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Spacecraft Doppler tracking with possible violations of LLI and LPI: a theoretical modeling *

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Abstract Currently two-way and three-way spacecraft Doppler tracking techniques are widely used and play important roles in control and navigation of deep space missions. Starting from a one-way Doppler model, we extend the theory to two-way and three-way Doppler models by making them include possible violations of the local Lorentz invariance (LLI) and the local position invariance (LPI) in order to test the Einstein equivalence principle, which is the cornerstone of general relativity and all other metric theories of gravity. After taking the finite speed of light into account, which is the so-called light time solution (LTS), we make these models depend on the time of reception of the signal only for practical convenience. We find that possible violations of LLI and LPI cannot affect two-way Doppler tracking under a linear approximation of LTS, although this approximation is sufficiently good for most cases in the solar system. We also show that, in three-way Doppler tracking, possible violations of LLI and LPI are only associated with two stations, which suggests that it is better to set the stations at places with significant differences in velocities and gravitational potentials to obtain a high level of sensitivity for the tests.

Key words: space vehicles — techniques: radial velocities — gravitation

1 INTRODUCTION

As one of the most important current methods for determining the motion of a spacecraft, the Doppler tracking technique has been successfully implemented in many deep space missions for control and navigation (Kruger 1965; Moyer & Yuen 2000). It can also be used for a variety of scientific applications, such as fundamental physics. The measurement of the frequency shift in signals relayed between the *Cassini* spacecraft and Earth yields a stringent test that demonstrates the validity of general relativity (GR) in the solar system (Bertotti et al. 2003). On the other hand, Kopeikin et al. (2007) point out that this test of GR is under a restrictive condition that the Sun's gravitational field is static, and if this restriction is removed, the test becomes less stringent. It is also known that

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Doppler tracking might be the only possible way to detect specific low-frequency (10^{-5} – 1 Hz) gravitational waves (see Armstrong 2006, for a recent review). In this work, we focus on another application in fundamental physics for testing the Einstein equivalence principle (EEP), which is the "heart and soul" of gravitational theory (see Will 1993, 2006, for reviews). It is worth mentioning that a number of notable scientists, including V. Fock, J. Synge, F. Rohlrich and others, do *not* support this "heart and soul" opinion (see Norton 1993, for a historical review of pros and cons related to EEP). Although this disagreement may even persist today, we support EEP in this work.

EEP is the cornerstone for building GR and all other metric theories of gravity. It states that (1) the trajectory of a freely falling test body is independent of its internal structure and composition, the so-called weak equivalence principle (WEP); (2) the outcome of any local non-gravitational experiment is independent of the velocity of the freely-falling reference frame where it is performed, the so-called local Lorentz invariance (LLI); and (3) the outcome of any local non-gravitational experiment is independent of where and when in the Universe it is performed, the so-called local position invariance (LPI) (see Will 1993, 2006, for more details). The second and third pieces of EEP, i.e. LLI and LPI, can be tested by measuring the frequency of a signal transmitted from a clock as it moves in the gravitational field of a massive body (e.g. Krisher 1990).

Gravity Probe A (GP-A) was launched by NASA in 1976. It carried a hydrogen maser oscillator nearly vertically upward to 10^7 m in the Earth's gravitational field and confirmed that the observed relativistic frequency shift agreed with prediction at the level of 7×10^{-5} (Vessot et al. 1980). The *Voyager* flybys of Saturn in 1980 made the first test of an extraterrestrial gravitational redshift and it verified the prediction of EEP to an accuracy of 1% as the spacecraft moved in and out of the gravitational field of Saturn (Krisher et al. 1990). During flybys of Venus and Earth in 1990, the *Galileo* mission performed an experiment on solar redshift and confirmed the total frequency shift predicted by EEP to an accuracy of 0.5% and the solar gravitational redshift to an accuracy of 1% (Krisher et al. 1993).

All these experiments relied on a one-way radio signal transmitted from the spacecraft to ground stations. The transmitted frequency was referred to as the onboard clock or frequency standard, while the received signal was referred to as these standards at the stations. However, one-way Doppler tracking has practical problems for precision tracking of spacecraft. Onboard frequency standards are significantly less stable than ground-based standards and they are limited by their own noise. One solution for this is to use two-way Doppler tracking. In the two-way mode, the ground station emits a radio signal referenced to a high-quality frequency standard. Then, the spacecraft receives this signal and phase-coherently retransmits it to Earth. The transponding process adds noise, but at negligible levels in current observations, and does not require a good oscillator on the spacecraft (see Armstrong 2006 for a review). The two-way Doppler tracking forms a closed-loop for the signal. The three-way mode has an open-loop because Station 1 emits the signal, but the transponded signal is received by Station 2.

Therefore, considering these advantages, we will theoretically extend relativistic models of two-way and three-way Doppler tracking by including possible violations of LLI and LPI as the first step. Case studies will be left for subsequent works.

In Section 2, starting from the one-way Doppler case, we will construct these models. Since the radio signals travel with finite speed, the light time solution will be corrected in Section 3. Conclusions and discussion will be presented in Section 4.

2 DOPPLER TRACKING WITH VIOLATIONS OF LLI AND LPI

In the following investigation, we will build our models of two-way and three-way Doppler tracking within the solar system barycentric reference system by starting from the one-way Doppler model (Krisher et al. 1993).

2.1 One-Way Doppler Tracking

It is well known that EEP predicts a shift in the frequency (Weinberg 1972; Misner et al. 1973). The observed redshift z is defined as

$$1 + z = \frac{\nu_{\rm R}(t_{\rm R})}{\nu_{\rm E}(t_{\rm E})}\,, \tag{1}$$

where $\nu_{\rm E}(t_{\rm E})$ is the frequency of an emitted signal at time $t_{\rm E}$ and $\nu_{\rm R}(t_{\rm R})$ is the frequency of a received signal at time $t_{\rm R}$. In the following parts of this work, we will omit these dependences on $t_{\rm E}$ and $t_{\rm R}$ in notations so that $\nu_{\rm E} \equiv \nu_{\rm E}(t_{\rm E})$ and $\nu_{\rm R} \equiv \nu_{\rm E}(t_{\rm R})$ unless we specify exceptional cases. Up to the order of ϵ^2 where $\epsilon \equiv c^{-1}$ and c is the speed of light, this relation can be written as (Brumberg 1991; Krisher et al. 1993; Kopeikin et al. 2011)

$$\begin{aligned}\frac{\nu_{\rm R}}{\nu_{\rm E}} \;=\;& 1 + \epsilon\boldsymbol{K}\cdot[\boldsymbol{v}_{\rm R}(t_{\rm R}) - \boldsymbol{v}_{\rm E}(t_{\rm E})] - \epsilon^2[\boldsymbol{K}\cdot\boldsymbol{v}_{\rm R}(t_{\rm R})][\boldsymbol{K}\cdot\boldsymbol{v}_{\rm E}(t_{\rm E})] + \epsilon^2[\boldsymbol{K}\cdot\boldsymbol{v}_{\rm E}(t_{\rm E})]^2\\ &+\frac{1}{2}\epsilon^2[\boldsymbol{v}^2_{\rm R}(t_{\rm R}) - \boldsymbol{v}^2_{\rm E}(t_{\rm E})] + \epsilon^2\{U[\boldsymbol{y}_{\rm R}(t_{\rm R})] - U[\boldsymbol{y}_{\rm E}(t_{\rm E})]\}\\ &+\mathcal{O}(\epsilon^3)\,,\end{aligned} \tag{2}$$

where $\boldsymbol{y}_{\rm E}$ and $\boldsymbol{y}_{\rm R}$ are the respective positional vectors of the emitter and receiver, $\boldsymbol{v}_{\rm E}$ and $\boldsymbol{v}_{\rm R}$ are their respective velocities, $t_{\rm E}$ and $t_{\rm R}$ are the respective times of emission and reception, and the unit vector $\boldsymbol{K}$ is

$$\boldsymbol{K} = -\frac{\boldsymbol{y}_{\rm R}(t_{\rm R}) - \boldsymbol{y}_{\rm E}(t_{\rm E})}{|\boldsymbol{y}_{\rm R}(t_{\rm R}) - \boldsymbol{y}_{\rm E}(t_{\rm E})|}\,. \tag{3}$$

Here, $U[\boldsymbol{y}_{\rm E}(t_{\rm E})]$ and $U[\boldsymbol{y}_{\rm R}(t_{\rm R})]$ are the Newtonian gravitational potentials at the emitter and receiver respectively, which can be written as

$$U[\boldsymbol{y}_{\rm R}(t_{\rm R})] = \sum_A U_A[\boldsymbol{y}_{\rm R}(t_{\rm R})] \quad \text{and} \quad U[\boldsymbol{y}_{\rm E}(t_{\rm E})] = \sum_A U_A[\boldsymbol{y}_{\rm E}(t_{\rm E})]\,. \tag{4}$$

In Equation (2), all velocity-dependent terms originate in special relativity, but the terms depending on the gravitational potentials are predicted by GR.

In order to test EEP, following Krisher et al. (1993), we adopt the parametrization of the one-way Doppler model [see Equation (2)] as

$$\begin{aligned}\frac{\nu_{\rm R}}{\nu_{\rm E}} \;=\;& 1 + \epsilon\boldsymbol{K}\cdot[\boldsymbol{v}_{\rm R}(t_{\rm R}) - \boldsymbol{v}_{\rm E}(t_{\rm E})] - \epsilon^2[\boldsymbol{K}\cdot\boldsymbol{v}_{\rm R}(t_{\rm R})][\boldsymbol{K}\cdot\boldsymbol{v}_{\rm E}(t_{\rm E})] + \epsilon^2[\boldsymbol{K}\cdot\boldsymbol{v}_{\rm E}(t_{\rm E})]^2\\ &+\frac{1}{2}\epsilon^2\beta_{\rm R}\boldsymbol{v}^2_{\rm R}(t_{\rm R}) - \frac{1}{2}\epsilon^2\beta_{\rm E}\boldsymbol{v}^2_{\rm E}(t_{\rm E}) + \epsilon^2\sum_A \alpha^A_{\rm R}U_A[\boldsymbol{y}_{\rm R}(t_{\rm R})] - \epsilon^2\sum_A \alpha^A_{\rm E}U_A[\boldsymbol{y}_{\rm E}(t_{\rm E})]\\ &+\mathcal{O}(\epsilon^3)\,.\end{aligned} \tag{5}$$

Equation (5) describes the shift in frequency with possible violations of LLI and LPI. Here, violations of LLI can be tested by fitting the dimensionless parameters $\beta_{\rm R}$ and $\beta_{\rm E}$. If LLI is valid, then $\beta_{\rm R/E} = 1$. Violations of LPI can be tested by fitting the dimensionless parameters $\alpha^A_{\rm R}$ and $\alpha^A_{\rm E}$. If LPI holds true, $\alpha^A_{\rm R/E} = 1$.

For separating these possible violations, we will also use notations $\bar{\beta}_{\rm R/E} \equiv \beta_{\rm R/E} - 1$ and $\bar{\alpha}^A_{\rm R/E} \equiv \alpha^A_{\rm R/E} - 1$. Equation (5) can be rewritten as

$$\left.\frac{\nu_{\rm R}}{\nu_{\rm E}}\right|_{\rm E\to R} \equiv \mathcal{F}_{\rm E\to R}(t_{\rm E}, t_{\rm R}) = \hat{\mathcal{F}}_{\rm E\to R}(t_{\rm E}, t_{\rm R}) + \bar{\mathcal{F}}_{\rm E\to R}(t_{\rm E}, t_{\rm R}) + \mathcal{O}(\epsilon^3)\,, \tag{6}$$

where $\hat{\mathcal{F}}_{\mathrm{E}\to\mathrm{R}}(t_{\mathrm{E}},t_{\mathrm{R}})$ represents the shift in frequency predicted by GR to be

$$\begin{aligned}\hat{\mathcal{F}}_{\mathrm{E}\to\mathrm{R}}(t_{\mathrm{E}},t_{\mathrm{R}}) \;=\;& 1+\epsilon\boldsymbol{K}\cdot[\boldsymbol{v}_{\mathrm{R}}(t_{\mathrm{R}})-\boldsymbol{v}_{\mathrm{E}}(t_{\mathrm{E}})]-\epsilon^2[\boldsymbol{K}\cdot\boldsymbol{v}_{\mathrm{R}}(t_{\mathrm{R}})][\boldsymbol{K}\cdot\boldsymbol{v}_{\mathrm{E}}(t_{\mathrm{E}})]+\epsilon^2[\boldsymbol{K}\cdot\boldsymbol{v}_{\mathrm{E}}(t_{\mathrm{E}})]^2\\ &+\frac{1}{2}\epsilon^2\boldsymbol{v}_{\mathrm{R}}^2(t_{\mathrm{R}})-\frac{1}{2}\epsilon^2\boldsymbol{v}_{\mathrm{E}}^2(t_{\mathrm{E}})+\epsilon^2\sum_A U_A[\boldsymbol{y}_{\mathrm{R}}(t_{\mathrm{R}})]-\epsilon^2\sum_A U_A[\boldsymbol{y}_{\mathrm{E}}(t_{\mathrm{E}})]\,, \qquad (7)\end{aligned}$$

and $\bar{\mathcal{F}}_{\mathrm{E}\to\mathrm{R}}(t_{\mathrm{E}},t_{\mathrm{R}})$ indicates the effects caused by possible violations in LLI and LPI to be

$$\bar{\mathcal{F}}_{\mathrm{E}\to\mathrm{R}}(t_{\mathrm{E}},t_{\mathrm{R}})=\frac{1}{2}\epsilon^2\bar{\beta}_{\mathrm{R}}\boldsymbol{v}_{\mathrm{R}}^2(t_{\mathrm{R}})-\frac{1}{2}\epsilon^2\bar{\beta}_{\mathrm{E}}\boldsymbol{v}_{\mathrm{E}}^2(t_{\mathrm{E}})+\epsilon^2\sum_A\bar{\alpha}_{\mathrm{R}}^A U_A[\boldsymbol{y}_{\mathrm{R}}(t_{\mathrm{R}})]-\epsilon^2\sum_A\bar{\alpha}_{\mathrm{E}}^A U_A[\boldsymbol{y}_{\mathrm{E}}(t_{\mathrm{E}})]\,. \qquad (8)$$

Equation (6) will be used to develop two-way and three-way Doppler tracking.

2.2 Two-Way Doppler Tracking

In two-way Doppler tracking, a ground station (S) emits a radio signal ν_{E} at time t_{E} and a spacecraft (P) receives the signal with frequency ν' at time t'; then, the spacecraft (P) immediately transmits the radio signal $q\nu'$ back, where q is a known ratio between two integers; and station (S) receives the signal with frequency ν_{R} at time t_{R}. The whole procedure can be decomposed into two one-way Doppler trackings and the shift in the frequency in this closed-loop can be easily and concisely expressed as

$$\left.\frac{\nu_{\mathrm{R}}}{q\nu_{\mathrm{E}}}\right|_{\mathrm{S}\to\mathrm{P}\to\mathrm{S}}=\frac{\nu'}{\nu_{\mathrm{E}}}\cdot\frac{\nu_{\mathrm{R}}}{q\nu'}=\mathcal{F}_{\mathrm{S}\to\mathrm{P}}(t_{\mathrm{E}},t')\cdot\mathcal{F}_{\mathrm{P}\to\mathrm{S}}(t',t_{\mathrm{R}})+\mathcal{O}(\epsilon^3)\,, \qquad (9)$$

whose explicit form can be written as

$$\begin{aligned}\left.\frac{\nu_{\mathrm{R}}}{q\nu_{\mathrm{E}}}\right|_{\mathrm{S}\to\mathrm{P}\to\mathrm{S}} \;=\;& 1+\epsilon\boldsymbol{K}'_{2\mathrm{w}}\cdot[\boldsymbol{v}_{\mathrm{P}}(t')-\boldsymbol{v}_{\mathrm{S}}(t_{\mathrm{E}})]+\epsilon\boldsymbol{K}''_{2\mathrm{w}}\cdot[\boldsymbol{v}_{\mathrm{S}}(t_{\mathrm{R}})-\boldsymbol{v}_{\mathrm{P}}(t')]\\ &+\epsilon^2\{\boldsymbol{K}'_{2\mathrm{w}}\cdot[\boldsymbol{v}_{\mathrm{P}}(t')-\boldsymbol{v}_{\mathrm{S}}(t_{\mathrm{E}})]\}\{\boldsymbol{K}''_{2\mathrm{w}}\cdot[\boldsymbol{v}_{\mathrm{S}}(t_{\mathrm{R}})-\boldsymbol{v}_{\mathrm{P}}(t')]\}\\ &-\epsilon^2[\boldsymbol{K}'_{2\mathrm{w}}\cdot\boldsymbol{v}_{\mathrm{P}}(t')][\boldsymbol{K}'_{2\mathrm{w}}\cdot\boldsymbol{v}_{\mathrm{S}}(t_{\mathrm{E}})]+\epsilon^2[\boldsymbol{K}'_{2\mathrm{w}}\cdot\boldsymbol{v}_{\mathrm{S}}(t_{\mathrm{E}})]^2\\ &-\epsilon^2[\boldsymbol{K}''_{2\mathrm{w}}\cdot\boldsymbol{v}_{\mathrm{S}}(t_{\mathrm{R}})][\boldsymbol{K}''_{2\mathrm{w}}\cdot\boldsymbol{v}_{\mathrm{P}}(t')]+\epsilon^2[\boldsymbol{K}''_{2\mathrm{w}}\cdot\boldsymbol{v}_{\mathrm{P}}(t')]^2\\ &+\frac{1}{2}\epsilon^2[\boldsymbol{v}_{\mathrm{P}}^2(t')-\boldsymbol{v}_{\mathrm{S}}^2(t_{\mathrm{E}})]+\epsilon^2\left\{\sum_A U_A[\boldsymbol{y}_{\mathrm{P}}(t')]-\sum_A U_A[\boldsymbol{y}_{\mathrm{S}}(t_{\mathrm{E}})]\right\}\\ &+\frac{1}{2}\epsilon^2[\boldsymbol{v}_{\mathrm{S}}^2(t_{\mathrm{R}})-\boldsymbol{v}_{\mathrm{P}}^2(t')]+\epsilon^2\left\{\sum_A U_A[\boldsymbol{y}_{\mathrm{S}}(t_{\mathrm{R}})]-\sum_A U_A[\boldsymbol{y}_{\mathrm{P}}(t')]\right\}\\ &+\frac{1}{2}\epsilon^2[\bar{\beta}_{\mathrm{P}}\boldsymbol{v}_{\mathrm{P}}^2(t')-\bar{\beta}_{\mathrm{S}}\boldsymbol{v}_{\mathrm{S}}^2(t_{\mathrm{E}})]+\epsilon^2\left\{\sum_A\bar{\alpha}_{\mathrm{P}}^A U_A[\boldsymbol{y}_{\mathrm{P}}(t')]-\sum_A\bar{\alpha}_{\mathrm{S}}^A U_A[\boldsymbol{y}_{\mathrm{S}}(t_{\mathrm{E}})]\right\}\\ &+\frac{1}{2}\epsilon^2[\bar{\beta}_{\mathrm{S}}\boldsymbol{v}_{\mathrm{S}}^2(t_{\mathrm{R}})-\bar{\beta}_{\mathrm{P}}\boldsymbol{v}_{\mathrm{P}}^2(t')]+\epsilon^2\left\{\sum_A\bar{\alpha}_{\mathrm{S}}^A U_A[\boldsymbol{y}_{\mathrm{S}}(t_{\mathrm{R}})]-\sum_A\bar{\alpha}_{\mathrm{P}}^A U_A[\boldsymbol{y}_{\mathrm{P}}(t')]\right\}\\ &+\mathcal{O}(\epsilon^3)\,, \qquad (10)\end{aligned}$$

where

$$\boldsymbol{K}'_{2\mathrm{w}}=-\frac{\boldsymbol{y}_{\mathrm{P}}(t')-\boldsymbol{y}_{\mathrm{S}}(t_{\mathrm{E}})}{|\boldsymbol{y}_{\mathrm{P}}(t')-\boldsymbol{y}_{\mathrm{S}}(t_{\mathrm{E}})|}\quad\text{and}\quad\boldsymbol{K}''_{2\mathrm{w}}=-\frac{\boldsymbol{y}_{\mathrm{S}}(t_{\mathrm{R}})-\boldsymbol{y}_{\mathrm{P}}(t')}{|\boldsymbol{y}_{\mathrm{S}}(t_{\mathrm{R}})-\boldsymbol{y}_{\mathrm{P}}(t')|}\,. \qquad (11)$$

In a special case that $t_{\mathrm{E}}=t'=t_{\mathrm{R}}$, we can omit them, so we have

$$\left.\frac{\nu_{\mathrm{R}}}{q\nu_{\mathrm{E}}}\right|_{\mathrm{S}\to\mathrm{P}\to\mathrm{S}}^{t_{\mathrm{E}}=t'=t_{\mathrm{R}}}=1-2\epsilon\boldsymbol{n}_{\mathrm{PS}}\cdot\boldsymbol{v}_{\mathrm{PS}}+2\epsilon^2(\boldsymbol{n}_{\mathrm{PS}}\cdot\boldsymbol{v}_{\mathrm{PS}})^2+\mathcal{O}(\epsilon^3)\,, \qquad (12)$$

where $\boldsymbol{v}_{\mathrm{PS}} = \boldsymbol{v}_{\mathrm{P}} - \boldsymbol{v}_{\mathrm{S}}$, $\boldsymbol{n}_{\mathrm{PS}} = \boldsymbol{R}_{\mathrm{PS}}/R_{\mathrm{PS}}$, $\boldsymbol{R}_{\mathrm{PS}} \equiv \boldsymbol{y}_{\mathrm{P}} - \boldsymbol{y}_{\mathrm{S}}$ and $R_{\mathrm{PS}} = |\boldsymbol{R}_{\mathrm{PS}}|$. When velocities of the spacecraft and the station are very small, this instantaneous approximation of a three-way equality in the above equation can be regarded as valid. The condition $t_{\mathrm{E}} = t' = t_{\mathrm{R}}$ also means the light time (see the next section for details) is not taken into account.

2.3 Three-Way Doppler Tracking

In three-way Doppler tracking, there are two stations. Station 1 (S_1) emits a signal and Station 2 (S_2) receives the signal transmitted by the spacecraft (P). In this open loop, the shift in frequency is

$$\left.\frac{\nu_{\mathrm{R}}}{q\nu_{\mathrm{E}}}\right|_{\mathrm{S}_1\to\mathrm{P}\to\mathrm{S}_2} = \frac{\nu'}{\nu_{\mathrm{E}}}\cdot\frac{\nu_{\mathrm{R}}}{q\nu'} = \mathcal{F}_{\mathrm{S}_1\to\mathrm{P}}(t_{\mathrm{E}},t')\cdot\mathcal{F}_{\mathrm{P}\to\mathrm{S}_2}(t',t_{\mathrm{R}}) + \mathcal{O}(\epsilon^3)\,, \tag{13}$$

whose explicit form can be written as

$$\begin{aligned}\left.\frac{\nu_{\mathrm{R}}}{q\nu_{\mathrm{E}}}\right|_{\mathrm{S}_1\to\mathrm{P}\to\mathrm{S}_2} =\; & 1 + \epsilon\boldsymbol{K}'_{3\mathrm{w}}\cdot[\boldsymbol{v}_{\mathrm{P}}(t') - \boldsymbol{v}_{\mathrm{S}_1}(t_{\mathrm{E}})] + \epsilon\boldsymbol{K}''_{3\mathrm{w}}\cdot[\boldsymbol{v}_{\mathrm{S}_2}(t_{\mathrm{R}}) - \boldsymbol{v}_{\mathrm{P}}(t')]\\
&+\epsilon^2\{\boldsymbol{K}'_{3\mathrm{w}}\cdot[\boldsymbol{v}_{\mathrm{P}}(t') - \boldsymbol{v}_{\mathrm{S}_1}(t_{\mathrm{E}})]\}\{\boldsymbol{K}''_{3\mathrm{w}}\cdot[\boldsymbol{v}_{\mathrm{S}_2}(t_{\mathrm{R}}) - \boldsymbol{v}_{\mathrm{P}}(t')]\}\\
&-\epsilon^2[\boldsymbol{K}'_{3\mathrm{w}}\cdot\boldsymbol{v}_{\mathrm{P}}(t')][\boldsymbol{K}'_{3\mathrm{w}}\cdot\boldsymbol{v}_{\mathrm{S}_1}(t_{\mathrm{E}})] + \epsilon^2[\boldsymbol{K}'_{3\mathrm{w}}\cdot\boldsymbol{v}_{\mathrm{S}_1}(t_{\mathrm{E}})]^2\\
&-\epsilon^2[\boldsymbol{K}''_{3\mathrm{w}}\cdot\boldsymbol{v}_{\mathrm{S}_2}(t_{\mathrm{R}})][\boldsymbol{K}''_{3\mathrm{w}}\cdot\boldsymbol{v}_{\mathrm{P}}(t')] + \epsilon^2[\boldsymbol{K}''_{3\mathrm{w}}\cdot\boldsymbol{v}_{\mathrm{P}}(t')]^2\\
&+\frac{1}{2}\epsilon^2[\boldsymbol{v}^2_{\mathrm{P}}(t') - \boldsymbol{v}^2_{\mathrm{S}_1}(t_{\mathrm{E}})] + \epsilon^2\Big\{\sum_A U_A[\boldsymbol{y}_{\mathrm{P}}(t')] - \sum_A U_A[\boldsymbol{y}_{\mathrm{S}_1}(t_{\mathrm{E}})]\Big\}\\
&+\frac{1}{2}\epsilon^2[\boldsymbol{v}^2_{\mathrm{S}_2}(t_{\mathrm{R}}) - \boldsymbol{v}^2_{\mathrm{P}}(t')] + \epsilon^2\Big\{\sum_A U_A[\boldsymbol{y}_{\mathrm{S}_2}(t_{\mathrm{R}})] - \sum_A U_A[\boldsymbol{y}_{\mathrm{P}}(t')]\Big\}\\
&+\frac{1}{2}\epsilon^2[\bar{\beta}_{\mathrm{P}}\boldsymbol{v}^2_{\mathrm{P}}(t') - \bar{\beta}_{\mathrm{S}_1}\boldsymbol{v}^2_{\mathrm{S}_1}(t_{\mathrm{E}})] + \epsilon^2\Big\{\sum_A \bar{\alpha}^A_{\mathrm{P}}U_A[\boldsymbol{y}_{\mathrm{P}}(t')] - \sum_A \bar{\alpha}^A_{\mathrm{S}_1}U_A[\boldsymbol{y}_{\mathrm{S}_1}(t_{\mathrm{E}})]\Big\}\\
&+\frac{1}{2}\epsilon^2[\bar{\beta}_{\mathrm{S}_2}\boldsymbol{v}^2_{\mathrm{S}_2}(t_{\mathrm{R}}) - \bar{\beta}_{\mathrm{P}}\boldsymbol{v}^2_{\mathrm{P}}(t')] + \epsilon^2\Big\{\sum_A \bar{\alpha}^A_{\mathrm{S}_2}U_A[\boldsymbol{y}_{\mathrm{S}_2}(t_{\mathrm{R}})] - \sum_A \bar{\alpha}^A_{\mathrm{P}}U_A[\boldsymbol{y}_{\mathrm{P}}(t')]\Big\}\\
&+\mathcal{O}(\epsilon^3)\,,\end{aligned} \tag{14}$$

where

$$\boldsymbol{K}'_{3\mathrm{w}} = -\frac{\boldsymbol{y}_{\mathrm{P}}(t') - \boldsymbol{y}_{\mathrm{S}_1}(t_{\mathrm{E}})}{|\boldsymbol{y}_{\mathrm{P}}(t') - \boldsymbol{y}_{\mathrm{S}_1}(t_{\mathrm{E}})|} \quad\text{and}\quad \boldsymbol{K}''_{3\mathrm{w}} = -\frac{\boldsymbol{y}_{\mathrm{S}_2}(t_{\mathrm{R}}) - \boldsymbol{y}_{\mathrm{P}}(t')}{|\boldsymbol{y}_{\mathrm{S}_2}(t_{\mathrm{R}}) - \boldsymbol{y}_{\mathrm{P}}(t')|}\,. \tag{15}$$

In the special case that $t_{\mathrm{E}} = t' = t_{\mathrm{R}}$, we can have

$$\begin{aligned}\left.\frac{\nu_{\mathrm{R}}}{q\nu_{\mathrm{E}}}\right|^{t_{\mathrm{E}}=t'=t_{\mathrm{R}}}_{\mathrm{S}_1\to\mathrm{P}\to\mathrm{S}_2} =\; & 1 - \epsilon\boldsymbol{n}_{\mathrm{PS}_1}\cdot\boldsymbol{v}_{\mathrm{PS}_1} - \epsilon\boldsymbol{n}_{\mathrm{PS}_2}\cdot\boldsymbol{v}_{\mathrm{PS}_2} + \epsilon^2(\boldsymbol{n}_{\mathrm{PS}_1}\cdot\boldsymbol{v}_{\mathrm{PS}_1})(\boldsymbol{n}_{\mathrm{PS}_2}\cdot\boldsymbol{v}_{\mathrm{PS}_2})\\
&-\epsilon^2(\boldsymbol{n}_{\mathrm{PS}_1}\cdot\boldsymbol{v}_{\mathrm{P}})(\boldsymbol{n}_{\mathrm{PS}_1}\cdot\boldsymbol{v}_{\mathrm{S}_1}) + \epsilon^2(\boldsymbol{n}_{\mathrm{PS}_1}\cdot\boldsymbol{v}_{\mathrm{S}_1})^2\\
&-\epsilon^2(\boldsymbol{n}_{\mathrm{PS}_2}\cdot\boldsymbol{v}_{\mathrm{P}})(\boldsymbol{n}_{\mathrm{PS}_2}\cdot\boldsymbol{v}_{\mathrm{S}_2}) + \epsilon^2(\boldsymbol{n}_{\mathrm{PS}_2}\cdot\boldsymbol{v}_{\mathrm{P}})^2\\
&+\frac{1}{2}\epsilon^2(\boldsymbol{v}^2_{\mathrm{S}_2} - \boldsymbol{v}^2_{\mathrm{S}_1}) + \epsilon^2\Big[\sum_A U_A(\boldsymbol{y}_{\mathrm{S}_2}) - \sum_A U_A(\boldsymbol{y}_{\mathrm{S}_1})\Big]\\
&+\frac{1}{2}\epsilon^2(\bar{\beta}_{\mathrm{S}_2}\boldsymbol{v}^2_{\mathrm{S}_2} - \bar{\beta}_{\mathrm{S}_1}\boldsymbol{v}^2_{\mathrm{S}_1}) + \epsilon^2\Big[\sum_A \bar{\alpha}^A_{\mathrm{S}_2}U_A(\boldsymbol{y}_{\mathrm{S}_2}) - \sum_A \bar{\alpha}^A_{\mathrm{S}_1}U_A(\boldsymbol{y}_{\mathrm{S}_1})\Big]\\
&+\mathcal{O}(\epsilon^3)\,,\end{aligned} \tag{16}$$

where $\boldsymbol{v}_{\mathrm{PS}_{1/2}} = \boldsymbol{v}_{\mathrm{P}} - \boldsymbol{v}_{\mathrm{S}_{1/2}}$, $\boldsymbol{n}_{\mathrm{PS}_{1/2}} = \boldsymbol{R}_{\mathrm{PS}_{1/2}}/R_{\mathrm{PS}_{1/2}}$, $\boldsymbol{R}_{\mathrm{PS}_{1/2}} \equiv \boldsymbol{y}_{\mathrm{P}} - \boldsymbol{y}_{\mathrm{S}_{1/2}}$ and $R_{\mathrm{PS}_{1/2}} = |\boldsymbol{R}_{\mathrm{PS}_{1/2}}|$. This equation can be expressed in the same form as equation (28) in Cao et al. (2011) when LLI and LPI are valid.

Although these theoretical models have been established (see Equations (6), (9) and (13)), they are still difficult to put into practice because of their dependences on t_{E} and/or t' which are usually unavailable in real measurements. In order to solve this problem and make these models only depend on the time of reception for the signal t_{R}, we need the light time solution (Moyer & Yuen 2000).

3 LIGHT TIME SOLUTION

The primary advantage of the light time solution (LTS) is to bridge the gaps between t_{E}, t' and t_{R} (see Chapter 8 in Moyer & Yuen 2000, for details). In the general case, t_{E} and t_{R} are related as

$$\Delta t \equiv (t_{\mathrm{R}} - t_{\mathrm{E}}) = \epsilon|\boldsymbol{y}_{\mathrm{R}}(t_{\mathrm{R}}) - \boldsymbol{y}_{\mathrm{E}}(t_{\mathrm{E}})| + \epsilon^3 \Delta\mathcal{T}_{\mathrm{Shapiro}} + \mathcal{O}(\epsilon^5)\,, \tag{17}$$

where the second term on the right-hand side is the Shapiro time delay caused by the curvature of spacetime (Shapiro 1964). The Shapiro delay is intensively studied in Moyer & Yuen (2000). For light traveling from Jupiter, grazing the surface of the Sun, and arriving at Earth, its delay due to the Sun is about 10^{-4} s. For light traveling from Saturn, grazing the surface of Jupiter, and arriving at Earth, its effect due to Jupiter's mass is $\sim 10^{-7}$ s. For a one-way case that light travels from Saturn, grazes the surface of Earth and then stops, this delay caused by the mass of the Earth is $\sim 10^{-10}$ s. The magnitudes of such Shapiro time delays are very much less than the timescales of translational and rotational motions of the emitters and receivers of the Doppler tracking links in the solar system, so that we can ignore the Shapiro time delays in the LTS and only keep

$$\Delta t \equiv (t_{\mathrm{R}} - t_{\mathrm{E}}) = \epsilon|\boldsymbol{y}_{\mathrm{R}}(t_{\mathrm{R}}) - \boldsymbol{y}_{\mathrm{E}}(t_{\mathrm{E}})| + \mathcal{O}(\epsilon^3)\,. \tag{18}$$

To numerically solve the above equation, one can use an iterative method. In this work, we prefer to obtain an explicit solution. Since, in Doppler tracking of a spacecraft, the timescales of orbital motions of an emitter and receiver are usually much larger than the timescales of light propagation Δt, we can express the Taylor expansion as

$$\boldsymbol{y}_{\mathrm{R}}(t_{\mathrm{R}}) = \boldsymbol{y}_{\mathrm{R}}(t_{\mathrm{E}} + \Delta t) = \boldsymbol{y}_{\mathrm{R}}(t_{\mathrm{E}}) + \boldsymbol{v}_{\mathrm{R}}(t_{\mathrm{E}})\Delta t + \frac{1}{2}\boldsymbol{a}_{\mathrm{R}}(t_{\mathrm{E}})\Delta t^2 + \mathcal{O}(\Delta t^3)\,, \tag{19}$$

and

$$\boldsymbol{y}_{\mathrm{E}}(t_{\mathrm{E}}) = \boldsymbol{y}_{\mathrm{E}}(t_{\mathrm{R}} - \Delta t) = \boldsymbol{y}_{\mathrm{E}}(t_{\mathrm{R}}) - \boldsymbol{v}_{\mathrm{E}}(t_{\mathrm{R}})\Delta t + \frac{1}{2}\boldsymbol{a}_{\mathrm{E}}(t_{\mathrm{R}})\Delta t^2 + \mathcal{O}(\Delta t^3)\,. \tag{20}$$

Moyer & Yuen (2000) argue that the maximum acceleration in the solar system occurs in a region near the Sun ($a \sim 25 - 274$ m s^{-2}) and a region near Jupiter ($a \sim 25$ m s^{-2}). As long as the spacecraft and station are outside of these regions, Moyer & Yuen (2000) suggest the acceleration terms in the two above equations can safely be dropped. If we assume all of the Doppler measurements are recorded in terms of t_{R}, a sufficiently good linear approximation to the LTS is

$$\boldsymbol{y}_{\mathrm{E}}(t_{\mathrm{E}}) = \boldsymbol{y}_{\mathrm{E}}(t_{\mathrm{R}}) - \boldsymbol{v}_{\mathrm{E}}(t_{\mathrm{R}})\Delta t + \mathcal{O}(\Delta t^2)\,, \tag{21}$$

and

$$\Delta t = \epsilon|\boldsymbol{y}_{\mathrm{R}}(t_{\mathrm{R}}) - \boldsymbol{y}_{\mathrm{E}}(t_{\mathrm{R}})| + \mathcal{O}(\epsilon^3)\,. \tag{22}$$

For practical convenience, we will make Doppler models only depend on the time of reception for the signal by using such a linear LTS, which is sufficient for most cases (Moyer & Yuen 2000).

3.1 One-Way Doppler Tracking with LTS

With Equation (22), the equation describing one-way Doppler tracking can formally be written as

$$\left.\frac{\nu_{\rm R}}{\nu_{\rm E}}\right|_{\rm E\to R} = \mathcal{F}_{\rm E\to R}(t_{\rm E},t_{\rm R}) = \mathcal{F}_{\rm E\to R}[t_{\rm R}-\epsilon|\boldsymbol{y}_{\rm R}(t_{\rm R})-\boldsymbol{y}_{\rm E}(t_{\rm R})|,t_{\rm R}]+\mathcal{O}(\epsilon^3)\,. \qquad (23)$$

To obtain its explicit expression, we need the expansion of the unit vector $\boldsymbol{K}$ [see Equation (3)] which is

$$\boldsymbol{K} = -\boldsymbol{n}_{\rm RE}(t_{\rm R}) - \epsilon\{\boldsymbol{v}_{\rm E}(t_{\rm R}) - [\boldsymbol{n}_{\rm RE}(t_{\rm R})\cdot\boldsymbol{v}_{\rm E}(t_{\rm R})]\boldsymbol{n}_{\rm RE}(t_{\rm R})\} + \mathcal{O}(\epsilon^2)\,, \qquad (24)$$

where $\boldsymbol{n}_{\rm RE}(t_{\rm R}) = \boldsymbol{R}_{\rm RE}(t_{\rm R})/R_{\rm RE}(t_{\rm R})$, $\boldsymbol{R}_{\rm RE}(t_{\rm R}) \equiv \boldsymbol{y}_{\rm R}(t_{\rm R})-\boldsymbol{y}_{\rm E}(t_{\rm R})$ and $R_{\rm RE}(t_{\rm R}) = |\boldsymbol{R}_{\rm RE}(t_{\rm R})|$. Thus, the second term on the right-hand side of Equation (5) can be rewritten as

$$\begin{aligned}\boldsymbol{K}\cdot[\boldsymbol{v}_{\rm R}(t_{\rm R})-\boldsymbol{v}_{\rm E}(t_{\rm E})] \;=\; & -\boldsymbol{n}_{\rm RE}(t_{\rm R})\cdot[\boldsymbol{v}_{\rm R}(t_{\rm R})-\boldsymbol{v}_{\rm E}(t_{\rm R})] - \epsilon\Big\{\boldsymbol{n}_{\rm RE}(t_{\rm R})\cdot\boldsymbol{a}_{\rm E}(t_{\rm R})R(t_{\rm R})\\ & +\boldsymbol{v}_{\rm E}(t_{\rm R})\cdot\boldsymbol{v}_{\rm R}(t_{\rm R}) - [\boldsymbol{n}_{\rm RE}(t_{\rm R})\cdot\boldsymbol{v}_{\rm E}(t_{\rm R})][\boldsymbol{n}_{\rm RE}(t_{\rm R})\cdot\boldsymbol{v}_{\rm R}(t_{\rm R})]\\ & -\boldsymbol{v}_{\rm E}^2(t_{\rm R}) + [\boldsymbol{n}_{\rm RE}(t_{\rm R})\cdot\boldsymbol{v}_{\rm E}(t_{\rm R})]^2\Big\} + \mathcal{O}(\epsilon^2)\,. \qquad (25)\end{aligned}$$

Finally, to the order ϵ^3, the shift in frequency with possible violations of LLI and LPI for one-way Doppler tracking in terms of $t_{\rm R}$ is

$$\begin{aligned}\left.\frac{\nu_{\rm R}}{\nu_{\rm E}}\right|_{\rm E\to R} =\; & \mathcal{F}_{\rm E\to R}[t_{\rm R}-\epsilon|\boldsymbol{y}_{\rm R}(t_{\rm R})-\boldsymbol{y}_{\rm E}(t_{\rm R})|,t_{\rm R}]+\mathcal{O}(\epsilon^3)\\ =\; & 1-\epsilon\boldsymbol{n}_{\rm RE}(t_{\rm R})\cdot\boldsymbol{v}_{\rm RE}(t_{\rm R}) - \epsilon^2\boldsymbol{v}_{\rm E}(t_{\rm R})\cdot\boldsymbol{v}_{\rm R}(t_{\rm R}) - \epsilon^2\boldsymbol{n}_{\rm RE}(t_{\rm R})\cdot\boldsymbol{a}_{\rm E}(t_{\rm R})R(t_{\rm R})\\ & +\frac{1}{2}\epsilon^2\boldsymbol{v}_{\rm R}^2(t_{\rm R}) + \frac{1}{2}\epsilon^2\boldsymbol{v}_{\rm E}^2(t_{\rm R}) + \epsilon^2\sum_A U_A[\boldsymbol{y}_{\rm R}(t_{\rm R})] - \epsilon^2\sum_A U_A[\boldsymbol{y}_{\rm E}(t_{\rm R})]\\ & +\frac{1}{2}\epsilon^2\bar\beta_{\rm R}\boldsymbol{v}_{\rm R}^2(t_{\rm R}) - \frac{1}{2}\epsilon^2\bar\beta_{\rm E}\boldsymbol{v}_{\rm E}^2(t_{\rm R}) + \epsilon^2\sum_A\bar\alpha_{\rm R}^A U_A[\boldsymbol{y}_{\rm R}(t_{\rm R})] - \epsilon^2\sum_A\bar\alpha_{\rm E}^A U_A[\boldsymbol{y}_{\rm E}(t_{\rm R})]\\ & +\mathcal{O}(\epsilon^3)\,, \qquad (26)\end{aligned}$$

where $\boldsymbol{v}_{\rm RE} = \boldsymbol{v}_{\rm R} - \boldsymbol{v}_{\rm E}$. When $\boldsymbol{v}_{\rm R} = 0$, $\boldsymbol{a}_{\rm E} = 0$, $U_A = 0$ and $\bar\beta_{\rm R/E} = \bar\alpha_{\rm R/E} = 0$, the above equation can be reduced to the case for special relativistic transverse Doppler tracking (Landau & Lifshitz 1975). A possible deviation in the redshift z from the prediction by EEP is

$$\begin{aligned}\left.\delta z\right|_{\rm E\to R} \equiv\; & \left.\frac{\nu_{\rm R}}{\nu_{\rm E}}\right|_{\rm E\to R} - \left.\frac{\nu_{\rm R}}{\nu_{\rm E}}\right|_{\rm E\to R}^{\rm EEP} + \mathcal{O}(\epsilon^3)\\ =\; & \frac{1}{2}\epsilon^2\bar\beta_{\rm R}\boldsymbol{v}_{\rm R}^2(t_{\rm R}) - \frac{1}{2}\epsilon^2\bar\beta_{\rm E}\boldsymbol{v}_{\rm E}^2(t_{\rm R}) + \epsilon^2\sum_A\bar\alpha_{\rm R}^A U_A[\boldsymbol{y}_{\rm R}(t_{\rm R})] - \epsilon^2\sum_A\bar\alpha_{\rm E}^A U_A[\boldsymbol{y}_{\rm E}(t_{\rm R})]\\ & +\mathcal{O}(\epsilon^3)\,. \qquad (27)\end{aligned}$$

3.2 Two-Way Doppler Tracking with LTS

In the case of two-way Doppler tracking, after considering LTS, we have

$$\begin{aligned}\left.\frac{\nu_{\rm R}}{q\nu_{\rm E}}\right|_{\rm S\to P\to S} =\; & \mathcal{F}_{\rm S\to P}(t_{\rm E},t')\cdot\mathcal{F}_{\rm P\to S}(t',t_{\rm R}) + \mathcal{O}(\epsilon^3)\\ =\; & \mathcal{F}_{\rm S\to P}[t'-\epsilon|\boldsymbol{y}_{\rm P}(t')-\boldsymbol{y}_{\rm S}(t')|,t']\cdot\mathcal{F}_{\rm P\to S}(t',t_{\rm R}) + \mathcal{O}(\epsilon^3)\,. \qquad (28)\end{aligned}$$

After substituting $t' = t_{\rm R} - \epsilon|\boldsymbol{y}_{\rm S}(t_{\rm R}) - \boldsymbol{y}_{\rm P}(t_{\rm R})|$ into the above expression and expanding it with respect to ϵ, we can obtain

$$\left.\frac{\nu_{\rm R}}{q\nu_{\rm E}}\right|_{\rm S\to P\to S} = 1 - \epsilon 2\boldsymbol{n}_{\rm PS}(t_{\rm R})\cdot\boldsymbol{v}_{\rm PS}(t_{\rm R}) + 2\epsilon^2\boldsymbol{v}^2_{\rm PS}(t_{\rm R}) + 2\epsilon^2[\boldsymbol{n}_{\rm PS}(t_{\rm R})\cdot\boldsymbol{a}_{\rm PS}(t_{\rm R})]R_{\rm PS}(t_{\rm R}) + \mathcal{O}(\epsilon^3)\,, \tag{29}$$

where $\boldsymbol{a}_{\rm PS} = \boldsymbol{a}_{\rm P} - \boldsymbol{a}_{\rm S}$. Since possible violations of LLI and LPI have opposite signs in the uplink and downlink portions of two-way Doppler tracking, they cancel out in this closed loop, which suggests that these violations cannot affect two-way Doppler tracking under such a linear approximation of LTS [Equations (21) and (22)], i.e. $\delta z|^{\rm EEP}_{\rm S\to P\to S} = \mathcal{O}(\epsilon^3)$. The effect of a more general approximation for LTS on two-way Doppler tracking will be investigated in our future work.

3.3 Three-Way Doppler Tracking with LTS

Applying a similar procedure as was applied to one-way and two-way Doppler tracking, we can obtain an expression for three-way Doppler tracking with LTS as

$$\begin{aligned}\left.\frac{\nu_{\rm R}}{q\nu_{\rm E}}\right|_{\rm S_1\to P\to S_2} &= \mathcal{F}_{\rm S_1\to P}(t_{\rm E}, t')\cdot\mathcal{F}_{\rm P\to S_2}(t', t_{\rm R}) + \mathcal{O}(\epsilon^3)\\ &= \mathcal{F}_{\rm S_1\to P}[t' - \epsilon|\boldsymbol{y}_{\rm P}(t') - \boldsymbol{y}_{\rm S_1}(t')|, t']\cdot\mathcal{F}_{\rm P\to S_2}(t', t_{\rm R}) + \mathcal{O}(\epsilon^3)\,,\end{aligned} \tag{30}$$

where $t' = t_{\rm R} - \epsilon|\boldsymbol{y}_{\rm S_2}(t_{\rm R}) - \boldsymbol{y}_{\rm P}(t_{\rm R})|$. After Taylor expansion with respect to ϵ, we have

$$\begin{aligned}\left.\frac{\nu_{\rm R}}{q\nu_{\rm E}}\right|_{\rm S_1\to P\to S_2} = &\ 1 - \epsilon\Big[\boldsymbol{n}_{\rm PS_1}(t_{\rm R})\cdot\boldsymbol{v}_{\rm PS_1}(t_{\rm R}) + \boldsymbol{n}_{\rm PS_2}(t_{\rm R})\cdot\boldsymbol{v}_{\rm PS_2}(t_{\rm R})\Big]\\ &+\epsilon^2\Big[\mathcal{R}^{\rm P}_{\rm S_2S_1}(t_{\rm R})\boldsymbol{v}^2_{\rm PS_1}(t_{\rm R}) - \boldsymbol{v}_{\rm P}(t_{\rm R})\cdot\boldsymbol{v}_{\rm S_1}(t_{\rm R}) - \boldsymbol{v}_{\rm P}(t_{\rm R})\cdot\boldsymbol{v}_{\rm S_2}(t_{\rm R})\\ &\quad+\boldsymbol{v}^2_{\rm P}(t_{\rm R}) + \frac{1}{2}\boldsymbol{v}^2_{\rm S_1}(t_{\rm R}) + \frac{1}{2}\boldsymbol{v}^2_{\rm S_2}(t_{\rm R})\Big]\\ &+\epsilon^2\Big\{[\boldsymbol{n}_{\rm PS_1}(t_{\rm R})\cdot\boldsymbol{v}_{\rm PS_1}(t_{\rm R})][\boldsymbol{n}_{\rm PS_2}(t_{\rm R})\cdot\boldsymbol{v}_{\rm PS_2}(t_{\rm R})]\\ &\quad-\mathcal{R}^{\rm P}_{\rm S_2S_1}(t_{\rm R})[\boldsymbol{n}_{\rm PS_1}(t_{\rm R})\cdot\boldsymbol{v}_{\rm PS_1}(t_{\rm R})]^2\Big\}\\ &+\epsilon^2\Big[\boldsymbol{n}_{\rm PS_1}(t_{\rm R})\cdot\boldsymbol{a}_{\rm PS_1}(t_{\rm R})R_{\rm PS_2}(t_{\rm R}) - \boldsymbol{n}_{\rm PS_1}(t_{\rm R})\cdot\boldsymbol{a}_{\rm S_1}(t_{\rm R})R_{\rm PS_1}(t_{\rm R})\\ &\quad+\boldsymbol{n}_{\rm PS_2}(t_{\rm R})\cdot\boldsymbol{a}_{\rm P}(t_{\rm R})R_{\rm PS_2}(t_{\rm R})\Big]\\ &+\epsilon^2\Big\{\sum_A U_A[\boldsymbol{y}_{\rm S_2}(t_{\rm R})] - \sum_A U_A[\boldsymbol{y}_{\rm S_1}(t_{\rm R})]\Big\}\\ &+\frac{1}{2}\epsilon^2\Big[\bar\beta_{\rm S_2}\boldsymbol{v}^2_{\rm S_2}(t_{\rm R}) - \bar\beta_{\rm S_1}\boldsymbol{v}^2_{\rm S_1}(t_{\rm R})\Big]\\ &+\epsilon^2\Big\{\sum_A \bar\alpha^A_{\rm S_2}U_A[\boldsymbol{y}_{\rm S_2}(t_{\rm R})] - \sum_A \bar\alpha^A_{\rm S_1}U_A[\boldsymbol{y}_{\rm S_1}(t_{\rm R})]\Big\}\\ &+\mathcal{O}(\epsilon^3)\,,\end{aligned} \tag{31}$$

where $\boldsymbol{a}_{\mathrm{PS}_{1/2}} = \boldsymbol{a}_{\mathrm{P}} - \boldsymbol{a}_{\mathrm{S}_{1/2}}$ and

$$\mathcal{R}^{\mathrm{P}}_{\mathrm{S}_2\mathrm{S}_1}(t_{\mathrm{R}}) \equiv \frac{R_{\mathrm{PS}_2}(t_{\mathrm{R}})}{R_{\mathrm{PS}_1}(t_{\mathrm{R}})} . \tag{32}$$

The possible deviation in redshift z resulting from the prediction by EEP is

$$\begin{aligned} \delta z\bigg|_{\mathrm{S}_1\to\mathrm{P}\to\mathrm{S}_2} &\equiv \frac{\nu_{\mathrm{R}}}{q\nu_{\mathrm{E}}}\bigg|_{\mathrm{S}_1\to\mathrm{P}\to\mathrm{S}_2} - \frac{\nu_{\mathrm{R}}}{q\nu_{\mathrm{E}}}\bigg|^{\mathrm{EEP}}_{\mathrm{S}_1\to\mathrm{P}\to\mathrm{S}_2} + \mathcal{O}(\epsilon^3) \\ &= \frac{1}{2}\epsilon^2\Big[\bar{\beta}_{\mathrm{S}_2}\boldsymbol{v}^2_{\mathrm{S}_2}(t_{\mathrm{R}}) - \bar{\beta}_{\mathrm{S}_1}\boldsymbol{v}^2_{\mathrm{S}_1}(t_{\mathrm{R}})\Big] \\ &\quad +\epsilon^2\Big\{\sum_A \bar{\alpha}^A_{\mathrm{S}_2} U_A[\boldsymbol{y}_{\mathrm{S}_2}(t_{\mathrm{R}})] - \sum_A \bar{\alpha}^A_{\mathrm{S}_1} U_A[\boldsymbol{y}_{\mathrm{S}_1}(t_{\mathrm{R}})]\Big\} \\ &\quad +\mathcal{O}(\epsilon^3) . \end{aligned} \tag{33}$$

This indicates that possible violations of LLI and LPI are only associated with two stations in three-way Doppler tracking, so that it is better to set the stations at places with significant differences in velocities and gravitational potentials to obtain a high level of sensitivity for the tests. In order to discuss the possibility of detection, we consider a special optimistic case here as the first step: the stations S_1 and S_2 are two ships respectively located at the north pole and on the equator of the Earth; only the gravitational potential of the Sun is taken into account; and it is assumed to be a sub-case in Krisher et al. (1993) such that $\bar{\beta}_{\mathrm{S}_1} = \bar{\beta}_{\mathrm{S}_2} = \bar{\beta} \sim 10^{-2}$ and $\alpha^{\odot}_{\mathrm{S}_1} = \alpha^{\odot}_{\mathrm{S}_2} = \bar{\alpha} \sim 10^{-2}$. Then we can have

$$\begin{aligned} \delta z\bigg|_{\mathrm{S}_1\to\mathrm{P}\to\mathrm{S}_2} &= \frac{1}{2}\epsilon^2\bar{\beta}\Big[\boldsymbol{v}^2_{\mathrm{S}_2}(t_{\mathrm{R}}) - \boldsymbol{v}^2_{\mathrm{S}_1}(t_{\mathrm{R}})\Big] + \epsilon^2\bar{\alpha}\Big\{\sum_A U_A[\boldsymbol{y}_{\mathrm{S}_2}(t_{\mathrm{R}})] - \sum_A U_A[\boldsymbol{y}_{\mathrm{S}_1}(t_{\mathrm{R}})]\Big\} \\ &\sim 10^{-12} , \end{aligned} \tag{34}$$

which also yields $\delta v = c\,\delta z|_{\mathrm{S}_1\to\mathrm{P}\to\mathrm{S}_2} \sim 3\times10^{-4}$ m s^{-1}. Although this magnitude of $\delta z|_{\mathrm{S}_1\to\mathrm{P}\to\mathrm{S}_2}$ may be able to be detected with current technology used in Doppler tracking, the configuration of the stations is too particular. In our future works, we will focus on case studies of some experiments conducted at real facilities.

4 CONCLUSIONS AND DISCUSSION

Currently, techniques for two-way and three-way Doppler tracking of spacecraft are widely used and play important roles in control and navigation for deep space missions. Starting from a one-way Doppler model (Krisher et al. 1993), we extend the models of two-way and three-way Doppler tracking by including [see Equations (10) and (14)] possible violations of LLI and LPI in order to test EEP, which is the cornerstone of GR and all other metric theories of gravity (Will 1993, 2006). After taking the finite speed of light into account, which is the so-called LTS (Moyer & Yuen 2000), we only extend these models to depend on the time of reception for the signal for practical convenience [see Equations (29) and (31)]. We find that possible violations of LLI and LPI cannot affect two-way Doppler tracking under a linear approximation of the LTS [Equations (21) and (22)], although this approximation is sufficiently good for most cases in the solar system (Moyer & Yuen 2000). We also show that, in three-way Doppler tracking, possible violations of LLI and LPI are only associated with two stations, which suggests that it is better to set the stations at places with significant differences in velocities and gravitational potentials to obtain a high level of sensitivity for the tests.

In practice, Doppler measurements certainly suffer various types of noise that can arise from frequency standards, plasma scintillation, tropospheric scintillation, antenna mechanics, ground electronics, the spacecraft transponder, thermal effects in the ground and spacecraft receivers, and unmodeled motion of the spacecraft (see Armstrong 2006, for a review). Although studies on these types of noise are out of the scope of this paper, they are extremely important for a positive detection. In our future work, we will focus on case studies from some specific missions.

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