RAA 2014 Vol. **14** No. **4**, 485–496 doi: 10.1088/1674–4527/14/4/009
http://www.raa-journal.org *http://www.iop.org/journals/raa*

***R**esearch in*
***A**stronomy and*
***A**strophysics*

Analysis of interval constants in calendars affiliated with the Shoushili

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Received 2013 July 8; accepted 2013 October 31

Abstract We study interval constants that are related to motions of the Sun and Moon, i.e., the Qi, Intercalation, Revolution and Crossing interval, in calendars affiliated with the Shoushi calendar (Shoushili), such as Datongli and Chiljeongsannaepyeon. It is known that these interval constants were newly introduced in the Shoushili calendar and revised afterward, except for the Qi interval constant, and the revised values were adopted in later calendars affiliated with the Shoushili. We first investigate the accuracy of these interval constants and then the accuracy of calendars affiliated with the Shoushili in terms of these constants by comparing times for the new moon and the maximum solar eclipse calculated by each calendar with modern methods of calculation. During our study, we found that the Qi and Intercalation interval constants used in the early Shoushili were well determined, whereas the Revolution and Crossing interval constants were relatively poorly measured. We also found that the interval constants used by the early Shoushili were better than those of the later one, and hence better than those of Datongli and Chiljeongsannaepyeon. On the other hand, we found that the early Shoushili is, in general, a worse calendar than Datongli for use in China but a better one than Chiljeongsannaepyeon for use in Korea in terms of times for the new moon and when a solar eclipse occurs, at least for the period 1281 – 1644. Finally, we verified that the times for sunrise and sunset in the Shoushili-Li-Cheng and Mingshi are those at Beijing and Nanjing, respectively.

Key words: history and philosophy of astronomy: general — celestial mechanics — ephemerides

1 INTRODUCTION

The Shoushi calendar (Shoushili, hereafter) was developed by Shoujing Guo and his colleagues from the Yuan dynasty (Bo 1997). It is known as one of most the famous calendars in Chinese history (Needham 1959). Calendars affiliated with it, including the Datongli in the Ming dynasty and the Chiljeongsannaepyeon (Naepyeon, hereafter) in the Joseon dynasty, are extremely similar.

Compared to previous Chinese calendars, the most distinguishing characteristic of the Shoushili is that it discontinues the use of a Super Epoch, an ancient epoch where the starting points of all interval constants are the same (Sivin 2009). Instead, the Shoushili adopted the winter solstice of 1280 (1280 December 14.06 in Julian date) as the start of an epoch and introduced seven interval constants from observations in order to reduce the large number of accumulated days in the calculations arising from the use of a Super Epoch. Of the seven interval constants, four are related to the motions of the Sun and the Moon: the Qi, Run (Intercalation), Zhuan (Revolution), and Jiao (Crossing) interval constants. According to the records of Mingshi (History of the Ming Dynasty), early values of these interval constants, except for the Qi interval constant, were revised later. Currently, it is known that Datongli and Naepyeon adopted the revised values of the early Shoushili; hence, both calendars are essentially identical to the later Shoushili. The main purpose of this study is to evaluate the effect of the values for these four interval constants on the accuracy of calendar calculations. In this paper, therefore, "Shoushili" refers to the early one unless otherwise stated.

In this study, we first estimate the accuracy of the values for the interval constants of the Qi, Intercalation, Revolution and Crossing in calendars affiliated with the Shoushili. We then investigate the accuracy of each calendar in terms of these interval constants by comparing the times of the new moon and the midpoint a solar eclipse calculated by each calendar with modern methods of calculation. Because times of sunrise (SR) and sunset (SS) are needed to calculate the times of the midpoint of a solar eclipse in calendars affiliated with the Shoushili, we verify those times presented in calendar books as well.

This paper is composed as follows. In Section 2, we briefly introduce calendars affiliated with the Shoushili and the modern method of astronomical calculations. In Section 3, we present the results on the analysis of the four interval constants, times of the new moon, times of SR and SS, and times for the midpoint a solar eclipse. Finally, we summarize our findings in Section 4.

2 CALENDARS AFFILIATED WITH THE SHOUSHILI

2.1 Shoushili

In the chapter of Yuanshi (History of the Yuan Dynasty) related to calendars, the Shoushili is described as covering two parts: Shoushili-Yi (Discussion) and Shoushili-Jing (Method). It is known that this calendar was used in the Goryeo dynasty of Korea since the reign of King Chungseon (1308 – 1313) (Jeon 1974). For this reason, the Shoushili is also discussed in the chapter related to calendars in the work Goryeosa (History of the Goryeo Dynasty). However, the Goryeosa does not include the Shoushili-Yi. Conversely, the Yuanshi contains no Li-Cheng (Ready-reference Astronomical Tables), which is presented in the Goryeosa. In addition, both historical books have no tables listing times of SR and SS.

A book entitled Shoushili-Li-Cheng (Ready-reference Astronomical Tables for the Shoushili) is preserved in the Kyuganggak Institute for Korean Study (Kyuganggak, hereinafter) in Korea. This book contains various tables for the use of calendar calculations by the Shoushili along with the times of SR and SS (for further details, see Lee & Jing 1998). We have to refer to these three other books (i.e., Yuanshi, Goryeosa and Shoushili-Li-Cheng) to completely understand the Shoushili. Kyuganggak also contains a book called Shoushili-Jie-Fa-Li-Cheng (Expeditious Ready-reference Astronomical Tables for the Shoushili). According to the preface of an edition in Kyuganggak, this book was brought from China by Bo Gang (an astronomer in the Goryeo court). It was printed in 1346, and reprinted in 1444, around the year of publication for Naepyeon.

2.2 Datongli

There are two versions of the work Datongli that were written in the Ming dynasty; Wushen-Datongli (Datongli of the Wushen Year) compiled by Ji Liu in 1368, and Datong-Lifa-Tonggui

(Comprehensive Guide to the Calendrical Method by Datongli) made by Tong Yuan in 1384. Although the epochs of the former and the latter calendars are the winter solstices of 1280 and 1383, respectively, both are based on the Shoushili (Lee 1996). In the chapter related to calendars in the Mingshi, Datongli is described as having three parts: the first is on the origin of the techniques, the second on Li-Cheng including tables for the timings of SR and SS, and the third on calendrical methods.

It is widely known that Datongli was used in the Goryeo dynasty since 1370, although this fact is arguable (refer to Lee et al. 2010). Unlike the Shoushili, Datongli is not included in the chapter related to calendars in the Goryeosa. Instead, a series with a name similar to Datong-Lifa-Tonggui of Tong Yuan, for example, Datong-Liri-Tonggui (Comprehensive Guide to the Calendar Day by Datongli), is preserved in the Kyuganggak. According to the work of Lee (1988), the series (called the Tonggui series hereafter) is related to the Datongli and was published around 1444, and its main purpose was being a reference during the compilation of the Naepyeon.

2.3 Naepyeon

King Sejong of the Joseon dynasty ordered In-Ji Jeong and others to compile the Naepyeon in 1433. Although the date when the compilation was completed is not clear, the oldest existing version is the one published in 1444 by Sun-Ji Yi and Dam Kim. In terms of the contents, each chapter of the Naepyeon corresponds to the Tonggui series and the timings of SR and SS are contained in the Taeum (Moon) chapter. However, these times are different in calendars affiliated with the Shoushili. Because the compilation of the Naepyeon was considered to be one of the greatest works of King Sejong, this book is also appended in his Veritable Record, unlike the Veritable Records of other kings (Lee et al. 2008).

A book entitled Jeongmyoyeon-Gyeosik-Garyeong (Example Supplement for the Calculations of the Solar and Lunar Eclipses that Occurred in 1447; hereafter referred to as Garyeong), which is related to the Naepyeon, also remains in the Kyuganggak. This book contains modified interval constants for the epoch of 1442 and is valuable for step-by-step calculations of not only the solar eclipse but also the new moon by the Naepyeon, and hence by the Shoushili or Datongli. In this study, we refer to Garyeong to calculate times for the new moon and the maximum solar eclipse by calendars affiliated with the Shoushili, and we refer to the work of Lee (1988), which examined differences among these calendars.

In Table 1, we summarize values of the four interval constants in each calendar. All dates are given in the Julian calendar and all values of interval constants are in units of Parts; one day is 10 000 Parts. The Qi interval constant is the interval between the epoch and the midnight of the first day in a sexagenary cycle when counting backwards from the epoch. This value is the same in all calendars affiliated with the Shoushili. The intercalation interval constant is the interval from the epoch to the ‘mean’ new moon of the month belonging to the epoch. The time of a new moon is determined by correcting the slowness or fastness of the solar and lunar motions on the day of the mean new moon, which is obtained by accumulating the Intercalation interval constant. Revolution and Crossing interval constants are the lengths between epochs, and the times of lunar perigee and passage of the descending node, respectively (see also Sivin 2009).

As can be seen in Table 1, the values of interval constants in Datongli and Naepyeon are identical to each other. Interestingly, the numbers for the Shoushili of the Goryeosa are the same as those for the Naepyeon and the Datongli, except for the Revolution interval constant.

2.4 Modern Calculations

In modern calculations, we use the astronomical algorithms of Meeus (1989, 1998) and the DE406 ephemeris of Standish et al. (1997). In addition, we use Besselian elements to calculate the time of

Table 1 Summary of the Four Interval Constants Adopted in Calendars Affiliated with the Shoushili

Interval	Shoushili		Datongli		Naepyeon
Constants	Yuanshi	Goryeosa	Mingshi[1]	Tonggui Series	
Qi	550 600	550 600	550 600	550 600	550 600
Intercalation	201 850	202 050	202 050	202 050	202 050
Revolution	131 904	131 904	130 205	130 205	130 205
Crossing	260 187.86	260 388	260 388	260 388	260 388

Notes: [1] Values when the epoch starts as measured from the winter solstice of 1280.

a solar eclipse for a local circumstance. Although Mucke & Meeus (1992) tabulated Besselian elements for solar eclipses ranging from -2003 to 2526, they presented only the first order coefficients in each element. Hence, we use Besselian elements extracted from the DE406 ephemeris to increase our accuracy (e.g., Lee 2008). One of the important parameters to calculate ancient astronomical phenomena is ΔT, the difference between universal time (UT) and terrestrial time (TT). To estimate ΔT for a given year, we employ a cubic spline interpolation method (see Press et al. 1992) using data recently obtained by Morrison & Stephenson (2004). To directly compare the results of modern calculations with those of calendars associated with the Shoushili, we convert UT into the local apparent solar time by correcting the equation of time. Lastly, we assume that the locations of Beijing, Nanjing and Seoul are at 39° 55′ N and 116° 25′ E, 32° 3′ N and 118° 53′ E, and 37° 34′ N and 126° 59′ E, respectively.

3 RESULTS

3.1 Values of Interval Constants

The winter solstice was used as an epoch in ancient Chinese calendars. In the Shoushili, the values of interval constants were based on the winter solstice of 1280, as mentioned earlier. It is known that Shoujing Guo determined the date of the winter solstice from measurements of a shadow with a gnomon (Chen 1983; Li 2005). He used a tall gnomon and estimated the moment when the length of the shadow, casted by the Sun, was the longest, based on observations spanning several days. In modern times, the winter solstice has been calculated as the time when the Sun passed through the ecliptic longitude (λ) of 270°, using an astronomical ephemeris such as DE406. For details on how Shoujing Guo determined the times of the winter solstice and of the lunar perigee and passages of the descending node, refer to Yuanshi.

In Table 2, we present the dates related to four interval constants in calendars affiliated with the Shoushili along with results from modern calculations. All dates are expressed in terms of the Julian calendar, in units of the apparent solar time at Beijing unless otherwise mentioned. The difference in the equation of time based on the different locations (at Beijing, Nanjing and Seoul) is negligible. Hence, we can easily convert the times at Beijing into times at other locations by only considering the difference in longitude. Therefore, the time at Seoul, for example, is obtained by adding +42.26 min to the time at Beijing (there is a difference of 10.566° in longitude between Beijing and Seoul).

In the table, the first column contains items related to the interval constants; WS1280 is the winter solstice of 1280, which is the epoch of calendars affiliated with the Shoushili. MFDSC is midnight of the first day in the sexagenary cycle (the Jiazi day) before the epoch. MNM and NM are the mean new moon and new moon, respectively, and LPP and LDNP are passages of lunar perigee and the descending node, respectively. The second column contains the Julian dates derived from the epoch and the values of interval constants in calendars affiliated with the Shoushili, except for WS1280, the epoch itself, and NM. The third column is the day number obtained by subtracting

Table 2 Summary of the Dates in Calendars affiliated with the Shoushili and Modern Calculations Relating to the Four Interval Constants

Item	Calendars affiliated with the Shoushili (A)		Modern Calculation (B)	B − A	Calendar[7]	Remark
	Julian Date	JD − 2188925.56	JD − 2188925.56	(min)		
WS1280[1]	Dec. 14.060000	0.000000	0.011638	16.8	S, D, N	Epoch
MFDSC[2]	Oct. 20.000000	−55.060000	−55.060000	0.0	S, D, N	Qi
MNM[3]	Nov. 23.875000	−20.185000	–	–	S	Intercalation
	Nov. 23.855000	−20.205000	–	–	D, N	
NM[4]	Nov. 24.211966	−19.848034	−19.840819	10.4	S	(Intercalation)
	Nov. 24.191856	−19.868144	−19.840819	39.3	D, N	
LPP[5]	Nov. 30.869600	−13.190400	−13.355960	238.4	S	Revolution
	Dec. 01.039500	−13.020500	−13.355960	483.1	D, N	
LDNP[6]	Nov. 18.041214	−26.018786	−25.863352	−223.8	S	Crossing
	Nov. 18.021200	−26.038800	−25.863352	−252.6	D, N	

Notes: [1] Winter solstice of 1280, [2] Midnight of the first day in a sexagenary cycle when counting backwards from the epoch, [3] Mean new moon, [4] New moon, [5] Passage of lunar perigee, [6] Passage of lunar descending node, [7] S: Shoushili, D: Datongli, N: Naepyeon.

2188925.56 d (the epoch) from the Julian day number (JD) corresponding to the date given in the second column. The fourth and fifth columns are the results of modern calculations and the difference between the modern calculations and the values derived from calendars affiliated with the Shoushili, respectively. The sixth column represents the calendar; S is Shoushili, D Datongli and N Naepyeon. The last column contains the interval constants related to the items in the first column.

According to modern calculations, ΔT in 1280 is 532.6 s and JD for the winter solstice of that year at Beijing is 2188925.571638 d (1280 December 14.071638), which is obtained by correcting the equation of time by −0.087 min. A difference of +16.8 min compared to the modern calculation shows that Shoujing Guo accurately estimated the epoch in the Shoushili (see Table 2). Although the difference in time between the Shoushili and modern calculations is 16.8 min, there is no change in the date. Hence, the MFDSCs in the Shoushili and modern calculations are the same as the 56th day in the sexagenary cycle (the Jiwei day), October 20.0. Because of this, the Qi interval constant, which is the length between the epoch and the MFDSC, has the same difference in the epoch, i.e., 16.8 min. To verify our calculation, we compute the winter solstice of 2010 using ΔT = 65.9 s[1], to compare the result with the data from CAS2009[2], and find a good agreement in the values, with the JD being 2455552.485037 d (2010 December 21.985037) in UT[3].

Although it is known that Shoujing Guo also determined the Intercalation interval constant based on observations, there are not many details on the calculations he used (Li & Zhang 1998a). Hence, we calculated the date of the new moon in November, 1280 as an indirect method to verify the Intercalation interval constant. The dates are calculated to be JD 2188905.711966 and JD 2188905.719181 d (or 1280 November 21.344808 in UT) by the Shoushili and modern calculations, respectively, which give a difference of only +10.4 min. To check our calculation, we also compare

[1] U.S. Nautical Almanac Office 2009, The Astronomical Almanac for the Year 2010 and Its Companion (Washington: U.S. Government Printing Office)

[2] Purple Mountain Observatory, Chinese Academy of Sciences 2009, Chinese Astronomical Almanac for the Year 2010 (Beijing: Science Press)

[3] also see Korea Astronomy and Space Science Institute 2009, Korean Astronomical Almanac for the Year 2010 (Seoul: Namsandang)

the time of the new moon with the data provided by NASA[4] and find that the difference is less than 1 min. The exact times at which the astronomers of the Yuan dynasty modified the values of the interval constants of the Shoushili are not known. When using Datongli's value of the Intercalation interval constant, the difference increases, becoming ∼39.3 min, at least for the time of the new moon in November, 1280. We discuss the values for other periods in the next subsection.

Unlike the Qi and Intercalation interval constants, we find relatively large differences in the remaining constants. According to modern calculations, times for the lunar perigee and passage of the descending node at that time are JD 2188912.204040 and JD 2188899.696648 d, respectively. That is, Revolution and Crossing interval constants show a difference of ∼238.4 (cf. ∼216 min; Chen 2006) and ∼223.8 min, respectively, compared with modern calculations. In particular, there is a large change in the Revolution interval constant in the Datongli, which is +1699 Parts (∼244.7 min). Hence, the difference is also larger in proportion to the amount, i.e., ∼483.1 (= 238.4 + 244.7) min in Datongli (refer to Tables 1 and 2).

3.2 Time of the New Moon

In calendars affiliated with the Shoushili, the time for any phase of the Moon, for example, new moon, is determined in the following manner. First calculate the mean time for new moon using the Intercalation interval constants and the length of the synodic month (i.e., 295305.93 Parts in calendars affiliated with the Shoushili). The time of the new moon is then determined by using the Revolution interval constant and by considering motions of the Sun and the Moon. Because there is no Li-Cheng in Yuanshi, we use the values of Shoushili-Li-Cheng preserved in Kyuganggak for the motions of the Sun and the Moon. For the sake of completeness, we check the values of Shoushili-Li-Cheng against Mingshi and Naepyeon and find that all values are identical to each other, except for a few typo graphical errors in each book.

Comparing the length of the synodic month in the Shoushili with that obtained by modern calculations (i.e., 29.530587 d in 1280; refer to CAS2009), the difference is less than 1 s. Hence, we ignore the error in the length of the synodic month in the Shoushili. Instead, it is worth noting that the motion of the Sun speeds up or slows down before and after perihelion or aphelion, but not around the winter or summer solstices as mentioned in the Shoushili. However, it is well known that the time of the winter solstice was very close to the time of perihelion passage for Earth around 1280. According to our investigations, the Earth passed perihelion on JD 2188925.078000 d resulting in a difference of 0.48200 d compared to the time of winter solstice in 1280.

To evaluate the effect of the Intercalation and the Revolution interval constants in determining times of the new moon, we compute those times for calendars associated with the Shoushili for years ranging from 1280 to 1644, compare them with the results of modern calculations, and present the number distribution of differences in Figure 1 along with values of the root-mean-square (RMS). In the figure, panels (a) and (b) show the results at Beijing and Seoul, respectively. The solid and dotted lines represent the results for Shoushili and Datongli (Naepyeon in Fig. 1(b)), respectively. In each panel, the horizontal and vertical axes represent the difference in units of minutes in intervals of 15 min and the number, respectively.

As shown in Figure 1, times of the new moon provided by the Datongli are on average more accurate than those provided by the Shoushili at Beijing, that is RMS values are ∼23.8 min (cf. ∼21 min; Li & Zhang 1998a,b) and ∼30.7 min, respectively. Therefore, it can be said that the Datongli (or the later Shoushili) is one of the better calendars in China. However, the Shoushili gives better results than Datongli in Korea, That is, the RMS values by the former and latter calendars are ∼27.9 and ∼46.6 min, respectively, in Korea. Interestingly, the RMS by the Shoushili in Korea (with an RMS of ∼27.9 min) is even less than that in China (with an RMS of ∼30.7 min).

[4] *http://eclipse.gsfc.nasa.gov/phase/phasecat.html*

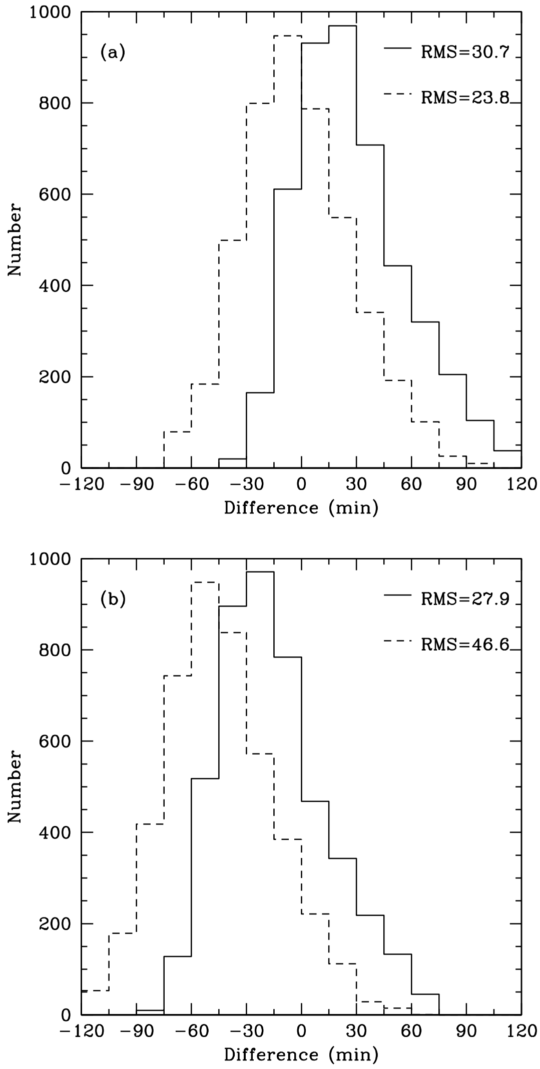

Fig. 1 The number distribution showing the differences in times for the new moon between calendars affiliated with the Shoushili and modern calculations at (a) Beijing and (b) Seoul. The solid and dotted lines represent the results for the Shoushili and Datongli (Naepyeon in (b)), respectively.

3.3 Time of Sunrise and Sunset

One way to assess the overall accuracy of the four interval constants is to check times for a solar eclipse. Before that, we verify the times of SR and SS presented in the calendar books because these times are used for calculating the times of solar eclipses.

In Figure 2, we depict the lengths of nighttime (i.e., period from SS to SR) from the Shoushili-Li-Cheng and the Mingshi along with the results of modern calculations. In the figure, the dotted lines represent the results (a) from Shoushili-Li-Cheng at Beijing and (b) from Mingshi at Nanjing, while the solid lines are the results from modern calculations at each location. In each figure, the

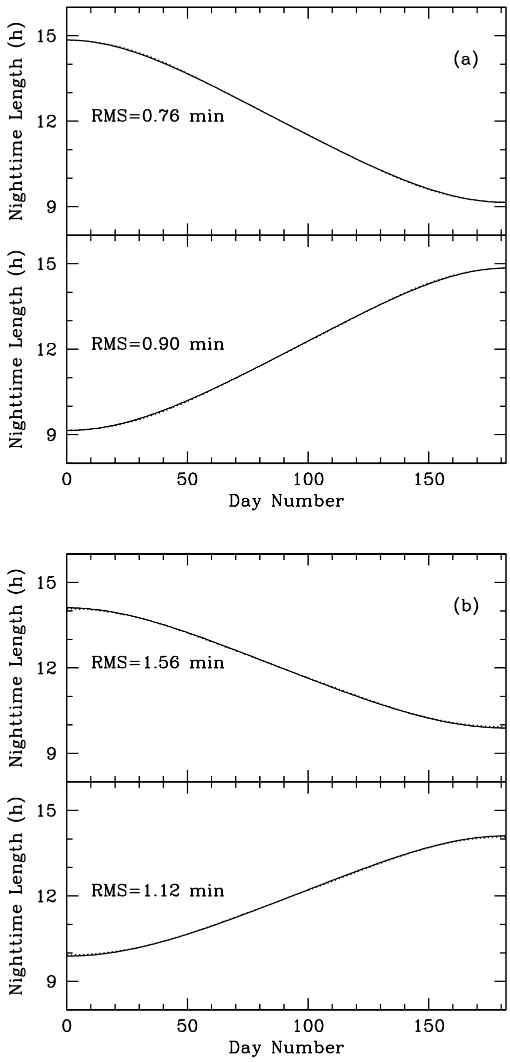

Fig. 2 Nighttime lengths (a) from Shoushili-Li-Cheng at Beijing and (b) from Mingshi (Taiyin-Tonggui) at Nanjing. The dotted and solid lines indicate results from the literature and the modern calculations, respectively, at each location. In each panel, the upper and lower parts are nighttime lengths after the winter and summer solstices, respectively. For more details, refer to the text.

upper and lower panels show the lengths of nighttime after the winter and summer solstices, respectively. The horizontal and vertical axes represent the day number and the nighttime length in units of hours, respectively. According to calendars affiliated with the Shoushili, the summer solstice of 1281 occurred on June 14, JD = 2189107.5 d (Lee et al. 2010). In this study, we define the SR/SS time as the zenith distance (z) of 90°, which is different from the modern definition of $z = 90° \; 50'$. The result for Naepyeon at Seoul is given in the work of Lee et al. (2011), and shows a difference of less than 1 min on average.

A table of times listing SR and SS is also contained in the Taiyin-Tonggui (Comprehensive Guide on the Sun), which is one volume in the Tonggui series. According to our observations, there were some disagreements between Mingshi and Taiyin-Tonggui in the SR and SS times. We can easily determine which document is incorrect by checking Daybreak (the period from midnight to SR or from SS to midnight) and Dusk (the period from midnight to SS) Parts because the sum of both parts should be 10 000, which is one day. In spite of this check, there were two discrepancies. However, these can be ignored because the differences were less than 1 Part. Hence, we can consider that the values of the SR and SS times are identical in both documents. Although there is no statement in the Taiyin-Tonggui, the Mingshi explicitly states that the SR and SS times are those at Nanjing. As shown in Figure 2, the RMS values are less than $\sim$1 min, compared to modern calculations, making it hard to distinguish them from each other. Therefore, we can confirm that the SR and SS times given in the Shoushili-Li-Cheng are those at Beijing and that the statement on the SR and SS times in Mingshi is true.

3.4 Time of a Solar Eclipse

Based on the four interval constants and the SR/SS times discussed above, we calculate the times for the maximum solar eclipse according to the calendars and compare these values with results using modern methods of computation. Prior to the comparison, we validate our calculations by calendars affiliated with the Shoushili using the records in historical literature. In the Garyeong, the procedure for calculating solar and lunar eclipses by Naepyeon is described in great detail. A calendar book entitled Jiaosi-Tonggui (Comprehensive Guide on the Eclipse), which is preserved in Kyuganggak, also lists step-by-step values in the process of calculating several eclipses by Datongli. We find that the results of our calculations of Naepyeon and Datongli show exact agreement with those of the documents. In particular, we find that the times of solar eclipse with Jiaosi-Tonggui come from the result that used the SR/SS times at Nanjing and not Beijing. We compare the calculations in the Shoushili with the records in the Mingshi. In the history book, the times for a total of 32 solar eclipses, calculated by the Shoushili, are recorded (see also Yabuuchi & Nakayama 2006). We find that all times match well except for four records: 707 June 1, 1059 January 1, 1061 June 1, and 1162 January 1 in the luni-solar calendar. According to our computations based on the Shoushili, there was no solar eclipse in 707, whereas the others show a difference of 1 Mark ($\sim$15 min). However, we think that the actual differences would be smaller than 1 Mark, which is the significant digit in the records. For the hour systems used in ancient China and Korea, refer to Saito (1995) and Lee et al. (2011).

In this study, we restrict ourselves to the cases where the maximum eclipse occurred during daytime, which means the Sun's altitude is greater than zero at the maximum eclipse.

Figure 3 shows the difference between the times given by calendars affiliated with the Shoushili (Ts) and by modern calculations (Tm), i.e., Ts − Tm, between 1281 and 1644, along with values for RMS. We use SR/SS times (a) at Beijing and (b) at Seoul. In each panel, the circles and crosses represent the results for the Shoushili and Datongli (Naepyeon in (b)), respectively.

According to this study, RMS values for the Shoushili and Datongli at Beijing are about $\sim$38.4 and $\sim$25.8 min, respectively. Meanwhile, Li & Zhang (1997) also studied the difference between the times of solar eclipse given by the Shoushili (according to this study, the later Shoushili) and by a modern ephemeris, and found that the RMS is $\sim$24 min. From the figure, we can easily see that the interval constants of the Datongli are better than those of the Shoushili at Beijing, which is similar to the case of times for the new moon. The situation is reversed at Seoul (RMS values for the Shoushili and Naepyeon are $\sim$50.8 and $\sim$77.1 min, respectively). Thus, the interval constants of the Shoushili give better results than those of Naepyeon for the calculating times for solar eclipses in Korea.

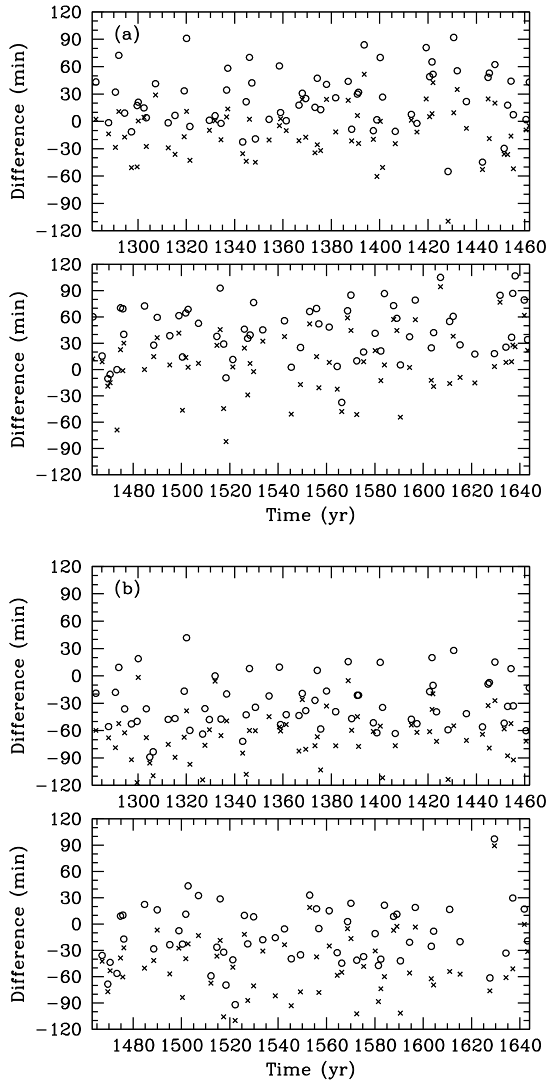

Fig. 3 Differences in the times for the maximum solar eclipse between calendars affiliated with the Shoushili and modern calculations according to the region (a) at Beijing and (b) at Seoul. The circles and crosses represent the results for the Shoushili and Datongli (Naepyeon in (b)), respectively.

4 SUMMARY

It is known that the Shoushili, from the Yuan dynasty, is one of the most accurate calendars in the history of China. The court of the next dynasty, the Ming dynasty, revised the calendar and entitled it Datongli. An example of the revisions is that the annual precession was abandoned. In the Joseon dynasty of Korea, both calendars were referred to for the compilation of Naepyeon by the Joseon royal astronomers. With regard to interval constants, the Joseon court adopted the values of Datongli (the later Shoushili) in Naepyeon. Although there are some differences, particularly in the values of the interval constants, the Datongli and the Naepyeon are basically identical to the (early) Shoushili.

In this paper, we study the four interval constants given in calendars affiliated with the Shoushili, which are related to motions of the Sun and the Moon: the Qi, Intercalation, Revolution, and Crossing interval constants. We first compare the values of those interval constants with the results of modern calculations, and then investigate the accuracy of times for the new moon and the maximum solar eclipse given by calendars affiliated with the Shoushili using the interval constants adopted in each calendar, along with times of SR and SS. The following is a summary of our findings.

(1) In the Shoushili, the Qi and Intercalation interval constants are well determined (with errors of ∼16.8 and ∼10.4 min, respectively) but the Revolution and Crossing ones are relatively poorly measured (with errors of ∼238.4 and ∼223.8 min, respectively). The latter two interval constants are worse in Datongli (with errors of ∼483.1 and ∼252.6 min, respectively).
(2) On calculating times for the new moon in the period from 1280 to 1644 using Intercalation and Revolution interval constants from calendars affiliated with the Shoushili, the results show that RMS values by the Shoushili and Datongli are 30.7 and 23.8 min, respectively, at Beijing, and 27.9 and 46.6 min, respectively, at Seoul. Therefore, it can be concluded that Datongli is the better calendar for China but not for Korea. Moreover, the Shoushili is, in general, more suitable for use in Korea rather than in China in terms of times for the new moon, at least for the period from 1280 to 1644.
(3) Unlike the interval constants, the timings of SR and SS are accurately determined in each calendar book, with errors of less than 1 min. In addition, the times listed in Shoushili-Li-Cheng are at Beijing, as we confirmed, and in Mingshi the times are at Nanjing, as noted in the book.
(4) The times of the the maximum solar eclipse by the Shoushili and Datongli, from 1281 to 1644, show RMS values of 38.4 and 25.8 min, respectively, at Beijing, and 50.8 and 77.1 min, respectively, at Seoul. Similar to the time for the new moon, the Shoushili gives better results than Datongli at Seoul in the calculation of a solar eclipse.

In the future, we think that more studies are needed to explain why the RMS values for the times of the new moon and the maximum solar eclipse are smaller, by a factor of one hundred, compared to the large errors in the Revolution and the Crossing interval constants of calendars affiliated with the Shoushili, which have much larger errors. One possibility might be the fact that both constants have opposite signs in the differences when compared to modern calculations.

Acknowledgements Ki-Won Lee is supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (2013R1A1A2013747).

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