

A class of transient acceleration models consistent with Big Bang cosmology *

Tian-Long Zu, Jie-Wen Chen and Yang Zhang

Key Laboratory for Researches in Galaxies and Cosmology, Department of Astronomy, University of Science and Technology of China, Hefei 230026, China; zutl@mail.ustc.edu.cn

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Abstract Is it possible that the current cosmic accelerating expansion will turn into a decelerating one? Can this transition be realized by some viable theoretical model that is consistent with the standard Big Bang cosmology? We study a class of phenomenological models with a transient acceleration, based on a dynamical dark energy with a very general form of equation of state $p_{\text{de}} = \alpha\rho_{\text{de}} - \beta\rho_{\text{de}}^m$. It mimics the cosmological constant $\rho_{\text{de}} \rightarrow \text{const}$ for a small scale factor a , and behaves as a barotropic gas with $\rho_{\text{de}} \rightarrow a^{-3(\alpha+1)}$ with $\alpha \geq 0$ for large a . The cosmic evolution of four models in the class has been examined in detail, and all yield a smooth transient acceleration. Depending on the specific model, the future universe may be dominated by either dark energy or by matter. In two models, the dynamical dark energy can be explicitly realized by a scalar field with an analytical potential $V(\phi)$. Moreover, a statistical analysis shows that the models can be as robust as Λ CDM in confronting the observational data of Type Ia supernovae, cosmic microwave background (CMB) and baryon acoustic oscillation. As improvements over previous studies, our models overcome the problem of over-abundance of dark energy during early eras, and satisfy the constraints on dark energy from WMAP observations of CMB.

Key words: cosmology: theory — methods: analytical

1 INTRODUCTION

Cosmological observations, such as Type Ia supernovae (SN Ia) (Riess et al. 1998; Perlmutter et al. 1999) and cosmic microwave background (CMB) anisotropies (Spergel et al. 2003, 2007; Komatsu et al. 2009, 2011; Hinshaw et al. 2013) have indicated that the universe is now in an accelerating expansion. Interpreted in the framework of general relativity, the acceleration can be attributed to the existence of some dark energy, which currently dominates the total cosmic energy in the Universe. There have been a number of candidates for dark energy, such as the cosmological constant Λ , various scalar field dynamical dark energy models (Ratra & Peebles 1988; van den Hoogen et al. 1999; Barreiro et al. 2000; Liddle & Scherrer 1999; Tong et al. 2011; Copeland et al. 1998, 2006), Chaplygin gas model (Kamenshchik et al. 2001), quantum Yang-Mills condensate models (Zhang 2003; Zhang et al. 2007; Xia & Zhang 2007), etc. So far, there is no observational evidence about whether the current acceleration is eternal or transient. In an eternally accelerating universe, there

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is an event horizon, and the S-matrix in string theory will be ill-defined (Hellerman et al. 2001; Fischler et al. 2001; Witten 2001). Recently, the analysis of combined data of SN Ia + baryon acoustic oscillation (BAO) + CMB (Shafieloo et al. 2009) seems to indicate that the acceleration may be slowing down. More data are needed to resolve the issue (Guimarães & Lima 2011). Thus the possibility of return to decelerating expansion in the future has been explored. There have been various models based on different possible mechanisms (Townsend & Wohlfarth 2003; Sahni & Shtanov 2003; Russo 2004; Srivastava 2007; Wu et al. 2008; Gong et al. 2008; Bento et al. 2008; Pavón 2007). In particular, a scalar field with an exponential potential as dark energy in Russo (2004), and two coupled scalar fields as dark energy in Blais & Polarski (2004) are studied, and, for a certain range of parameters, a transient acceleration occurs in these scalar models. Some references (Fabris et al. 2010; Chen et al. 2011; Costa 2010) consider possible interaction between matter and dark energy that can lead to a transient accelerating expansion. A coupling between Chaplygin gas and a scalar field is studied in Bilic et al. (2005). Based on certain ansatz on the dark energy density to achieve a return to deceleration, some scalar field models are proposed, which have an exponential type of potential with a quadratic dependence on the scalar field (Carvalho et al. 2006; Alcaniz et al. 2009; Alcaniz 2010). However, as has been checked (Cui et al. 2013), when extended back to earlier stages, these exponential types of scalar dark energy would be dominant over the matter component, jeopardizing the standard Big Bang cosmological scenario.

To handle this over-abundance problem within the whole class of scalar field models, one might directly design some special form of scalar potential and go ahead by trial and error to see if it works. This is essentially the method used in the references (Carvalho et al. 2006; Alcaniz et al. 2009; Alcaniz 2010) which has not worked for their chosen form of potential. Moreover, there are an infinite number of possible forms of scalar potentials, and it is not practical to try each of them. In view of this, we adopt a parameterization approach instead. That is, we take some simple form of dynamic dark energy density $\rho_{\text{de}}(t)$ which is less dominant than the matter density $\rho_{\text{m}}(t)$ during the early stage of cosmic expansion. If this works, it will automatically overcome the over-abundance problem and provide a possible model of transient acceleration. In this paper, we will specifically work with those ρ_{de} which mimic the cosmological constant for a small scale factor a and behave, for large a , like a barotropic gas with $\rho_{\text{de}} \rightarrow a^{-3(\alpha+1)}$ and $\alpha \geq 0$. In certain cases, an explicit expression of scalar potential $V(\phi)$ is obtained analytically. Depending on specific models in the class, the future universe may be dominated by either dark energy or by matter. The interesting characteristic of our simple scalar models is that the dark energy will always be less dominant than the matter when extended to earlier stages, allowing for a transient acceleration within the framework of the standard Big Bang cosmology.

2 MODELS

The spacetime background is the homogeneous and isotropic flat Friedmann-Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2). \quad (1)$$

We will set $a = 1$ as the current value. The dynamical expansion of spacetime is determined by the Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_{\text{de}} + \rho_{\text{m}}), \quad (2)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_{\text{m}} + \rho_{\text{de}} + 3p_{\text{de}}), \quad (3)$$

where ρ_{de} is the dark energy density to be discussed in the following, $\rho_{\text{m}} = \Omega_{\text{m}}\rho_c a^{-3}$ is the matter density and $\Omega_{\text{m}} + \Omega_{\text{de}} = 1$.

First, we consider a dynamical dark energy density as a function of the scale factor a

$$\rho_{\text{de}}(t) = \Omega_{\text{de}}\rho_c \frac{1+r}{1+ra(t)^3}, \quad (4)$$

where Ω_{de} is the current dark energy fraction, ρ_c is the critical density and r is a dimensionless parameter of the model. It behaves as $\rho_{\text{de}} \propto \Omega_{\text{de}}\rho_c(1+r)$ for $a^3 \ll 1/r$, and $\rho_{\text{de}} \propto a^{-3}$ for $a^3 \gg 1/r$, similar to that of the matter component. However, we do not regard ρ_{de} as matter. For simplicity, we do not include the coupling between dark energy and matter. From the equation of energy conservation $a \frac{d\rho_{\text{de}}}{da} + 3(\rho_{\text{de}} + p_{\text{de}}) = 0$, the pressure of dark energy is given by

$$\begin{aligned} p_{\text{de}}(t) &= -\Omega_{\text{de}}\rho_c \frac{(1+r)}{(1+ra(t)^3)^2} \\ &= -\frac{\rho_{\text{de}}^2(t)}{\Omega_{\text{de}}\rho_c(1+r)}. \end{aligned} \quad (5)$$

The equation of state of dark energy is $w_{\text{de}} = p_{\text{de}}/\rho_{\text{de}} = -\frac{\rho_{\text{de}}}{\Omega_{\text{de}}\rho_c(1+r)}$. Equation (5) turns out to be similar to a model $p \propto -\rho^{-\alpha}$ with $\alpha < -1$ in Sen & Scherrer (2005), but is different from the generalized Chaplygin gas model $p \propto -\rho^{-\alpha}$ with $\alpha \geq -1$ (Kamenshchik et al. 2001). Equations (2) and (3) follow the deceleration parameter explicitly

$$\begin{aligned} q &= -\frac{\ddot{a}a}{(\dot{a})^2} \\ &= \frac{1}{2} - \frac{3\Omega_{\text{de}}(1+r)a^3}{2(1+ra^3)[\Omega_{\text{m}} + (r + \Omega_{\text{de}})a^3]}. \end{aligned} \quad (6)$$

We plot q as a function of a in Figure 1. Here $\Omega_{\text{de}} = 0.74$ and $\Omega_{\text{m}} = 0.26$ are taken for concreteness. It is seen that $q \rightarrow \frac{1}{2}$ as the asymptotic values in both the limits $a \rightarrow 0$ and $a \rightarrow \infty$. Therefore, this model predicts a decelerating expansion for both the past and the future, and, in the interval $a \sim (0.5, 5)$ with $q < 0$, the current acceleration is transient, as shown in Figure 1 for various values of r . In fact, for a finite value of $r > 0$ and a constraint $\frac{3r}{4r+1} < \Omega_{\text{de}}$, the acceleration is transient and is always followed by a decelerating expansion. By repeating the calculation, a larger Ω_{de} yields an earlier entry into the current acceleration, and a larger value of parameter r yields an earlier return to deceleration. In the limiting case $r = 0$, the model reduces to the Λ CDM model. On the other hand, for $r \neq 0$, the model predicts a dark energy fraction $\sim \Omega_{\text{de}}(1+r)$ for the early stage. However, based on the observations of CMB, WMAP7 has given an error band $\pm 4\%$ for Ω_{Λ} in the Λ CDM model (Spergel et al. 2003, 2007; Komatsu et al. 2009, 2011; Hinshaw et al. 2013). To be consistent with observations, we impose an upper limit $r \simeq 0.04$ for our model. That is, for the parameter $r \sim (0, 0.04)$, our model is within the constraint from the observations by WMAP.

In Figure 2 we plot the ratio $\rho_{\text{de}}/\rho_{\text{m}}$ as a function of a . At early times when $a \ll 1$ and $\rho_{\text{de}}/\rho_{\text{m}} \rightarrow 0$, the dark energy is comparatively small and the universe is matter dominated, as desired. In the distant future when $a \gg 1$, the ratio $\rho_{\text{de}}/\rho_{\text{m}} \rightarrow (1+r)\Omega_{\Lambda}/r\Omega_{\text{m}} > 1$ will asymptotically approach a constant. Thus the universe in the future will be dominated by ρ_{de} , which is decreasing as $\propto a(t)^{-3}$, and $a(t) \propto t^{2/3}$, expanding like the matter-dominated case.

The dark energy in this model can also be explicitly realized by a scalar field ϕ . For simplicity, we do not consider the effects of the matter component (Barrow 1990). We start with the Lagrangian of the scalar field

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - V(\phi), \quad (7)$$

where $V(\phi)$ is the potential to be determined. The energy density and the pressure are $\rho_{\phi} = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ and $p_{\phi} = \frac{1}{2}\dot{\phi}^2 - V(\phi)$ respectively, and satisfy the conservation equation $a \frac{d\rho_{\phi}}{da} + 3(\rho_{\phi} + p_{\phi}) =$

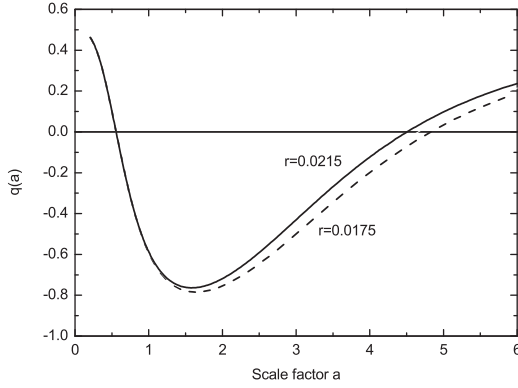


Fig. 1 The deceleration parameter q in Eq. (6).

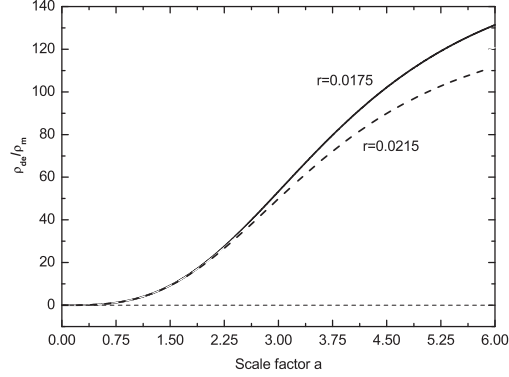


Fig. 2 The ratio $\rho_{\text{de}}/\rho_{\text{m}}$ in the model of Eq. (5). ρ_{de} will still dominate over ρ_{m} in the future.

0, which can be written as the scalar field evolution equation

$$\dot{\phi}^2 = -\frac{1}{3}a \frac{d\rho_{\phi}}{da}. \quad (8)$$

Now we require that ρ_{ϕ} behaves as ρ_{de} in Equation (4)

$$\rho_{\phi} = \Omega_{\text{de}}\rho_c \frac{1+r}{1+ra^3}. \quad (9)$$

Using Equation (2) without the matter, Equation (8) can be written as

$$\sqrt{\frac{8\pi G}{3}} \frac{d\phi}{da} = \pm \sqrt{\frac{ra}{1+ra^3}}. \quad (10)$$

By integrating, one obtains

$$\sqrt{6\pi G}(\phi - \phi_0) = \pm \ln \frac{\sqrt{ra^3} + \sqrt{1+ra^3}}{\sqrt{ra_0^3} + \sqrt{1+ra_0^3}}, \quad (11)$$

where a_0 and ϕ_0 are constants. For simplicity, we set $\phi_0 = 0$ and $a_0 = 0$. Then Equation (11) shrinks to

$$\sqrt{6\pi G}\phi = \pm \ln \left[\sqrt{ra^3} + \sqrt{1+ra^3} \right]. \quad (12)$$

Taking the + sign yields

$$2\sqrt{ra^3} = e^{\sqrt{6\pi G}\phi} - e^{-\sqrt{6\pi G}\phi}. \quad (13)$$

By $V = (\rho_{\phi} - p_{\phi})/2$, one has

$$V = \frac{1}{2}\Omega_{\text{de}}\rho_c(1+r) \left[\frac{2+ra^3}{(1+ra^3)^2} \right]. \quad (14)$$

Substituting Equation (13) into Equation (14), we finally obtain V in terms of the field ϕ

$$V(\phi) = 2\Omega_{\text{de}}\rho_c(1+r) \left[\frac{1}{(e^{\sqrt{6\pi G}\phi} + e^{-\sqrt{6\pi G}\phi})^2} + \frac{4}{(e^{\sqrt{6\pi G}\phi} + e^{-\sqrt{6\pi G}\phi})^4} \right]. \quad (15)$$

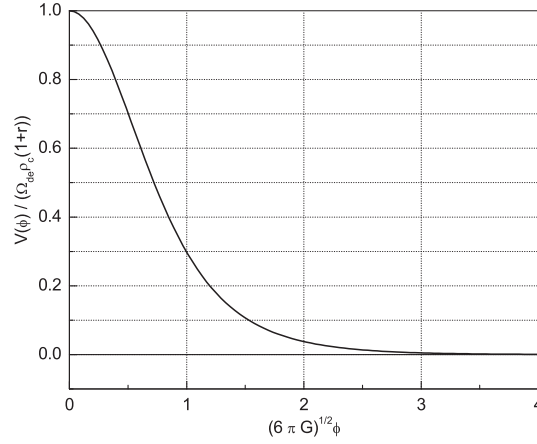


Fig. 3 The re-scaled potential $V(\phi)/\Omega_{\text{de}}\rho_c(1+r)$ is shown.

Note that $V(\phi)$ in Equation (15) is proportional to the factor $\Omega_{\text{de}}\rho_c(1+r)$. In Figure 3 the re-scaled potential $V(\phi)/\Omega_{\text{de}}\rho_c(1+r)$ is plotted.

It is worth noticing that the same $V(\phi)$ holds if a “ $-$ ” sign is taken in Equation (12). Indeed, the resulting Lagrangian \mathcal{L} is symmetric under $\phi \rightarrow -\phi$. When $\sqrt{6\pi G}\phi \gg 1$ the potential reduces to $V(\phi) \propto e^{-2\sqrt{6\pi G}\phi}$, a simple exponential function of ϕ , which is similar to what Ratra and Peebles (Ratra & Peebles 1988) used for dark energy and dark matter in a different context. Notice that the expression of Equation (15) is a combination of exponential functions of ϕ , but differs from the exponential potential with a quadratic ϕ^2 in Carvalho et al. (2006); Alcaniz et al. (2009). In particular, the profile of our $V(\phi)$ is more sloped around $\phi = 0$ than that in Carvalho et al. (2006); Alcaniz et al. (2009).

To examine the observational viability of this simple model of transient acceleration, we perform a joint analysis involving the data of SN Ia, CMB and BAO. We use the distance modulus $\mu_{\text{obs}}(z_i)$ data of 557 SN Ia (Amanullah et al. 2010), the shift parameter of CMB by the WMAP observations (Komatsu et al. 2011), and the BAO measurement from the Sloan Digital Sky Survey (SDSS, Percival et al. 2010, 2007). We shall follow the computational method in Wang et al. (2008); Fu et al. (2011); Tong et al. (2011). Assuming that these three data sets of observations are mutually independent and that the measurement errors for each set are Gaussian, the likelihood function then has the form

$$\mathcal{L} \propto e^{-\chi^2/2} \quad (16)$$

with

$$\chi^2 = \chi_{\text{SN}}^2 + \chi_{\text{BAO}}^2 + \chi_{\text{CMB}}^2. \quad (17)$$

$$\chi_{\text{SN}}^2 = \sum_{i=1}^{557} \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i)]^2}{\sigma_i^2}, \quad (18)$$

$$\chi_{\text{BAO}}^2 = \frac{(A - A_{\text{obs}})^2}{\sigma_A^2}, \quad (19)$$

$$\chi_{\text{CMB}}^2 = \frac{(R - R_{\text{obs}})^2}{\sigma_R^2}. \quad (20)$$

The detailed specifications of these formulae have been given in Fu et al. (2011). Variations in values of the model parameters yield respective values of χ^2 . For demonstration, we take the model

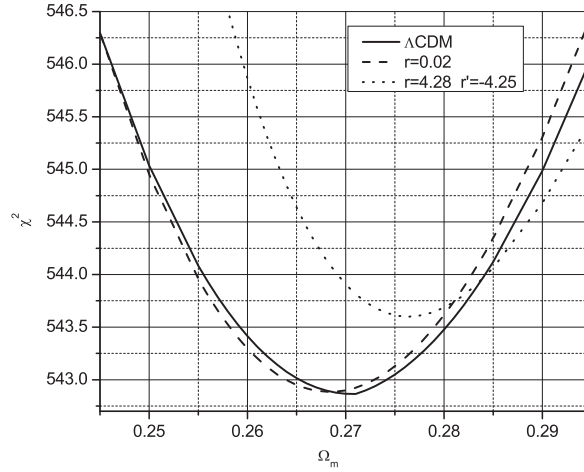


Fig. 4 The χ^2 of the transient acceleration models based upon the joint data of SN Ia, BAO and CMB. The dashed line is from the first model with $r = 0.02$ in Eq. (4). The dotted line is from the second model in Eq. (21) with $r = 4.28$ and $r' = -4.25$. For comparison, χ^2 of Λ CDM ($r = 0$) is also shown in the solid line.

of $r = 0.02$ and let Ω_m vary. The resulting $\chi^2(\Omega_m)$ is plotted (in the dashed line) in Figure 4. To compare with the standard Λ CDM, we also plot the case of $r = 0$ (corresponding to Λ CDM). The minimum is $\chi^2 = 542.88$ at $\Omega_m = 0.268$ for the model with $r = 0.02$, whereas Λ CDM has $\chi^2 = 542.86$ at $\Omega_m = 0.271$. Its corresponding likelihood \mathcal{L} is 0.99 times that of Λ CDM. Thus the joint analysis shows that the model with $r = 0.02$ is quite close to Λ CDM in confronting the observational data, and is robust in providing a transient acceleration.

The above model can be extended so that the universe in the future is dominated by the matter term ρ_m , while retaining the return to deceleration. We consider a dark energy density

$$\rho_{\text{de}} = \Omega_{\text{de}} \rho_c \frac{1 + r + r'}{1 + r a^3 + r' a^{\epsilon(a)}}, \quad (21)$$

where r and r' are constant parameters, and $\epsilon(a)$ is some function of the scale factor a . By energy conservation, one obtains

$$p_{\text{de}} = -\frac{\theta}{\Omega_{\text{de}} \rho_c (1 + r + r')} \rho_{\text{de}}^2, \quad (22)$$

where

$$\theta \equiv 1 + r' a^{\epsilon(a)} - \frac{1}{3} r' a \frac{d}{da} a^{\epsilon(a)}. \quad (23)$$

Then the deceleration parameter is

$$q = \frac{1}{2} - \frac{3}{2} \frac{\Omega_{\text{de}} (1 + r + r') \theta a^3}{(1 + r a^3 + r' a^{\epsilon(a)}) [\Omega_m + (r + (1 + r') \Omega_{\text{de}}) a^3 + \Omega_m r' a^{\epsilon(a)}]}. \quad (24)$$

This model allows a return to deceleration as long as $\epsilon(a) > 3$ for a small a , and $\epsilon(a) < 3$ for a large a . For instance, taking $\epsilon(a) = (2a + 4)/(a + 1)$, one has $\epsilon(a) \rightarrow 2$ for $a \rightarrow \infty$, and $\epsilon(a) \rightarrow 4$ for $a \rightarrow 0$, and the ratio $\frac{\rho_{\text{de}}}{\rho_m} \rightarrow \frac{\Omega_{\text{de}}}{\Omega_m} \frac{1+r+r'}{r}$ as $a \rightarrow \infty$. A future matter domination is achieved if $\frac{1+r+r'}{r} < \frac{\Omega_m}{\Omega_{\text{de}}}$. To be specific, we take $r = 4.28$ and $r' = -4.25$ for $\Omega_{\text{de}} = 0.74$ and $\Omega_m = 0.26$ respectively, yielding $\frac{\rho_{\text{de}}}{\rho_m} \rightarrow 0.685$ in the future.

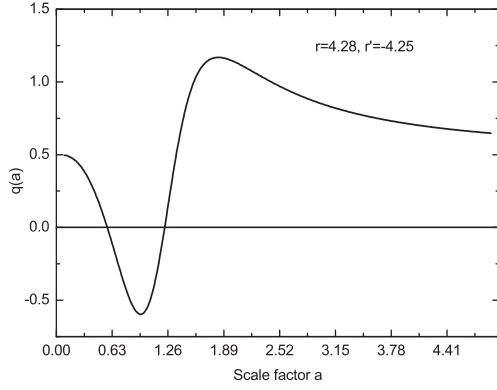


Fig. 5 The deceleration parameter q in Eq. (24) in the second model.

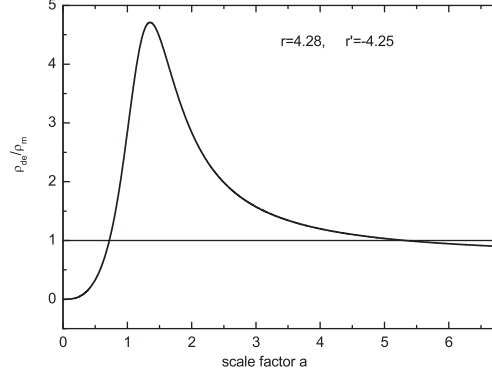


Fig. 6 The ratio ρ_{de}/ρ_m of Eq. (21) in the second model. ρ_m will dominate over ρ_{de} in the future.

Figures 5 and 6 demonstrate q and $\frac{\rho_{de}}{\rho_m}$, respectively. It is also checked that larger values of r or r' yield an earlier return to the decelerated expansion. To be consistent with the error band $\pm 4\%$ for Ω_Λ by WMAP7 (Komatsu et al. 2011), the allowed value of the parameters should be constrained to $r + r' < 0.04$. In confronting the joint data of SN Ia, BAO and CMB, the χ^2 of this model with $r = 4.28$ and $r' = -4.25$ is also obtained and shown (in the dotted line) in Figure 4. Its minimum is $\chi^2 = 543.60$ at $\Omega_m = 0.276$. Its corresponding likelihood \mathcal{L} is 0.69 times that of Λ CDM. Thus, this second model is also close to Λ CDM by statistical analysis, although it is not as good as the first model.

In the above two models, the dark energy has a negative pressure $p_{de} < 0$. In fact, the first model can be generalized so that a positive pressure $p_{de} > 0$ for $a \gg 1$ can be achieved. Consider

$$\rho_{de} = \Omega_{de}\rho_c \frac{1+r}{1+ra^{3(\alpha+1)}}, \quad (25)$$

where α is a positive constant. In the limit $\alpha \rightarrow 0$, this reduces to Equation (4) of the first model. From energy conservation, the pressure is given by

$$p_{de} = \alpha\rho_{de} - \frac{1+\alpha}{\Omega_{de}\rho_c(1+r)}\rho_{de}^2, \quad (26)$$

consisting of two terms. When $a \gg 1$, one has $\rho_{de} \ll \Omega_{de}\rho_c(1+r)\frac{\alpha}{1+\alpha}$, and $p_{de} \simeq \alpha\rho_{de}$, which is positive. From Equations (25) and (26) follows the deceleration parameter

$$q = \frac{1}{2} - \frac{3}{2} \frac{\Omega_{de}(1+r)(1-\alpha ra^{3\alpha+3})a^3}{(1+ra^{3\alpha+3})[\Omega_m(1+ra^{3\alpha+3}) + \Omega_{de}(1+r)a^3]}, \quad (27)$$

which is shown in Figure 7 for $\alpha = 1/3$ and $r = 0.0287$. When $a \gg 1$, the matter will dominate, similar to the second model.

This dark energy can also be realized by a scalar field ϕ . By calculation, we obtain

$$2\sqrt{ra^{3(1+\alpha)}} = e^{\sqrt{6\pi G(1+\alpha)}\phi} - e^{-\sqrt{6\pi G(1+\alpha)}\phi}, \quad (28)$$

and the potential

$$V(\phi) = 2\Omega_{de}\rho_c(1+r) \left[\frac{1-\alpha}{(e^{\sqrt{6\pi G(1+\alpha)}\phi} + e^{-\sqrt{6\pi G(1+\alpha)}\phi})^2} + 4 \frac{1+\alpha}{(e^{\sqrt{6\pi G(1+\alpha)}\phi} + e^{-\sqrt{6\pi G(1+\alpha)}\phi})^4} \right]. \quad (29)$$

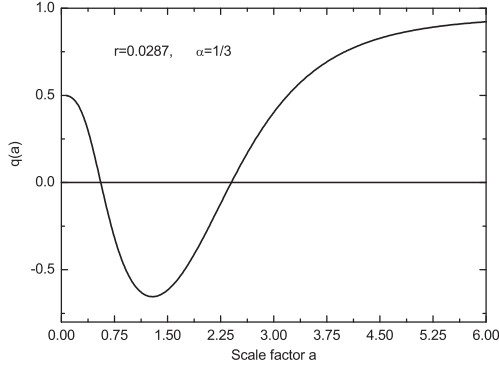


Fig. 7 The deceleration parameter q in Eq. (27) in the third model.

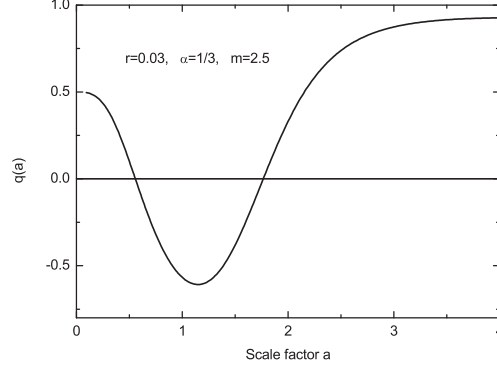


Fig. 8 The deceleration parameter q in Eq. (31) in the general model.

In the special case $\alpha = 0$, Equation (29) reduces to Equation (15) of the first model.

In fact, Equation (26) can be further extended into the following most general form

$$p_{\text{de}} = \alpha \rho_{\text{de}} - \beta \rho_{\text{de}}^m, \quad (30)$$

where $\alpha, \beta > 0$ and $m > 1$ are constant. By energy conservation, Equation (30) yields

$$\rho_{\text{de}} = \Omega_{\text{de}} \rho_c \left(\frac{1+r}{1+r a^{3(\alpha+1)(m-1)}} \right)^{\frac{1}{m-1}}, \quad (31)$$

where the parameter $r \equiv \frac{\sigma(\alpha+1)}{\beta}$, and σ is an integral constant and can be fixed by the initial condition $\rho_{\text{de}}|_{a=1} = \Omega_{\text{de}} \rho_c$. The deceleration parameter is

$$q(a) = \frac{1}{2} - \frac{3}{2} \frac{\Omega_{\text{de}} (1 - \alpha r a^{3(\alpha+1)(m-1)}) a^3}{(1 + r a^{3(\alpha+1)(m-1)}) [\Omega_{\text{de}} a^3 + \Omega_m \left(\frac{1+r a^{3(\alpha+1)(m-1)}}{1+r} \right)^{\frac{1}{m-1}}]}. \quad (32)$$

When $m = 2$, this general model reduces to the third model, and reduces to the first model if further $\alpha = 0$. In the limit $\rho_{\text{de}} \ll (\alpha/\beta)^{1/m-1}$, Equation (30) reduces to $p_{\text{de}} \simeq \alpha \rho_{\text{de}}$, a barotropic gas. The general model in Equation (31) has the following asymptotic behavior: in the limit $a \rightarrow 0$, $\rho_{\text{de}} \rightarrow (\frac{\alpha+1}{\beta})^{1/(m-1)}$ like the cosmological constant, and, in the limit $a \gg 1$, $\rho_{\text{de}} \rightarrow a^{-3(1+\alpha)}$, which mimics a barotropic gas. As we have checked by detailed computations, shown in Figure 8 for $\alpha = 1/3$ and $m = 2.5$, the general model also allows a transient accelerating expansion, in which the matter will dominate for $a \gg 1$, similar to the second and third models. Štefančić (2005) also discussed a special case of $\alpha = -1$ for Equation (30) in a different context.

3 CONCLUSIONS AND DISCUSSION

We have demonstrated, by explicit model constructions, that the current cosmic acceleration driven by some dynamical dark energy can be transient, and can smoothly transit into re-deceleration. Four specific models have been examined in detail, each being a special case of the most general one with the equation of state: $p_{\text{de}} = \alpha \rho_{\text{de}} - \beta \rho_{\text{de}}^m$. In all the models, the dark energy behaves like a barotropic gas with $\rho_{\text{de}} \rightarrow a^{-3(\alpha+1)}$ with $\alpha \geq 0$ for $a \gg 1$, and the total cosmic energy can be dominated by either ρ_m as in the last three models, or by ρ_{de} as in the first model. Our dark energy models can be realized by a scalar field ϕ . In two cases, by analytical integration, we have also obtained the explicit expressions of scalar field potential $V(\phi)$, which is as simple as a combination of the exponential

functions of ϕ , and approaches $V(\phi) \propto e^{-2\sqrt{6\pi G}\phi}$ for $a \gg 1$. This function $V(\phi)$ differs from the previous exponential type of potential in literature.

The interesting feature of these models is that the dark energy density will always show $\rho_{\text{de}} \rightarrow \text{const}$, and is less dominant than the matter during earlier stages. This is an improvement over the previous models of transient acceleration that employed the exponential type of scalar field potentials. Moreover, in all our models the fraction of dark energy at $a \ll 1$ is very close to the value of the cosmological constant Ω_Λ in Λ CDM, and is within the error band from WMAP observations. Besides, the joint likelihood analysis also shows that the transient acceleration models can be as robust as Λ CDM in confronting the observational data of SN Ia, CMB and BAO. Therefore, our models can be further incorporated into the framework of the standard Big Bang cosmology to achieve the possible transient acceleration.

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