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# The general class of Bianchi cosmological models with viscous fluid and particle creation in Brans-Dicke theory

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**Abstract** This paper deals with the general class of Bianchi cosmological models with bulk viscosity and particle creation described by full causal thermodynamics in Brans-Dicke theory. We discuss three types of average scale-factor solutions for the general class of Bianchi cosmological models by using a special law for the deceleration parameter which is linear in time with a negative slope. The exact solutions to the corresponding field equations are obtained in quadrature form and solutions to the Einstein field equations are obtained for three different physically viable cosmologies. All the physical parameters are calculated and discussed in each model.

Key words: cosmology: theory

## **1 INTRODUCTION**

Brans-Dicke theory, which is a generalization of general relativity, is more consistent with Mach's principle and less reliant on the absolute properties of space. In this theory, the gravitational effects are part geometrical and part due to a scalar interaction. There is a formal connection between this theory and one class of Jordan's theory (Jordan 1955, 1959; Brill 1962), but the interpretation is quite different. For example, Jordan's aspect of mass creation and non-conservation of energy and momentum is absent from this theory.

This new theory (called Brans-Dicke theory or BD theory) involves a violation of the strong principle of equivalence, but the weak principle of equivalence is satisfied. Thus we have a theory for which matter moves gravitationally on geodesics, and the locally measured value of the gravitational constant depends on a scalar field determined by the mass distribution of the universe.

If the gravitational "constant" is to vary, it should be a function of some scalar field variable. We postulate the existence of a new scalar field and the gravitational phenomenon is to be described by both this scalar field and the Riemannian metric tensor. The primary function of this field is the determination of the local value of the gravitational constant.

The possibility of Brans-Dicke theory with negative coupling constant  $\omega$  as a viable alternative to general relativity has been discussed by Smalley & Eby (1976). They have also shown that this type of theory fits the measurement of electromagnetic-signal deflection and of the time variation of the gravitational constant. The large negative value of  $\omega$  now seems to be consistent with the observations of small solar oblateness by Hill et al. (1974). The experimental results of the Robertson parameter  $\gamma$  and the time variation of the gravitational constant *G* also seem to demand such an alternative, even though the magnitude of  $\omega$  is large.

Among the various modifications of the general theory of relativity, the scalar tensor theory of Brans-Dicke is treated most seriously (Brans & Dicke 1961; Weinberg 1972). In this modified form of gravitational theory, a dynamical scalar field  $\phi$  is introduced, which corresponds to the variation in gravitational constant with respect to cosmic time, i.e.  $\phi \approx G^{-1}$ . The Brans-Dicke theory also contains a dimensionless free coupling parameter,  $\omega$ , between the scalar and tensor components of gravitation. It has already been pointed out in the literature that Brans-Dicke theory is consistent with the observations as long as  $\omega \geq 500$  (Will 1993). However, there are no *a priori* theoretical reasons for excluding other values of  $\omega$ . Adhav et al. (2007) observed that for a string Bianchi type-I metric of Kasner form, it is not possible to describe an anisotropic physical model of the universe in the Brans-Dicke scalar tensor theory of gravity with matter fluid satisfying the second law of thermodynamics and various energy conditions. The cosmological evolution of the Brans-Dicke field reduces the possible primordial anisotropy of the universe. Rathore & Mandawat (2009) obtained a five-dimensional Bianchi type-I string cosmological model in Brans-Dicke theory.

In the construction of a cosmological model, the assumption of homogeneity and isotropy of the universe are motivated by the cosmological principle and mathematical tractability of the resulting Friedmann-Robertson-Walker (FRW) models. Therefore, these symmetries can only be approximate. There are theoretical arguments (Chimento 2004; Misner 1968) and recent experimental data regarding cosmic background radiation anisotropies that support the existence of an anisotropic phase that approaches an isotropic one (Land & Magueijo 2005). These observations led us to consider more general anisotropic cosmologies, while retaining the assumption of large-scale spatial homogeneity. Wainwright & Ellis (1997) discussed how the spatially homogeneous and anisotropic cosmological models, which provide a richer structure, both geometrically and physically, than the FRW model, play a significant role in the description of early universe.

The spatially homogeneous cosmological models allow an extension of the cosmological investigations of distorted and rotating universes, giving estimates of the effects of anisotropy on primordial element production and on the measured isotropy in the spectrum of the cosmic microwave background radiation (Ellis & van Elst 1999). Aside from the observational reasons, there are various theoretical considerations that have motivated the study of anisotropic cosmologies. Among these are (i) some kind of singularity in our "past" is strongly indicated if certain reasonable conditions hold (Hawking & Ellis 1973). However, this could differ greatly from the type found in FRW models (Belinskij et al. 1970). (ii) The "Chaotic Cosmology" program of Misner (1968) sought a mechanism to explain why the observed isotropy and homogeneity should exist regardless of the initial conditions (MacCallum 1979; Ellis 1993; Kolb & Turner 1990).

A wide class of anisotropic cosmological models exist, and these are often studied in cosmology (Misner et al. 1973). There are theoretical arguments that sustain the existence of an anisotropic phase that approaches an isotropic case (Misner 1968) (Chaotic Cosmology), and anisotropic cosmological models are found to be a suitable candidate in avoiding the assumption of specific initial conditions in FRW models. The early universe could also be characterized by an irregular expansion mechanism. Therefore, it would be useful to explore cosmological models in which anisotropies, existing at an early stage of expansion, are damped out in the course of evolution. Such models have received interest (Hu & Parker 1978).

It is believed that the early universe was characterized by a highly irregular expansion mechanism, which isotropized later. The level of anisotropy left out by the era of decoupling is only about  $10^{-5}$ , as revealed by the cosmic microwave background (CMB) observations (Vishwakarma 2005). In many cosmological and astrophysical situations, an idealized fluid model of the matter is inappropriate. Dissipative effects, including both the bulk and shear viscosities, are supposed to play an important role in the early evolution of the universe. From a physical point of view, the inclusion of dissipative terms in the energy momentum tensor of the cosmological fluid seems to be the best motivated generalization of the matter term in the gravitational field equations. The dynamics of the universe may be governed by the creation of matter. Particle creation corresponds to an irreversible energy flow from the gravitational field to the created matter constituents. On a phenomenological level, particle production has also been described in terms of effective bulk viscosity coefficients.

Particle production processes in the early universe are supposed to have considerably influenced cosmological history. These processes are quantum in nature (Birrell & Davies 1982). Their back reaction on the cosmological dynamics, however, are frequently studied phenomenologically. As was observed by Zel'dovich (1970) and Murphy (1973) and later discussed by Hu (1982), Montani (2011) and Zimdahl et al. (1996), a non-vanishing particle production rate is equivalent to a bulk viscous pressure in fluid cosmology. It has been shown by Triginer et al. (1996) that the effective viscous pressure approach is compatible with kinetic theory in homogeneous spacetimes. Zel'dovich (1970) suggested that vacuum viscosity arises from the fluctuation (particle creation) and polarization (trace anomaly) of quantized fields interacting with dynamical spacetime. This differs from the classical viscosity and particle (matter) creation generates cosmological models with the same physical properties. So in the same sense, they are equivalent processes and hence have been considered by many authors (Hu 1982; Lima & Germano 1992; Belinskii et al. 1979).

Particle production that arises from the energy of the gravitational field (Prigogine et al. 1989; Barrow 1988) may also play an important role in Brans-Dicke theory when studying the behavior of the expanding universe (Sen & Banerjee 2000). Quantum field theories in curved spacetime should provide the mechanism that accounts for such processes (Birrell & Davies 1982). On a phenomenological level, particle production has been described in terms of bulk viscous stress (Zel'dovich 1970; Hu 1982; Lima & Germano 1992). Belinskii et al. (1979) studied the cosmological evolution of viscoelastic matter with causal thermodynamics in Bianchi type-I spacetime geometry. They found that the effect of matter creation near the initial singularity is preserved. Romano & Pavón (1993) studied the evolution of a Bianchi type-I universe with viscous dissipation. The effect of bulk viscosity on the evolution of FRW models was investigated by Desikan (1997), and Singh et al. (2002) presented FRW cosmological models with particle production and bulk viscosity in Brans-Dicke theory. Meanwhile, Singh & Kale (2011) studied anisotropic Bianchi type-I bulk viscous cosmological models with particle creation in Brans-Dicke theory, and recently, Chaubey (2012) obtained a Bianchi type-V bulk viscous cosmological model with particle creation in Brans-Dicke theory. The role of bulk viscosity in cosmic evolution, especially in its early stages, seems to be significant. One can use bulk viscosity to evaluate the rate of cosmological entropy production. Initially it was believed that the neutrino viscosity could smooth out initial anisotropies and lead to the isotropic universe that we observe today. In this paper, we study the general class of Bianchi bulk viscous cosmological models with particle creation in Brans-Dicke theory. The exact solutions to the Einstein field equations with particle creation in Brans-Dicke theory are obtained for three different physically viable cosmologies in Bianchi type-III, V,  $VI_0$  and  $VI_h$  spacetimes. All the physical parameters are calculated and discussed in each model.

#### 2 MODEL AND BASIC EQUATIONS

The well known Einstein field equations in Brans-Dicke theory are

$$G_{ij} + \frac{\omega}{\phi^2} \left[ \phi_{,i} \phi_{,j} - \frac{1}{2} g_{ij} \phi_{,l} \phi^{,l} \right] + \frac{1}{\phi} \left[ \phi_{,i;j} - \Box \phi g_{ij} \right] = \frac{8\pi}{\phi} T_{ij} , \qquad (1)$$

where  $\Box \phi = -\frac{8\pi T}{3+2\omega}$ . The energy momentum tensor  $(T_{ij})$  is defined as

$$T_{ij} = (\rho + p + p_c + \Pi)u_i u_j - (p + p_c + \Pi)g_{ij}, \qquad (2)$$

where  $p_c$  is the creation pressure and  $\Pi$  is the bulk viscosity of the cosmic fluid.

WMAP data have indicated that the universe was not isotropic in early times. Further proof of this is given by the anisotropies of order 20  $\mu$ K (microKelvin) in the CMB. Even though the CMB is

almost isotropic, it has been previously demonstrated that this does not guarantee that the universe was always isotropic (Coley 2003; Nilsson et al. 1999). The Bianchi models must now be introduced, and these have less symmetry than the standard Friedmann model. Such models can be examined to include the effects of shear in the early universe.

The diagonal form of the metric for a general class in the Bianchi cosmological model is given by

$$ds^{2} = dt^{2} - a_{1}^{2} dx^{2} - a_{2}^{2} e^{-2x} dy^{2} - a_{3}^{2} e^{-2mx} dz^{2}.$$
 (3)

We have additional classes of Bianchi models as follows: type III corresponds to m = 0, type V to m = 1, type VI<sub>0</sub> to m = -1, and all other m give VI<sub>h</sub>, where m = h - 1.

The functions  $a_1(t)$ ,  $a_2(t)$  and  $a_3(t)$  are the three anisotropic directions of expansion in normal three-dimensional space, where as earlier, we were considering only one expansion parameter, a(t), in the Friedmann model (i.e. we were assuming radial symmetry, and as such,  $a_1(t) = a_2(t) = a_3(t)$  in that scenario).

We first define the expressions for the average scale factor and volume scale factor, defining the generalized Hubble parameter H in analogy with a flat FRW model.

The average scale factor 'a' and spatial volume 'V' of the homogeneous, anisotropic general class of Bianchi cosmological models (Eq. (3)) are defined as

$$V = a^3 = a_1 a_2 a_3 \,. \tag{4}$$

We define the generalized Hubble parameter H as

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right),$$
(5)

where  $H_1 = \frac{\dot{a_1}}{a_1}$ ,  $H_2 = \frac{\dot{a_2}}{a_2}$  and  $H_3 = \frac{\dot{a_3}}{a_3}$  are the directional Hubble parameters. An overhead dot denotes differentiation with respect to cosmic time t. By using Equations (2) and (3), we get the following Einstein field equations:

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} - \frac{m}{a_1^2} = -\frac{8\pi(p+p_c+\Pi)}{\phi} - \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{\ddot{\phi}}{\phi} - \left(\frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right)\frac{\dot{\phi}}{\phi}, \quad (6)$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1\dot{a}_3}{a_1a_3} - \frac{m^2}{a_1^2} = -\frac{8\pi(p+p_c+\Pi)}{\phi} - \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{\ddot{\phi}}{\phi} - \left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_3}{a_3}\right)\frac{\dot{\phi}}{\phi}, \quad (7)$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1\dot{a}_2}{a_1a_2} - \frac{1}{a_1^2} = -\frac{8\pi(p+p_c+\Pi)}{\phi} - \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{\ddot{\phi}}{\phi} - \left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2}\right)\frac{\dot{\phi}}{\phi}, \quad (8)$$

$$\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1} - \frac{m^2 + m + 1}{a_1^2} = \frac{8\pi\rho}{\phi} + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 - \left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right)\frac{\dot{\phi}}{\phi}, \quad (9)$$

$$\frac{\dot{a}_2}{a_2} + m \frac{\dot{a}_3}{a_3} = (m+1)\frac{\dot{a}_1}{a_1},$$
(10)

and

$$\frac{\ddot{\phi}}{\phi} + \left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right)\frac{\dot{\phi}}{\phi} = -\frac{8\pi}{(3+2\omega)\phi}\left[\rho - 3(p+p_c+\Pi)\right].$$
(11)

By using Equations (6) to (9), we obtain the continuity equation as follows

$$\dot{\rho} + \left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right)(\rho + p) = -(p_c + \Pi)\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right).$$
(12)

Now, by adding three times Equation (9) and Equations (6), (7) and (8), one can easily obtain

$$\frac{\ddot{a_1}}{a_1} + \frac{\ddot{a_2}}{a_2} + \frac{\ddot{a_3}}{a_3} + 2\left(\frac{\dot{a_1}\dot{a_2}}{a_1a_2} + \frac{\dot{a_2}\dot{a_3}}{a_2a_3} + \frac{\dot{a_3}\dot{a_1}}{a_3a_1} - \frac{m^2 + m + 1}{a_1^2}\right)$$
$$= -\frac{12\pi(\rho - p - p_c - \Pi)}{\phi} - \frac{3\ddot{\phi}}{2\phi} - \frac{5}{2}\left(\frac{\dot{a_1}}{a_1} + \frac{\dot{a_2}}{a_2} + \frac{\dot{a_3}}{a_3}\right)\frac{\dot{\phi}}{\phi}.$$
 (13)

From Equation (4), we find

$$\frac{\ddot{V}}{V} = \frac{\ddot{a_1}}{a_1} + \frac{\ddot{a_2}}{a_2} + \frac{\ddot{a_3}}{a_3} + 2\left(\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1}\right).$$
(14)

From Equations (13) and (14), one can easily obtain

$$\frac{\ddot{V}}{V} - \frac{2(m^2 + m + 1)}{a_1^2} = \frac{12\pi(\rho - p - p_c - \Pi)}{\phi} - \frac{3\ddot{\phi}}{2\phi} - \frac{5}{2}\left(\frac{\dot{V}}{V}\right)\frac{\dot{\phi}}{\phi}.$$
(15)

The cosmological parameters such as the expansion parameter  $(\theta)$ , shear scalar  $(\sigma^2)$  and mean anisotropy parameter (A) are defined as:

$$\theta = u^i_{;i} = 3H, \qquad (16)$$

$$\sigma^{2} = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left[\left(\frac{\dot{a}_{1}}{a_{1}}\right)^{2} + \left(\frac{\dot{a}_{2}}{a_{2}}\right)^{2} + \left(\frac{\dot{a}_{3}}{a_{3}}\right)^{2}\right] - \frac{\theta^{2}}{6},$$
(17)

and

$$A = \frac{1}{3} \sum_{i=1}^{3} \left(\frac{\Delta H_i}{H}\right)^2 \,, \tag{18}$$

where  $u^i = (0, 0, 0, 1)$  is the co-moving velocity vector,  $\Delta H_i = H_i - H$  (i = 1, 2, 3) and

$$\sigma_{ij} = \frac{1}{2} (u_i;_{\alpha} p_j^{\alpha} + u_j;_{\alpha} P_i^{\alpha}) - \frac{1}{3} \theta P_{ij}.$$
(19)

The projection tensor  $P_{ij}$  is given by

$$P_{ij} = g_{ij} - u_i u_j \,. \tag{20}$$

Recently, Akarsu & Dereli (2012) proposed a special law for the deceleration parameter, which is linear in time with a negative slope. This law covers the Berman law (where the deceleration parameter is constant), which is used to obtain exact cosmological models, in the context of dark energy, to account for the current acceleration of the universe. According to this law, only spatially closed and flat universes with cosmological fluid exhibiting quintom like behavior are allowed, and the universe ends with a Big Rip. This new law provides the opportunity to generalize many of these dark energy models, and therefore has better consistency with the cosmological observations. The linearly varying deceleration parameter q is defined as (Akarsu & Dereli 2012)

$$q = -\frac{\ddot{a}\ddot{a}}{\dot{a}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -\frac{\dot{H}}{H^2} - 1 = -kt + n - 1,$$
(21)

where k and n are positive constants. We see that the deceleration parameter q is linear in time with a negative slope. The sign q indicates whether the model accelerates or not, and the positive sign of q corresponds to the standard decelerating model, whereas the negative sign indicates accelerated

expansion. For n > 1 + kt, q > 0, therefore the model represents a decelerating model, whereas for  $kt < n \le 1 + kt$ , we get  $-1 \le q < 0$ , which describes an accelerating model of the universe.

Solving Equation (21) for the scale factor, we obtain the law of variation for average scale factor a as

$$a = (nlt + c_1)^{1/n}, \qquad k = 0, \ n > 0; \tag{22}$$

$$a = c_2 e^{lt}, \qquad k = 0, \ n = 0;$$
 (23)

$$a = c_3 e^{\frac{2}{n} \tanh^{-1}(\frac{\kappa_l}{n} - 1)}, \qquad k > 0, \ n > 1,$$
(24)

where  $c_1$ ,  $c_2$  and  $c_3$  are constants of integration.

#### **3 BULK VISCOSITY WITH PARTICLE CREATION**

The balance equation is defined as

$$N^{\mu};_{\mu} = \dot{\eta} + 3\eta H = \Gamma, \qquad (25)$$

where  $N^{\mu}$  is the particle number density flow vector,  $\eta$  is the particle number density and  $N^{\mu} = \eta u^{\mu}$  satisfies the above Equation (25).

The covariant derivative of the entropy flux vector is defined as

$$S^{\mu};_{\mu} = \eta \dot{\nu} + \nu \Gamma \ge 0, \qquad (26)$$

where  $S^{\mu} = S \eta u^{\mu}$  is the entropy flux vector,  $\nu$  is the entropy per particle and  $\Gamma$  is the source term. For the production of particles, the source term should be positive, and for the annihilation of particles, the source term should be negative.

For no particle production or annihilation, the source term becomes zero. For an open thermodynamical system of temperature (T), the Gibbs equation is

$$\eta T \dot{\nu} = \dot{\rho} - (\rho + p) \frac{\dot{\eta}}{\eta}, \qquad (27)$$

where  $\nu$  is given as

$$\dot{\nu} = -\frac{3Hp_c}{\nu T} - \frac{3H\Pi}{\nu T} - \frac{(\rho+p)}{\eta^2 T}\Gamma.$$
(28)

 $\nu$  is uniform due to the adiabatic process, so  $\nu$  may change only by vicious phenomena, and therefore the creation pressure (the pressure arising from particle production) is given by

$$p_c = -\frac{(\rho+p)}{3H}\Gamma = -\frac{(\rho+p)}{3H}\left(3H + \frac{\dot{\eta}}{\eta}\right).$$
(29)

From Equations (25), (28) and (29), we get

$$\dot{\nu} = -\frac{3H\Pi}{\eta T} \,. \tag{30}$$

Using Equation (26) in Equation (30), we obtain

$$\Gamma \ge \frac{3H\Pi}{T\nu} \,. \tag{31}$$

By using Equations (27) and (30), we find

$$\frac{\dot{\eta}}{\eta} = \frac{\dot{\rho} + 3H\Pi}{\rho + p} \,. \tag{32}$$

Here, we consider the barotropic fluid, and the equation of state for this fluid is given by

$$p = \gamma \rho, \quad 0 \le \gamma \le 1. \tag{33}$$

From Equations (32) and (33), the particle number density ( $\eta$ ) is given by

$$\eta^{1+\gamma} = B\rho \, \exp\left(\int 3H\Pi\rho^{-1}dt\right)\,,\tag{34}$$

where B is a constant of integration.

The causal evolution equation of bulk viscosity for full causal non-equilibrium thermodynamics is given by

$$\Pi + \tau \dot{\Pi} = -\xi \left( \frac{\dot{a_1}}{a_1} + \frac{\dot{a_2}}{a_2} + \frac{\dot{a_3}}{a_3} \right) - \frac{\epsilon \tau \Pi}{2} \left( \frac{\dot{a_1}}{a_1} + \frac{\dot{a_2}}{a_2} + \frac{\dot{a_3}}{a_3} + \frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{T}}{T} \right) , \qquad (35)$$

where  $\xi$  and  $\tau$  are the bulk viscosity coefficient and relaxation time, respectively.

For  $\epsilon = 0, 1$  and  $\tau = 0$ , the above Equation (35) reduces to truncated theory, full causal theory and non-causal theory (Eckart's theory) respectively.

#### 3.1 Bulk Viscosity in the Eckart, Truncated and Full Causal Theories

The role of bulk viscosity in cosmic evolution, especially in its early stages, seems to be significant. Here, we discuss the viscosity in the Eckart, truncated and full causal theories.

Several authors (Maartens 1995; Zimdahl 1996) have studied the bulk viscosity in Eckart's non causal theory. This is the most common and simplest relation between the bulk viscous stress  $\Pi$  and the Hubble parameter (*H*). For ( $\tau = 0$ ), Equation (35) reduces to

$$\Pi = -3\xi H \,. \tag{36}$$

For ( $\epsilon = 0$ ), the above evolution Equation (35) reduces in truncated theory as follows

$$\Pi + \tau \dot{\Pi} = -3\xi H \,, \tag{37}$$

where

$$\tau = \frac{\xi}{\rho} \,. \tag{38}$$

Gibb's integrability condition (Maartens 1995) is defined as

$$T \propto \exp\left(\int \frac{dp(\rho)}{\rho + p(\rho)}\right),$$

where T is the temperature of the barotropic fluid  $(T = T(\rho))$ . By using Equation (33) in the above expression, we get

$$T = T_0 \rho^{\frac{\gamma}{1+\gamma}} \,, \tag{39}$$

where  $T_0$  is a constant.

By using Equations (38) and (39) in Equation (35), the evolution Equation (35) then reduces to

$$\Pi + \frac{\xi}{\rho}\dot{\Pi} = -3\xi H - \frac{\xi\Pi}{2\rho} \left[ 3H - \left(\frac{1+2\gamma}{1+\gamma}\right)\frac{\dot{\rho}}{\rho} \right].$$
(40)

## **4 COSMOLOGICAL SOLUTIONS**

Here, we discuss three different physically viable cosmologies k = 0, n > 0; k = 0, n = 0; and k > 0, n > 1, respectively, which motivate physical interests in describing the decelerating and accelerating phases of the universe. Here, we take scalar potential  $\phi$  as

$$\phi = \phi_0 t^\beta \,, \tag{41}$$

where  $\phi_0$  and  $\beta$  are constants.

*Case 1.* When k = 0, n > 0 and  $a_3 = V^b$ , where b is any constant number.

Using Equations (4), (10) and (22), we obtain

$$a_1(t) = (nlt + c_1)^{(3+3mb-3b)/n(m+2)}, \qquad (42)$$

$$a_2(t) = (nlt + c_1)^{(3+3m-3b-6mb)/n(m+2)}, \qquad (43)$$

$$a_3(t) = (nlt + c_1)^{3b/n}. (44)$$

The directional Hubble parameters  $H_1$ ,  $H_2$  and  $H_3$  have values given by

$$H_1 = \left(\frac{3+3mb-3b}{m+2}\right) \frac{l}{nlt+c_1},$$
(45)

$$H_2 = \left(\frac{3+3m-3b-6mb}{m+2}\right) \frac{l}{nlt+c_1},$$
(46)

$$H_3 = \frac{3bl}{nlt+c_1}.\tag{47}$$

From Equation (5), the average generalized Hubble parameter H has the value given by

$$H = \frac{l}{nlt + c_1} \,. \tag{48}$$

From Equations (16), (17) and (18), the dynamical scalars are given by

$$\theta = \frac{3l}{nlt + c_1} \,,\tag{49}$$

$$\sigma^{2} = (3 + 3m - 18b - 18mb + 27mb^{2} - 18m^{2}b + 27m^{2}b^{2} + 27b^{2} + 3m^{2}) \times \frac{l^{2}}{(m+2)^{2}(nlt+c_{1})^{2}},$$
(50)

and

$$A = (2 + 2m - 12b - 12mb + 18mb^2 - 12m^2b + 18m^2b^2 + 18b^2 + 2m^2)\frac{1}{(m+2)^2}.$$
 (51)

The scalar curvature R for the general class of a Bianchi cosmological model is defined as

$$R = 2\left[\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1} - \frac{(m^2 + m + 1)}{a_1^2}\right].$$
(52)

From Equations (42), (43), (44) and (52), we get

$$R = \frac{2l^2}{(m+2)^2(nlt+c_1)^2} \left[ (m+1)(27-18b+27b^2-12n) + m^2(9-18b+27b^2-3n) \right] + 2(m^2+m+1)(nlt+c_1)^{-\frac{2(3+3mb-3b)}{n(m+2)}}.$$
(53)

From Equation (9), we obtain

$$\rho = \frac{9\phi_0 l^2 k_1}{8\pi} \frac{t^{\beta}}{(nlt+c_1)^2} - \frac{\phi_0 (m^2+m+1)}{8\pi} \frac{t^{\beta}}{(nlt+c_1)^{\frac{k_2}{n}}} - \frac{\phi_0 \omega \beta^2}{16\pi} t^{\beta-2} + \frac{3\phi_0 l\beta}{8\pi} \frac{t^{\beta-1}}{nlt+c_1},$$
(54)

where  $k_1$  and  $k_2$  are constants and defined as

$$k_1 = \frac{1 + m + 2b + 2mb + 2m^2b - 3mb^2 - 3m^2b^2 - 3b^2}{(m+2)^2},$$
  

$$k_2 = \frac{3 + 3mb - 3b}{m+2}.$$

From Equations (33) and (54), we get

$$p = \gamma \left[ \frac{9\phi_0 l^2 k_1}{8\pi} \frac{t^{\beta}}{(nlt+c_1)^2} - \frac{\phi_0 (m^2+m+1)}{8\pi} \frac{t^{\beta}}{(nlt+c_1)^{\frac{k_2}{n}}} - \frac{\phi_0 \omega \beta^2}{16\pi} t^{\beta-2} + \frac{3\phi_0 l\beta}{8\pi} \frac{t^{\beta-1}}{nlt+c_1} \right].$$
(55)

We now investigate the behavior of the above cosmological model by analyzing the different physical parameters. Here, we observe the above results showing that the spatial volume (V) is zero at  $t = t_0 = -\frac{c_1}{nl}$ . The scalar curvature R, the energy density and pressure are infinite at this epoch. The rate of expansion and the mean anisotropy parameter are infinite at  $t \to t_0$ . Thus the universe starts evolving with zero volume at  $t = t_0 = 0$ , which implies that  $c_1 = 0$  and it expands with cosmic time, 't'.

For large cosmic time t, the spatial volume, expansion parameter, shear scalar and mean anisotropic parameter tend to zero. Here,  $\lim_{t\to\infty} \frac{\sigma^2}{\theta} = 0$ , so the model approaches isotropy for large cosmic time t. The conditions of homogeneity and isotropization, formulated by Collins & Hawking (1973), are satisfied in the present model.

Here we discuss four subcases. The values of creation pressure and bulk viscosity are given in tabular form in Appendix 1 (see the online version) for each subcase.

#### Subcase 1A: Model With Bulk Viscosity Energy Density Law

The power-law relation between the bulk viscosity coefficient and the energy density has already been considered by several authors (Johri & Sudharsan 1988; Maartens 1996; Pavon et al. 1991).

$$\xi = \xi_0 \rho^\alpha \,, \tag{56}$$

where  $\xi_0 \ge 0$  and  $\alpha \ge 0$  are constants.

Several authors (Murphy 1973; Santos et al. 1985) suggested, for  $\alpha = 1$  and  $\alpha = 1.5$ , that the above Equation (56) represents a radiative fluid and a string dominated universe, respectively.

Here we take a relation between the bulk viscous pressure and energy density as

$$\Pi = \Pi_0 \rho^{\alpha} \,. \tag{57}$$

Here the particle number density is given by

$$\frac{\dot{\eta}}{\eta} = \left[\frac{3l^2\phi_0(3k_1 - \beta n)}{8\pi} \frac{t^{\beta-1}}{(nlt+c_1)^3} - \frac{9\phi_0l^2nk_1}{4\pi} \frac{t^\beta}{(nlt+c_1)^3} - \frac{\phi_0(m^2+m+1)}{8\pi} \right] \\
\times \left(\frac{\beta t^{\beta-1}}{(nlt+c_1)^{\frac{k_2}{n}}} - \frac{lk_2t^\beta}{(nlt+c_1)^{\frac{k_2}{n}+1}}\right) - \frac{\phi_0\omega\beta^2(\beta-2)}{16\pi} t^{\beta-3} + \frac{3\phi_0l\beta(\beta-1)}{8\pi} \frac{t^{\beta-2}}{nlt+c_1}\right] \\
\times \frac{1}{(1+\gamma)\left(\frac{9\phi_0l^2k_1}{8\pi} \frac{t^\beta}{(nlt+c_1)^2} - \frac{\phi_0(m^2+m+1)}{8\pi} \frac{t^\beta}{(nlt+c_1)^{\frac{k_2}{n}}} - \frac{\phi_0\omega\beta^2}{16\pi} t^{\beta-2} + \frac{3\phi_0l\beta}{8\pi} \frac{t^{\beta-1}}{nlt+c_1}\right)}{nlt+c_1} + \frac{3\pi_0l}{(1+\gamma)(nlt+c_1)} \left[\frac{9\phi_0l^2k_1}{8\pi} \frac{t^\beta}{(nlt+c_1)^2} - \frac{\phi_0(m^2+m+1)}{8\pi} \frac{t^\beta}{(nlt+c_1)^{\frac{k_2}{n}}} - \frac{\phi_0(m^2+m+1)}{8\pi} \frac{t^\beta}{(nlt+c_1)^{\frac{k_2}{n}}} - \frac{\phi_0(m^2+m+1)}{8\pi} \frac{t^\beta}{(nlt+c_1)^{\frac{k_2}{n}}}\right] \\
- \frac{\phi_0\omega\beta^2}{16\pi} t^{\beta-2} + \frac{3\phi_0l\beta}{8\pi} \frac{t^{\beta-1}}{nlt+c_1}\right]^{\alpha-1}.$$
(58)

## Subcase 1B: Model With Uniform Particle Number Density

Here,  $\dot{\eta} = 0$  for the uniform particle number density model. From Equation (25), the particle production source ( $\Gamma$ ) is determined as

$$\Gamma = 3H\eta \,. \tag{59}$$

#### Subcase 1C: Model For Ideal Gas

In this model,  $\Gamma = 0$  and  $p_c = 0$ . Then Equation (25) is reduced to

$$N^{\mu};_{\mu} = \dot{\eta} + 3\eta H = 0.$$
(60)

From Equation (48) and Equation (60), the expression for the particle number density is obtained as

$$\eta = E(nlt + c_1)^{-3/n}, \tag{61}$$

where E is a constant of integration.

#### Subcase 1D: Creation With Second-Order Correction in H

In this case, the total particle number is considered in the Taylor expansion of  $\frac{\dot{\eta}}{\eta} = f(H)$  up to the second order in H as (Triginer & Pavón 1994)

$$\frac{\dot{\eta}}{\eta} = -3H + b_1 H^2 \,, \tag{62}$$

where  $b_1$  is a constant. From Equations (25) and (62), one can obtain

$$\Gamma = b_1 \eta H^2 \,. \tag{63}$$

For creation, no creation or annihilation of particles,  $b_1$  should be greater than zero, equal to zero, or less than zero, respectively. In the context of open thermodynamic systems, we take  $b_1$  as greater than or equal to zero, i.e., there is either creation or no creation.

The behavior of creation pressure  $(p_c)$  with a bulk viscosity energy density law, uniform particle number density, ideal gas and second-order correction in H for Case 1 can be seen in Figure 1. Here, we consider some very small values ( $\alpha = 1, \beta = 0.25, \gamma = 1/3, \Pi_0 = -1, \frac{\phi_0}{16\pi} = 1, n = 1, l = 1, c_1 = 1, b_1 = 1, m = 2$  and b = 1/3) that do not give rise to singularity. It is observed that all the parameters are a function of time and the solutions tend asymptotically to zero. The rapidity of



**Fig.1** The variation in creation pressure  $(p_c)$  with cosmic time t for the subcases 1A, 1B and 1D. Here, the creation pressure is measured in Pa (pascals) and the unit of time is Gyrs.



Fig. 2 The variation of bulk viscosity ( $\Pi$ ) with cosmic time t for the subcases 1A, 1B, 1C and 1D. Here, the bulk viscosity is measured on the Hubble scale (km s<sup>-1</sup> Mpc<sup>-1</sup>) and the unit of time is inverse Hubble scale.

growth at an early stage depends on the different models of the universe. Later on, this tends to zero for each model of the universe.

Figure 2 shows the variation in bulk viscous stress ( $\Pi$ ) for various models against cosmic time 't' for Case 1. It is observed that all the parameters are a function of time, and the solutions tend asymptotically to zero. The rapidity of its growth at an early stage depends on the different models of the universe. Later on, this tends to zero for each model of the universe.

The behavior of bulk viscosity ( $\xi$ ) with a bulk viscosity energy density law, uniform particle number density, ideal gas and second-order correction in H for Case 1 can be seen in Figures 3, 4, and 5. Here, we consider a very small value of the constants that does not give rise to singularity. It is observed that the bulk viscosities in Eckart's theory and truncated theory are a function of time, and the solutions tend asymptotically to zero. The rapidity of growth at an early stage depends on the different models of the universe. Later on, this tends to zero for each model of the universe. However, the bulk viscosity in full causal theory has a small value for large cosmic time t. From Figures 3, 4, and 5, it is observed that initially the bulk viscosity is large, and for large cosmic time t becomes almost equal to zero. Hence, we can say that bulk viscosity can play the role of an agent that drives the present acceleration of the universe.

Case 2. When n = 0 and  $a_3 = V^b$ 



**Fig. 3** The variation of bulk viscosity in Eckart's theory ( $\xi_e$ ) with cosmic time t for the subcases 1A, 1B, 1C and 1D. Here, the bulk viscosity in Eckart's theory is measured on the Hubble scale (km s<sup>-1</sup> Mpc<sup>-1</sup>) and the unit of time is inverse Hubble scale.



**Fig.4** The variation of bulk viscosity in truncated theory  $(\xi_t)$  with cosmic time t for the subcases 1A, 1B and 1D. Here, the bulk viscosity in truncated theory is measured on the Hubble scale (km s<sup>-1</sup> Mpc<sup>-1</sup>) and the unit of time is inverse Hubble scale.



**Fig. 5** The variation of bulk viscosity in full causal theory  $(\xi_f)$  with cosmic time t for the subcases 1A, 1B, 1C and 1D. Here, the bulk viscosity in full causal theory is measured on the Hubble scale (km s<sup>-1</sup> Mpc<sup>-1</sup>) and the unit of time is inverse Hubble scale.

Using Equations (4), (10) and (23), we obtain

$$a_1(t) = c_2^{\frac{3+3mb-3b}{m+2}} e^{\frac{(3+3mb-3b)lt}{m+2}},$$
(64)

$$a_2(t) = c_2^{\frac{3+3m-3b-6mb}{m+2}} e^{\frac{(3+3m-3b-6mb)lt}{m+2}},$$
(65)

$$a_3(t) = c_2^{3b} e^{3blt} \,. \tag{66}$$

The directional Hubble parameters  $H_1$ ,  $H_2$  and  $H_3$  have values given by

$$H_1 = \left(\frac{3+3mb-3b}{m+2}\right)l,\tag{67}$$

$$H_2 = \left(\frac{3+3m-3b-6mb}{m+2}\right)l,$$
(68)

$$H_3 = 3bl. (69)$$

From Equation (5), the average generalized Hubble parameter H has the value given by

$$H = l. (70)$$

From Equations (16), (17) and (18), the dynamical scalars are given by

$$\theta = 3l, \qquad (71)$$

- 0

$$\sigma^{2} = \left(3 + 3m - 18b - 18mb + 27mb^{2} - 18m^{2}b + 27m^{2}b^{2} + 27b^{2} + 3m^{2}\right)\frac{l^{2}}{(m+2)^{2}},$$
 (72)

and

$$A = \left(2 + 2m - 12b - 12mb + 18mb^2 - 12m^2b + 18m^2b^2 + 18b^2 + 2m^2\right)\frac{1}{(m+2)^2}.$$
 (73)

From Equations (64), (65), (66) and (52), we get

$$R = \frac{2l^2}{(m+2)^2} (27 - 18mb - 18b + 27m + 27b^2 + 9m^2 + 27mb^2 - 18m^2b + 27m^2b^2) -2(m^2 + m + 1)c_2^{\frac{-2(3+3mb-3b)}{m+2}} e^{-\frac{2(3+3mb-3b)lt}{(m+2)}}.$$
(74)

From Equation (9), we obtain

$$\rho = \frac{9\phi_0 l^2 k_1}{8\pi} t^\beta - \frac{\phi_0 (m^2 + m + 1)}{8\pi c_2^{2k_2}} \frac{t^\beta}{e^{2k_2 lt}} - \frac{\phi_0 \omega \beta^2}{16\pi} t^{\beta - 2} + \frac{3\phi_0 l\beta}{8\pi} t^{\beta - 1} \,. \tag{75}$$

From Equations (33) and (75), we find

$$p = \gamma \left[ \frac{9\phi_0 l^2 k_1}{8\pi} t^\beta - \frac{\phi_0 (m^2 + m + 1)}{8\pi c_2^{2k_2}} \frac{t^\beta}{e^{2k_2 l t}} - \frac{\phi_0 \omega \beta^2}{16\pi} t^{\beta - 2} + \frac{3\phi_0 l\beta}{8\pi} t^{\beta - 1} \right].$$
(76)

We now investigate the behavior of the above cosmological model by analyzing the different physical parameters. For large cosmic time t, the shear scalar and mean anisotropic parameter tend to zero. Here,  $\lim_{t\to\infty} \frac{\sigma^2}{\theta} = 0$ , so the model approaches isotropy for large cosmic time t. The conditions of homogeneity and isotropization, formulated by Collins & Hawking (1973), are satisfied in the present model.



Fig. 6 The same as Fig. 1 but for subcases 2A, 2B and 2D which are given in Appendix 2.



Fig. 7 The same as Fig. 2 but for subcases 2A, 2B, 2C and 2D which are given in Appendix 2.



Fig. 8 The same as Fig. 3 but for subcases 2A, 2B, 2C and 2D which are given in Appendix 2.

Here we discuss four subcases. The values of creation pressure and bulk viscosity are given in tabular form in Appendix 2 (see the online version) for each subcase.

The behavior of creation pressure  $(p_c)$  with a bulk viscosity energy density law, uniform particle number density, ideal gas and second-order correction in H for Case 2 can be seen in Figure 6. Here we consider a very small value of the constants that does not give rise to singularity. It is observed that all the parameters are a function of time, and the solutions tend asymptotically to zero. The



Fig. 9 The same as Fig. 4 but for subcases 2A, 2B, 2C and 2D which are given in Appendix 2.



Fig. 10 The same as Fig. 5 but for subcases 2A, 2B, 2C and 2D which are given in Appendix 2.

rapidity of growth at an early stage depends on the different models of the universe. Later on, this tends to zero for each model of the universe.

Figure 7 shows the variation in bulk viscous stress ( $\Pi$ ) for various models against cosmic time 't' for Case 2. It is observed that all the parameters are a function of time, and the solutions tend asymptotically to zero. The rapidity of growth at an early stage depends on the different models of the universe. Later on, this tends to zero for each model of the universe.

The behavior of bulk viscosity ( $\xi$ ) with a bulk viscosity energy density law, uniform particle number density, ideal gas and second-order correction in H for Case 2 can be seen in Figures 8, 9 and 10. Here we consider a very small value of the constants that does not give rise to singularity. It is observed that the bulk viscosity in Eckart's theory and truncated theory are a function of time, and the solutions tend asymptotically to zero. The rapidity of growth at an early stage depends on the different models of the universe. Later on, this tends to zero for each model of the universe. However, the bulk viscosity in full causal theory has a small value for large cosmic time t. From Figures 8, 9 and 10, it is observed that initially the bulk viscosity is large, and for large cosmic time t it becomes almost equal to zero. Hence, we can say that bulk viscosity could play the role of an agent that drives the present acceleration of the universe.

*Case 3.* When k > 0 and n > 1

Using Equations (4), (10) and (24), we obtain

$$a_1(t) = c_3^{\frac{3+3mb-3b}{m+2}} e^{\frac{2(3+3mb-3b)}{n(m+2)} \tanh^{-1}(\frac{kt}{n}-1)},$$
(77)

$$a_2(t) = c_3^{\frac{3+3m-3b-6mb}{m+2}} e^{\frac{2(3+3m-3b-6mb)}{n(m+2)} \tanh^{-1}(\frac{kt}{n}-1)},$$
(78)

$$a_3(t) = c_3^{3b} e^{\frac{6b}{n} \tanh^{-1}(\frac{kt}{n} - 1)} .$$
(79)

The directional Hubble parameters  $H_1$ ,  $H_2$  and  $H_3$  have values given by

$$H_{1} = \left(\frac{3+3mb-3b}{m+2}\right)\frac{2}{t(2n-kt)},$$
(80)

$$H_2 = \left(\frac{3+3m-3b-6mb}{m+2}\right)\frac{2}{t(2n-kt)},$$
(81)

$$H_3 = \frac{6b}{t(2n-kt)}.$$
 (82)

From Equation (5), the average generalized Hubble parameter H has the value given by

$$H = \frac{2}{t(2n-kt)} \,. \tag{83}$$

From Equations (16), (17) and (18), the dynamical scalars are given by

$$\theta = \frac{6}{t(2n-kt)},\tag{84}$$

$$\sigma^{2} = \left(1 + m - 6b - 6mb + 9mb^{2} - 6m^{2}b + 9m^{2}b^{2} + 9b^{2} + m^{2}\right)\frac{12}{t^{2}(2n - kt)^{2}(m + 2)^{2}},$$
(85)

and

$$A = \left(1 + m - 6b - 6mb + 9mb^2 - 6m^2b + 9m^2b^2 + 9b^2 + m^2\right)\frac{2}{(m+2)^2}.$$
 (86)

From Equations (77), (78), (79) and (52), we get

$$R = \frac{8}{t^2(2n-kt)^2(m+2)^2}(27+27m-18b-27mb+36mb^2-18m^2b+63m^2b^2 + 12mkt-12mn+12kt-12n+27b^2+9m^2+3m^2kt-3m^2n) -2(m^2+m+1)c_3\frac{-2(3+3mb-3b)}{m+2}e^{-\frac{4(3+3mb-3b)}{n(m+2)}\tanh^{-1}(\frac{kt}{n}-1)}.$$
(87)

From Equation (9), we obtain

$$\rho = \frac{9\phi_0 k_1}{2\pi} \frac{t^{\beta-2}}{(2n-kt)^2} - \frac{\phi_0(m^2+m+1)}{8\pi c_3^{2k_2}} \frac{t^{\beta}}{e^{\frac{4k_2}{n}\tanh^{-1}(\frac{kt}{n}-1)}} - \frac{\phi_0\omega\beta^2}{16\pi} t^{\beta-2} + \frac{3\phi_0\beta}{4\pi} \frac{t^{\beta-2}}{2n-kt}.$$
(88)

From Equations (33) and (88), we get

$$p = \gamma \left[ \frac{9\phi_0 k_1}{2\pi} \frac{t^{\beta-2}}{(2n-kt)^2} - \frac{\phi_0(m^2+m+1)}{8\pi c_3^{2k_2}} \frac{t^{\beta}}{e^{\frac{4k_2}{n}\tanh^{-1}(\frac{kt}{n}-1)}} - \frac{\phi_0\omega\beta^2}{16\pi} t^{\beta-2} + \frac{3\phi_0\beta}{4\pi} \frac{t^{\beta-2}}{2n-kt} \right].$$
(89)



Fig. 11 The same as Fig. 1 but for subcases 3A, 3B and 3D which are given in Appendix 3.



Fig. 12 The same as Fig. 2 but for subcases 3A, 3B, 3C and 3D which are given in Appendix 3.

We will now investigate the behavior of the above cosmological model by analyzing the different physical parameters. For large cosmic time t, the shear scalar and mean anisotropic parameter tend to zero. Here,  $\lim_{t\to\infty} \frac{\sigma^2}{\theta} = 0$ , so the model approaches isotropy for large cosmic time t. The conditions of homogeneity and isotropization, formulated by Collins & Hawking (1973), are satisfied in the present model.

Here we discuss four subcases. The values of creation pressure and bulk viscosity are given in tabular form in Appendix 3 (see the online version) for each subcase.

The behavior of the creation pressure  $(p_c)$  with a bulk viscosity energy density law, uniform particle number density, ideal gas and second-order correction in H for Case 3 can be seen in Figure 11. Here, we consider a very small value of the constants that does not give rise to singularity. It is observed that all the parameters are a function of time, and the solutions tend asymptotically to zero. The rapidity of growth at the early stage depends on the different models of the universe. Later on, this tends to zero for each model of the universe.

Figure 12 shows the variation in the bulk viscous stress ( $\Pi$ ) for various models against cosmic time 't' for Case 3. It is observed that all the parameters are a function of time, and the solutions tend asymptotically to zero. The rapidity of growth at an early stage depends on the different models of the universe. Later on, this tends to zero for each model of the universe.

The behavior of bulk viscosity ( $\xi$ ) in bulk viscosity energy density law, uniform particle number density, ideal gas and second-order correction in H for Case 3 can be seen in Figures 13, 14, and 15. Here, we consider a very small value of the constants that does not give rise to singularity. It is observed that the bulk viscosities in Eckart's theory and truncated theory are a function of time, and



Fig. 13 The same as Fig. 3 but for subcases 3A, 3B, 3C and 3D which are given in Appendix 3.



Fig. 14 The same as Fig. 4 but for subcases 3A, 3B, 3C and 3D which are given in Appendix 3.



Fig. 15 The same as Fig. 5 but for subcases 3A, 3B, 3C and 3D which are given in Appendix 3.

the solutions tend asymptotically to zero. The rapidity of growth at an early stage depends on the different models of the universe. Later on, this tends to zero for each model of the universe. However, the bulk viscosity in full causal theory has a small value for large cosmic time t. From Figures 13, 14 and 15, it is observed that initially the bulk viscosity is large, and for large cosmic time t it becomes almost equal to zero. Hence, we can say that bulk viscosity could play the role of an agent that is driving the present acceleration of the universe.

#### **5** CONCLUSIONS

This paper dealt with the general class of Bianchi cosmological models with bulk viscosity and particle creation described by full causal thermodynamics in Brans-Dicke theory. We used the general class of cosmological models for different values of m as follows: Bianchi type-III corresponded to m = 0, Bianchi type-V to m = 1, Bianchi type-VI<sub>0</sub> to m = -1 and all other value of m gave Bianchi type-VI<sub>h</sub>. The exact solutions to the corresponding field equations were obtained in quadrature form. Three different cases were discussed, depending on the nature of the relation between the scale factor and the cosmic time, t. All the models provided the solution which suggests a decreasing form of energy density, pressure and creation pressure, and bulk viscosity with the evolution of the universe. Here, we observed that bulk viscosity plays the role of an agent driving the present acceleration of the universe. Hence, we concluded that the bulk viscosity has played an important role in the evolution of the universe. In each case, the spatial volume, expansion parameter, shear scalar and mean anisotropic parameter tended to zero for large cosmic time, t. All the physical parameters were calculated and discussed for each model, and in each case, the cosmological model approached isotropy for a large value of cosmic time t. The model had a point singularity, and the rate of expansion slowed down and vanished as  $t \to \infty$ . This model represents a shearing, non-rotating and expanding universe, which approaches isotropy for large values of t. The results of this paper agree with the observational features of the universe.

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