RAA 2014 Vol. 14 No. 2, 139–143 doi: 10.1088/1674–4527/14/2/002
http://www.raa-journal.org *http://www.iop.org/journals/raa*

Research in Astronomy and Astrophysics

New upper limits on deviation from the inverse-square law of gravity in the solar system: a Yukawa parameterization *

Zi-Wei Li[1], Sheng-Feng Yuan[1], Chang Lu[1] and Yi Xie[1,2]

[1] School of Astronomy and Space Science, Nanjing University, Nanjing 210093, China
[2] Key Laboratory of Modern Astronomy and Astrophysics, Nanjing University, Ministry of Education, Nanjing 210093, China; *yixie@nju.edu.cn*

Received 2013 September 9; accepted 2013 October 7

Abstract New physics beyond the standard model of particles might cause deviation from the inverse-square law of gravity. In many theoretical models of modified gravity, it is parameterized by the Yukawa correction to the Newtonian gravitational force in terms of two parameters α and λ. Here α is a dimensionless strength parameter and λ is a length scale. Using the supplementary advances in perihelia provided by INPOP10a and EPM2011 ephemerides, we obtain new upper limits on the deviation from the inverse-square law when the uncertainty of the Sun's quadrupole moment is taken into account. We find that INPOP10a yields the upper limits as $\alpha = 3.1 \times 10^{-11}$ and $\lambda = 0.15$ au, and EPM2011 gives $\alpha = 5.2 \times 10^{-11}$ and $\lambda = 0.21$ au. In both of them, α is at least 10 times less than the previous results.

Key words: gravitation — ephemerides — celestial mechanics

1 INTRODUCTION

Although gravitation was the first known fundamental force in the universe, it still cannot be included into a quantum framework such as the standard model of strong, weak and electromagnetic interactions. It is an undoubtedly grand challenge to unify gravitation with the three others. Some candidate theories of quantum gravity predict there may be some possible deviation from the inverse-square law of gravity. Therefore, searching for such deviation experimentally and observationally might shed light on new physics (see Adelberger et al. 2003, for a review). In many theoretical models of modified gravity, one way to parameterize the deviation is the Newtonian gravitational potential with an additional Yukawa correction (Fischbach et al. 1986, 1992). That is

$$V = V_{\rm N}(r) + V_{\rm YK}(r), \tag{1}$$

where

$$V_{\rm N}(r) = \frac{Gm_1m_2}{r}, \tag{2}$$

$$V_{\rm YK}(r) = \frac{Gm_1m_2}{r}\alpha \exp\left(-\frac{r}{\lambda}\right). \tag{3}$$

* Supported by the National Natural Science Foundation of China.

Here G is the gravitational constant, m_i $(i = 1, 2)$ is the mass of the ith body and r is the distance between them. α is a dimensionless strength parameter and λ is a length scale (see Fischbach & Talmadge 1999, for a review of constraints on α and λ).

Inspired by the idea of tests of modified gravity using orbital motions of celestial bodies and artificial objects (e.g. Damour & Esposito-Farèse 1994; Iorio 2002, 2007, 2008; Deng et al. 2009; Deng 2011; Iorio 2012b; Iorio & Saridakis 2012; Deng & Xie 2013; Xie & Deng 2013), we will try to find new constraints on α and λ by making use of the supplementary advances in perihelia provided by INPOP10a (Fienga et al. 2011) and EPM2011 (Pitjeva 2013) ephemerides. It is worth mentioning that in the previous work of Iorio (2007), the upper bounds were found as $\alpha = 1.3\times10^{-9}$ and $\lambda = 0.18$ au[1] with EPM2004 ephemeris (Pitjeva 2005). Since INPOP10a and EPM2011 are significantly improved compared with EPM2004, we expect to obtain tighter upper limits.

In Section 2, we will calculate advances in the perihelia $\dot{\omega}_{\rm YK}$ of planets in the solar system by treating the Yukawa correction as a small disturbance and connecting them with the data of ephemerides. In Section 3, the supplementary advances in perihelia provided by INPOP10a and EPM2011 will be used to constrain the Yukawa parameters when the uncertainty of the Sun's quadrupole moment is taken into account. Conclusions and discussion will be presented in Section 4.

2 YUKAWA-TYPE $\dot{\omega}_{\rm YK}$ AND CONFRONTATION WITH DATA

Following the previous works (e.g. Iorio 2007, 2012b; Deng & Xie 2013), we consider the Yukawa correction as a perturbation on the Newtonian inverse-square law of gravity. It will cause secular advances in the perihelia (Iorio 2012b; Deng & Xie 2013)[2]

$$\dot{\omega}_{\rm YK} = \alpha\frac{na\sqrt{1-e^2}}{e\lambda}\exp\left(-\frac{a}{\lambda}\right)I_1\left(\frac{ae}{\lambda}\right), \tag{4}$$

where $I_1(z) = \mathrm{d}I_0(z)/\mathrm{d}z$ and $I_0(z)$ is the modified Bessel function of the first kind (Arfken & Weber 2005). It is closely connected with supplementary advances in perihelia $\dot{\omega}_{\rm suppl}$ provided by modern ephemerides, such as EPM2004 (Pitjeva 2005), INPOP10a (Fienga et al. 2010, 2011) and EPM2011 (Pitjeva 2013; Pitjeva & Pitjev 2013; Pitjev & Pitjeva 2013).

These $\dot{\omega}_{\rm suppl}$ might represent possible mismodeled or unmodeled parts of perihelion advances according to the inverse-square law of gravity and general theory of relativity. They are *almost all* compatible with zero so that they can be used to draw upper bounds on quantities parameterizing unmodeled forces like α and λ in this case. Nonetheless, the latest results by EPM2011 (Pitjeva & Pitjev 2013; Pitjev & Pitjeva 2013) returned non-zero values for Venus and Jupiter. Although the level of their statistical significance was not too high and further investigations are required, we still take them into account in this work. In the recent past, an extra non-zero effect on Saturn's perihelion was studied (Iorio 2009).

In the construction of $\dot{\omega}_{\rm suppl}$ (see Fienga et al. 2010, for details), the effects caused by the Sun's quadrupole mass moment $J_2^{\odot}$ are considered and isolated in the final results, but the perihelion shifts caused by the Lense-Thirring effect (Lense & Thirring 1918) due to the Sun's angular momentum $S_{\odot}$ are absent. Therefore, the entire relation between $\dot{\omega}_{\rm YK}$ and $\dot{\omega}_{\rm suppl}$ is

$$\dot{\omega}_{\rm suppl} = \dot{\omega}_{\rm YK} + \dot{\omega}_{\rm LT} + \dot{\omega}_{\delta J_2^{\odot}}. \tag{5}$$

Here, the Lense-Thirring term $\dot{\omega}_{\rm LT}$ is

$$\dot{\omega}_{\rm LT} = -\frac{6GS_{\odot}\cos i}{c^2a^3(1-e^2)^{3/2}}, \tag{6}$$

[1] We use lower-case "au" to represent the astronomical unit, according to International Astronomical Union 2012 Resolution B2: *http://www.iau.org/static/resolutions/IAU2012_English.pdf*.

[2] Here, we adopt widely used notations in celestial mechanics: a is the semi-major axis, e is the eccentricity, i is the inclination, ω is the argument of periastron and n is the Keplerian mean motion.

where $S_\odot = 1.9 \times 10^{41}$ kg m^2 s^{-1} (Pijpers 2003) and c is the speed of light. This effect of the Sun on planetary motions has been studied in several works (e.g. Iorio 2005b; Iorio et al. 2011; Iorio 2012a). Equation (6) only holds in a coordinate system whose z axis is aligned with the Sun's angular momentum. A general formula for an arbitrary orientation can be found in Iorio (2012c). It is useful in extrasolar planets and black holes, for which the orientation of the spin axis is generally unknown. We add the third term in Equation (5) to include the uncertainty of the Sun's quadrupole moment $\delta J_2^\odot$, which is currently about $\pm 10\%$ of $J_2^\odot$ (Damiani et al. 2011; Pireaux & Rozelot 2003; Rozelot et al. 2004; Rozelot & Damiani 2011; Rozelot & Fazel 2013). The Sun's quadrupole moment in INPOP10a is fitted to observations as $J_2^\odot = (2.40 \pm 0.25) \times 10^{-7}$ (Fienga et al. 2011) and its value in EPM2011 is $J_2^\odot = 2 \times 10^{-7}$ (Pitjeva & Pitjev 2013). This uncertainty of $J_2^\odot$ can cause an extra precession for a planet as described by Kozai (1959),

$$\dot{\omega}_{\delta J_2^\odot} = \frac{3}{2} \frac{\delta J_2^\odot R_\odot^2}{p^2} n \left(2 - \frac{5}{2} \sin^2 i \right), \tag{7}$$

where $p = a(1-e)$. Like Iorio (2005a), we will include this uncertainty in our estimation.

3 UPPER LIMITS ON α AND λ

The INPOP10a (Fienga et al. 2011) ephemeris provides $\dot{\omega}_{\rm suppl}$ for some planets in the solar system: Mercury, Venus, Earth-Moon Barycenter (EMB), Mars, Jupiter and Saturn. Similarly, EPM2011 (Pitjeva 2013) also gives those values of the planets from Mercury to Saturn. These numbers are taken from table 5 in Fienga et al. (2011) and tables 4 and 5 in Pitjeva & Pitjev (2013) and Pitjev & Pitjeva (2013) respectively (see Table 1 for details). It can be found that $\dot{\omega}_{\rm suppl}$ of Mercury and Venus from EPM2011 are considerably larger than those of INPOP10a, while Venus and Jupiter have non-zero values of $\dot{\omega}_{\rm suppl}$ in EPM2011.

We divide our estimation into two cases: Case I and Case II. In Case I, we neglect the uncertainty of the Sun's quadrupole moment, i.e. $\dot{\omega}_{\delta J_2^\odot} = 0$. By using the method of weighted least squares, we estimate the upper limits of Yukawa parameters as (i) $\alpha = 2.4 \times 10^{-11}$ and $\lambda = 0.15$ au by INPOP10a and (ii) $\alpha = 2.1 \times 10^{-11}$ and $\lambda = 0.23$ au by EPM2011. In Case II, we take $\delta J_2^\odot$ into account and simultaneously estimate α, λ and $\delta J_2^\odot / J_2^\odot$. We find that (i) INPOP10a yields the upper limits as $\alpha = 3.1 \times 10^{-11}$, $\lambda = 0.15$ au and $\delta J_2^\odot / J_2^\odot = -1.54\%$; and (ii) EPM2011 gives $\alpha = 5.2 \times 10^{-11}$, $\lambda = 0.21$ au and $\delta J_2^\odot / J_2^\odot = -12.2\%$. These results are summarized in Table 2.

Both INPOP10a and EPM2011 indicate the dimensionless strength of the possible deviation from the inverse-square law of gravity cannot exceed the level of a few parts in 10^{11}. Our estimations of α are at least 10 times less than the result of Iorio (2007) who uses EPM2004 (Pitjeva 2005) (see Table 2). This is a natural outcome because $\dot{\omega}_{\rm suppl}$ provided by INPOP10a (Fienga et al. 2011) and EPM2011 (Pitjeva & Pitjev 2013; Pitjev & Pitjeva 2013) are improved by at least one order of magnitude than those given by EPM2004 (Pitjeva 2005). Furthermore, the values of $\delta J_2^\odot / J_2^\odot$ by INPOP10a and EPM2011 (see Table 2) are compatible with the current uncertainty of $\pm 10\%$.

Like the method used in previous works (Iorio 2007, 2008), to check our estimation, we will calculate the ratios of the supplementary precession of perihelion for different pairs of planets A and B and will compare them to the corresponding ratios of the precession on the right-hand side of Equation (5). A quantity can be constructed as (Iorio 2007, 2008)

$$\eta_{\rm AB} = \left| \frac{\dot{\omega}^{\rm A}_{\rm suppl}}{\dot{\omega}^{\rm B}_{\rm suppl}} - \frac{\dot{\omega}^{\rm A}_{\rm YK} + \dot{\omega}^{\rm A}_{\rm LT} + \dot{\omega}^{\rm A}_{\delta J_2^\odot}}{\dot{\omega}^{\rm B}_{\rm YK} + \dot{\omega}^{\rm B}_{\rm LT} + \dot{\omega}^{\rm B}_{\delta J_2^\odot}} \right|. \tag{8}$$

If Equation (5) is correct, $\eta_{\rm AB}$ must be compatible with zero. Like in Iorio (2008), they can be calculated in terms of two groups of planets: the inner ones and the outer ones.

Table 3 shows some values of $\eta_{\rm AB}$, which are obtained by using the results of Case II. Their close-to-zero values demonstrate the validity of our estimations.

Table 1 Supplementary Advances in the Perihelia $\dot{\omega}_{\rm suppl}$ Given by INPOP10a and EPM2011

	$\dot{\omega}_{\rm suppl}$ (mas cy^{-1})	
	INPOP10a [a]	EPM2011 [b]
Mercury	0.4 ± 0.6	-2.0 ± 3.0
Venus	0.2 ± 1.5	2.6 ± 1.6
EMB	-0.2 ± 0.9	—
Earth	—	0.19 ± 0.19
Mars	-0.04 ± 0.15	-0.020 ± 0.037
Jupiter	-41 ± 42	58.7 ± 28.3
Saturn	0.15 ± 0.65	-0.32 ± 0.47

Notes: [a] Taken from table 5 in Fienga et al. (2011). [b] Provided by table 4 in Pitjeva & Pitjev (2013) and table 5 in Pitjev & Pitjeva (2013).

Table 2 Summary of α, λ and $\delta J_2^{\odot}$ Estimated by $\dot{\omega}_{\rm suppl}$

	α	λ (au)	$\delta J_2^{\odot}/J_2^{\odot}$	Adopted Ephemeris
Iorio (2007)	1.3×10^{-9}	0.18	–	EPM2004 (Pitjeva 2005)
Case I	2.4×10^{-11}	0.15	–	INPOP10a (Fienga et al. 2011)
	2.1×10^{-11}	0.23	–	EPM2011 (Pitjeva & Pitjev 2013)
Case II	3.1×10^{-11}	0.15	-1.54%	INPOP10a (Fienga et al. 2011)
	5.2×10^{-11}	0.21	-12.2%	EPM2011 (Pitjeva & Pitjev 2013)

Table 3 $\eta_{\rm AB}$ Obtained by the Results of Case II

		$\eta_{\rm AB}$	
A	B	INPOP10a	EPM2011
Venus	Mercury	2.3×10^{-1}	7.2×10^{-1}
EMB/Earth	Mercury	3.1×10^{-1}	6.6×10^{-2}
Mars	Mercury	2.3×10^{-2}	6.5×10^{-3}
EMB/Earth	Venus	2.9×10^{-1}	2.0×10^{-1}
Mars	Venus	2.7×10^{-1}	2.1×10^{-2}
Mars	EMB/Earth	4.5×10^{-1}	2.7×10^{-3}
Saturn	Jupiter	1.6×10^{-1}	1.5×10^{-1}

4 CONCLUSIONS AND DISCUSSION

Using the supplementary advances in perihelia provided by INPOP10a (Fienga et al. 2011) and EPM2011 (Pitjeva 2013) ephemerides, we estimate new upper limits on the deviation from the inverse-square law of gravity which is parameterized by the Yukawa correction to the Newtonian gravitational force with two parameters: a dimensionless strength parameter α and a length scale λ. After taking the uncertainty of the Sun's quadrupole moment into account, we find that INPOP10a yields the upper limits as $\alpha = 3.1 \times 10^{-11}$ and $\lambda = 0.15$ au and EPM2011 gives $\alpha = 5.2 \times 10^{-11}$ and $\lambda = 0.21$ au. In both of them, α is at least 10 times less than the previous results of Iorio (2007).

With tremendous advances in techniques for deep space exploration in the solar system, ephemerides are going to be increasingly improved by high-precision datasets provided from tracking spacecrafts and by sophisticated data analysis (e.g. Fienga et al. 2013; Verma et al. 2013a,b). The resulting upper limits on deviation from the inverse-square law of gravity are expected to be much tighter in the near future.

Acknowledgements This work is funded by the National Natural Science Foundation of China (Grant Nos. 11103010 and J1210039), the Fundamental Research Program of Jiangsu Province of China under No. BK2011553, the Research Fund for the Doctoral Program of Higher Education of China (No. 20110091120003) and the Innovation Training Project for Undergraduates of Nanjing University, China.

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