\odot}/J_2^{\odot}$ is included and has the values from Table 2. It clearly shows that the uncertainty in $J_2^{\odot}$ can barely affect the estimation made by the outer planets, such as Jupiter and Saturn, but it will significantly change the value of Λ estimated with the inner planets, such as Mercury. Saturn plays important roles by providing the tightest limits in both INPOP10a and EPM2011 when the uncertainty in the Sun's quadrupole moment is taken into account (see Table 3 for details).

Both INPOP10a and EPM2011 indicate an upper limit on the absolute value of Λ in the solar system is on the order of 10^{-43} m^{-2}. It is improved by at least 10 times compared to previous results (Cardona & Tejeiro 1998; Kagramanova et al. 2006; Iorio 2006a; Jetzer & Sereno 2006) (see Table 2). This is a natural outcome because values of $\dot{\omega}_{\rm sup}$ provided by INPOP10a (Fienga et al. 2011) and EPM2011 (Pitjeva & Pitjev 2013; Pitjev & Pitjeva 2013) are improved by at least one order of magnitude compared to those used previously. Furthermore, the values of $\delta J_2^{\odot}/J_2^{\odot}$ by INPOP10a and EPM2011 (see Table 2) are compatible with the current uncertainty of $\pm 10\%$.

4 CONCLUSIONS AND DISCUSSION

Using the supplementary advances in the perihelia provided by INPOP10a (Fienga et al. 2011) and EPM2011 (Pitjeva 2013) ephemerides, we estimate a new upper limit on the cosmological constant Λ for the planetary scale of the solar system. After taking the Lense-Thirring effect due to the Sun's angular momentum and the uncertainty in the Sun's quadrupole moment into account, we find that INPOP10a yields an upper limit of $\Lambda = (0.26 \pm 1.45) \times 10^{-43}$ m^{-2} and $\delta J_2^{\odot}/J_2^{\odot} = (5.6 \pm 1.0)\%$ and EPM2011 gives $\Lambda = (-0.44 \pm 8.93) \times 10^{-43}$ m^{-2} and $\delta J_2^{\odot}/J_2^{\odot} = (8.2 \pm 4.8)\%$. Their absolute values are improved by at least 10 times compared to previous results (Cardona & Tejeiro 1998; Kagramanova et al. 2006; Iorio 2006a) (see Table 2).

With tremendous advances in techniques for deep space exploration in the solar system, ephemerides are going to be increasingly improved by high-precision datasets provided from tracking spacecrafts and by sophisticated data analysis (e.g. Fienga et al. 2013; Verma et al. 2013, 2014). The resulting upper limits on the cosmological constant are expected to be tighter in the future.

It will also be necessary and important to make a similar analysis for Λ with other local systems by using proper observable quantities (e.g. radial velocities, timing, eclipsing times and so on). Like the investigation on the possibility of testing new physics in exoplanets using transit timing variations (Xie & Deng 2014), extrasolar planets (in particular, those with the widest orbits known so far) may serve as local test-beds for detecting Λ. Other local systems might be some wide compact binaries, hosting neutron stars and/or white dwarfs, and other binary systems, such as α Centauri AB.

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