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Relativistic transformation between τ and TCG for Mars missions under IAU Resolutions *

Jun-Yang Pan[1] and Yi Xie[1,2]

[1] Department of Astronomy, Nanjing University, Nanjing 210093, China; *yixie@nju.edu.cn*
[2] Key Laboratory of Modern Astronomy and Astrophysics, Nanjing University, Ministry of Education, Nanjing 210093, China

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Abstract Considering the fact that the general theory of relativity has become an inextricable part of deep space missions, we investigate the relativistic transformation between the proper time of an onboard clock τ and the Geocentric Coordinate Time (TCG) for Mars missions. By connecting τ with this local timescale associated with the Earth, we extend previous works which focus on the transformation between τ and the Barycentric Coordinate Time (TCB). (TCB is the global coordinate time for the whole solar system.) For practical convenience, the relation between τ and TCG is recast to directly depend on quantities which can be read from ephemerides. We find that the difference between τ and TCG can reach the level of about 0.2 seconds in a year. To distinguish various sources in the transformation, we numerically calculate the contributions caused by the Sun, eight planets, three large asteroids and the spacecraft. It is found that if the threshold of 1 microsecond is adopted, this transformation must include effects due to the Sun, Venus, the Moon, Mars, Jupiter, Saturn and the velocities of the spacecraft and Earth.

Key words: reference systems — time — method: numerical — space vehicles

1 INTRODUCTION

With tremendous advances in modern techniques, Einstein's general relativity (GR) has become an inextricable part of deep space missions. It has gone far beyond theoretical astronomy and physics into practice and engineering (Nelson 2011). Effects due to GR clearly show up in the radio signals of some space missions (e.g. Bertotti et al. 2003; Jensen & Weaver 2007), which provide the tightest constraint on GR (Bertotti et al. 2003).

In GR, one important idea is to abandon the concept of absolute time in Newton's absolute framework of space and time. There exist different kinds of times: proper time and coordinate times (Misner et al. 1973). The readings of an ideal clock form the proper time τ, which is an observable and is associated with the clock itself. The coordinate times cannot be measured directly, but they might be used as variables in the equations of motion of celestial and artificial bodies and light rays. The coordinate times are connected with the proper time through a four-dimensional space-time interval, which depends on kinematics and dynamics of the clock. This dramatically changes the way clock synchronization and time transfer are considered (Nelson 2011).

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In exploration missions to Mars and other planets, the synchronization of the clock onboard a spacecraft and the clock on the ground is critical for control, navigation and scientific operation. According to International Astronomical Union (IAU) Resolutions (Soffel et al. 2003), two intermediate steps are required. *Step 1* is to relativistically transform τ to the Barycentric Coordinate Time (TCB), which is the global time of the solar system. Then, in *Step 2*, TCB is converted to the Geocentric Coordinate Time (TCG), which is the coordinate time belonging to the local reference system of the Earth. Then, TCG can be easily changed to other timescales on the Earth, such as Terrestrial Time (TT), International Atomic Time (TAI) and the Coordinated Universal Time (UTC).

Taking the YingHuo-1 Mission (Ping et al. 2010a,b) as a technical example of future Chinese Mars explorations, some works are devoted to investigating *Step 1*. Deng (2012) studies this transformation by analytic and numerical methods and finds two main effects associated with it: the gravitational field of the Sun and the velocity of the spacecraft in the barycentric reference system. The combined contribution of these two effects can reach a few sub-seconds in one year (Deng 2012). Pan & Xie (2013) take a clock offset into account in this transformation and find that if an onboard clock can be calibrated to achieve an accuracy better than $\sim 10^{-6}$–10^{-5} s in one year (depending on the type of clock offset), the relativistic transformation between τ and TCB must be carefully handled.

This paper extends previous works by examining the local reference system of the Earth. We will focus on the relativistic transformation between τ and TCG, which means combining *Step 1* and *Step 2*. In Section 2, we will establish the relativistic transformation between τ and TCG for a mission like YingHuo-1 according to IAU Resolutions (Soffel et al. 2003) and make the transformation explicitly depend on variables provided by ephemerides (see Appendix A for details). In Section 3, we will numerically calculate this transformation and show contributions from various sources. Conclusions and discussion will be presented in Section 4.

2 TRANSFORMATION BETWEEN τ AND TCG

Within IAU Resolutions (Soffel et al. 2003), the proper time τ of a clock onboard a spacecraft and $t \equiv$TCB have a relation up to the first order post-Newtonian (1PN) approximation as

$$\tau - t = -\epsilon^2 \int_{t_0}^{t} \bigg(\sum_A \frac{Gm_A}{r_{sA}} + \frac{1}{2} v_s^2 \bigg) \mathrm{d}t + \mathcal{O}(\epsilon^2 J_n^{(A)}, \epsilon^4), \tag{1}$$

where $\epsilon \equiv 1/c$ and the non-spherically symmetric parts of the each body's gravitational field are omitted. The index "s" stands for the spacecraft and the index "A" enumerates each body whose gravitational effect needs to be considered. Also, $T \equiv$TCG can be transformed to t by

$$t - T = \epsilon^2 \bigg\{ \int_{t_0}^{t} \bigg[\sum_{A \neq E} \frac{Gm_A}{r_{EA}} + \frac{v_E^2}{2} \bigg] \mathrm{d}t + \boldsymbol{v}_E \cdot (\boldsymbol{x} - \boldsymbol{x}_E) \bigg\} + \mathcal{O}(\epsilon^2 J_n^{(A)}, \epsilon^4), \tag{2}$$

where "E" means the Earth. Adding Equations (1) and (2), we can immediately derive the relation between τ and T as

$$\begin{aligned} \tau - T \;=\; & -\epsilon^2 \int_{t_0}^{t} \bigg(\sum_A \frac{Gm_A}{r_{sA}} + \frac{v_s^2}{2} \bigg) \mathrm{d}t + \epsilon^2 \int_{t_0}^{t} \bigg(\sum_{A \neq E} \frac{Gm_A}{r_{EA}} + \frac{v_E^2}{2} \bigg) \mathrm{d}t \\ & + \epsilon^2 \boldsymbol{v}_E \cdot (\boldsymbol{x} - \boldsymbol{x}_E) + \mathcal{O}(\epsilon^2 J_n^{(A)}, \epsilon^4). \end{aligned} \tag{3}$$

In order to obtain values of $\tau - T$, one possible way is to numerically integrate the right-hand side (RHS) of Equation (3) with the help of ephemerides, which can provide positions and velocities of bodies in the solar system with respect to their own times. However, it is not so convenient to perform this calculation because the time variable in the integrals on the RHS is TCB, instead of the Barycentric Dynamical Time (TDB). TDB is the widely used independent variable of ephemerides describing the solar system. Therefore, for practical purposes, it is necessary to make the RHS of Equation (3) explicitly depend on TDB. By making use of the relation between TCB and TDB (Petit & Luzum 2010), we can obtain the new relation as

$$\tau - T = \sum_{i=1}^{3} \Delta T_i + \sum_{j=1}^{7} \Delta T'_j + \mathcal{O}(\epsilon^2 J_n^{(A)}, \epsilon^4, L_B^2), \tag{4}$$

whose RHS directly depends on TDB. The structures of ΔT_i ($i = 1, 2, 3$) are identical to those of all components in Equation (3), except for their dependence on TDB. ΔT_1 comes from the relativistic transformation between τ and t; ΔT_2 and ΔT_3 originate from the relativistic transformation between t and T. The detailed derivation of Equation (4) and its full expressions are given in Appendix A. In the next section, we will numerically investigate each component in this equation. It is worth noting that Equation (4) still keeps the difference between TCB and TDB, which is ignored in previous works (Deng 2012; Pan & Xie 2013).

3 NUMERICAL ANALYSIS ON $\tau - T$

Like the context assumed by Pan & Xie (2013), we suppose a spacecraft orbits around Mars from 2017-Jan-01 00:00:00.0000 (TDB) to 2018-Jan-01 00:00:00.0000 (TDB). All of the time variables are rescaled from this starting point in the remaining parts of this paper. The orbital inclination of the spacecraft with respect to the Martian equator is 5°. The apoapsis altitude is 80 000 km and the periapsis altitude is 800 km, with a period of about 3.2 days. To evaluate Equation (4), we numerically integrate it by Simpson's rule (Stoer & Bulirsch 2002) and using the ephemeris DE405. In particular, the positions and velocities of celestial bodies are taken from DE405 and the orbit of the spacecraft is solved by numerically integrating the Einstein-Infeld-Hoffmann equation (Einstein et al. 1938) with the Runge-Kutta 7 method (Stoer & Bulirsch 2002). The stepsize is one-hundredth of its Keplerian period. In the calculation, we include the gravitational contributions from the Sun, eight planets, the Moon and three large asteroids: Ceres, Pallas and Vesta.

Figure 1 shows the contributions of ΔT_1 [Eq. (A.8)], ΔT_2 [Eq. (A.9)] and $\Delta T_1 + \Delta T_2$. ΔT_1, which has the component $\tau - t$ that can decrease to about -0.3 s in a year. ΔT_2 describes the component $t - T$ that can reach about 0.5 s in a year. Thus, $\Delta T_1 + \Delta T_2$ can reach the level of ~ 0.2 s in a year. An issue of theoretical and practical importance is to separate contributions from different sources.

Figure 2 shows these contributions due to the gravitational effects from celestial bodies and the kinematic effect from the spacecraft. If we take ± 1 microsecond (μs) in a year as the threshold and neglect all terms that have absolute values less than 1 μs, it is shown that only the Sun, Mars, Jupiter, Saturn and the velocity of the spacecraft need to be considered in ΔT_1. Similarly, Figure 3 shows the various contributions caused by different sources in ΔT_2, except for the sub-figure at the bottom right corner. Again, if we take 1 μs in a year as the threshold, what we need to consider are the effects caused by the Sun, Venus (marginally), the Moon, Jupiter, Saturn and the velocity of Earth.

The term ΔT_3 [Eq. (A.10)] is at the end of the communication link of the spacecraft and depends on the location of the observer. If we consider a tracking station on the ground, ΔT_3 will show a strong effect caused by the rotation of the Earth, which can reach the level of about 2 μs with the period of a day. In the bottom right corner, the sub-figure gives the curve of ΔT_3 on the assumption that the station is located in Beijing, China. In the calculation, we take values describing the direction of the pole of rotation and the prime meridian of the Earth from the report of the IAU Working Group on Cartographic Coordinates and Rotational Elements (Archinal et al. 2011), which is a sufficiently good approximation for our purposes.

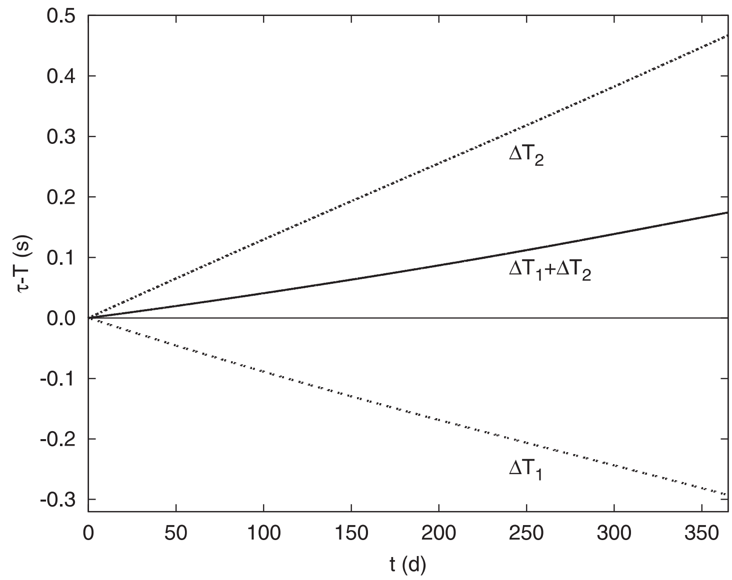

Fig. 1 Curves of ΔT_1, ΔT_2 and $\Delta T_1+\Delta T_2$. Their mathematical descriptions are given in Eqs. (A.8) and (A.9).

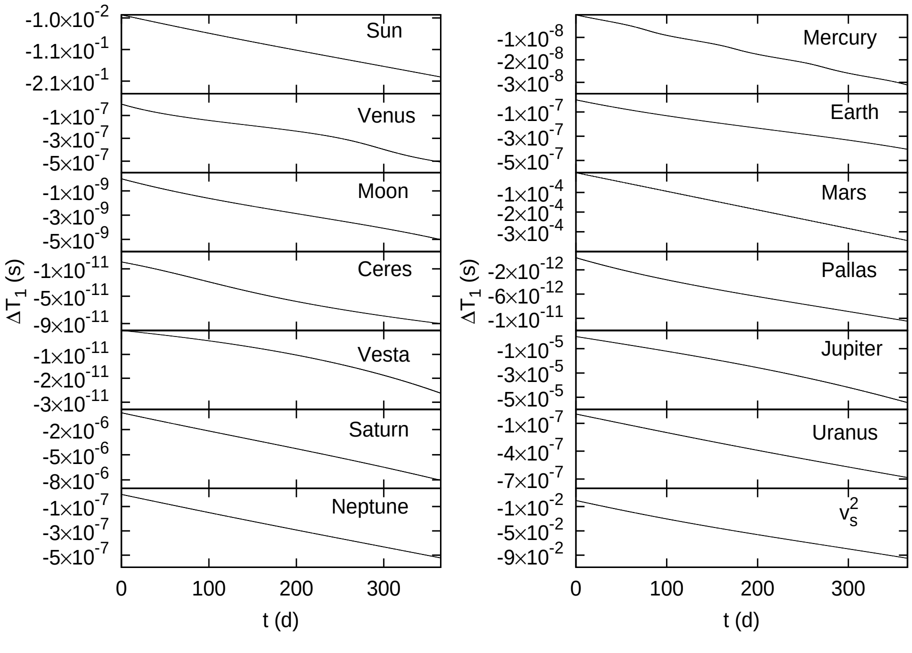

Fig. 2 In ΔT_1, contributions from the Sun, Mercury, Venus, Earth, the Moon, Mars, Ceres, Pallas, Vesta, Jupiter, Saturn, Uranus, Neptune and the velocity of the spacecraft.

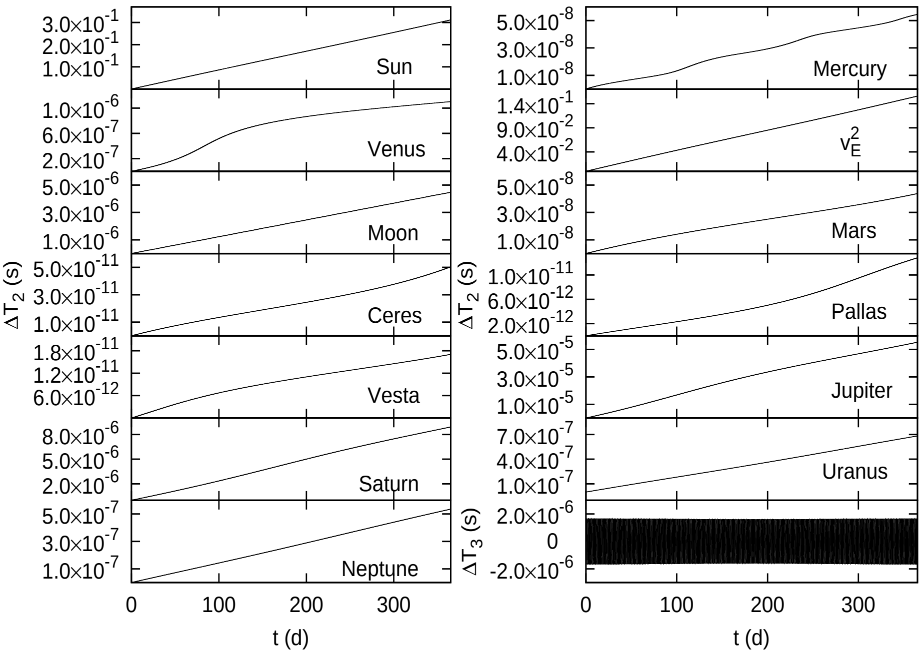

Fig. 3 In ΔT_2, contributions from the Sun, Mercury, Venus, the velocity of the Earth, the Moon, Mars, Ceres, Pallas, Vesta, Jupiter, Saturn, Uranus, and Neptune; the curve of ΔT_3 is shown in the bottom right corner for a station located in Beijing, China.

Table 1 The Maximum Contributions of $|\Delta T'_j|$ $(j = 1, \cdots, 7)$

Terms	Sources	$\max(\lvert\Delta T'_j\rvert)$ (s)	Terms	Sources	$\max(\lvert\Delta T'_j\rvert)$ (s)
$\Delta T'_1$		$\sim 5 \times 10^{-9}$	$\Delta T'_4$		
$\Delta T'_2$		$\sim 7 \times 10^{-9}$		Sun	$\sim 6 \times 10^{-9}$
$\Delta T'_3$				Mercury	$\sim 2 \times 10^{-14}$
	Sun	$\sim 2 \times 10^{-8}$		Venus	$\sim 2 \times 10^{-14}$
	Mercury	$\sim 6 \times 10^{-15}$		Mars	$\sim 2 \times 10^{-15}$
	Venus	$\sim 2 \times 10^{-14}$		Ceres	$\sim 2 \times 10^{-17}$
	Earth	$\sim 2 \times 10^{-13}$		Pallas	$\sim 7 \times 10^{-18}$
	Mars	$\sim 2 \times 10^{-9}$		Vesta	$\sim 8 \times 10^{-18}$
	Ceres	$\sim 2 \times 10^{-17}$		Jupiter	$\sim 8 \times 10^{-12}$
	Pallas	$\sim 5 \times 10^{-18}$		Saturn	$\sim 1 \times 10^{-12}$
	Vesta	$\sim 2 \times 10^{-17}$		Uranus	$\sim 2 \times 10^{-14}$
	Jupiter	$\sim 2 \times 10^{-11}$		Neptune	$\sim 5 \times 10^{-15}$
	Saturn	$\sim 4 \times 10^{-11}$		$\tilde{\boldsymbol{v}}_E \cdot \tilde{\boldsymbol{a}}_E$	$\sim 6 \times 10^{-9}$
	Uranus	$\sim 6 \times 10^{-14}$	$\Delta T'_5$		$\sim 2 \times 10^{-7}$
	Neptune	$\sim 3 \times 10^{-14}$	$\Delta T'_6$		$\sim 8 \times 10^{-12}$
	$\tilde{\boldsymbol{v}}_s \cdot \tilde{\boldsymbol{a}}_s$	$\sim 6 \times 10^{-8}$	$\Delta T'_7$		$\sim 2 \times 10^{-14}$

For $\Delta T'_j$ $(j = 1, \cdots, 7)$, whose mathematical expressions are given in Equations (A.11)–(A.17), we numerically calculate their contributions in the same case and find the maximum absolute values of their contributions are all less than 1 μs (see Table 1 for details).

4 CONCLUSIONS AND DISCUSSION

In this work, we take a mission like YingHuo-1 as an example and investigate the relativistic transformation between the proper time τ of the onboard clock and TCG, extending previous works which focus on the transformation between τ and TCB. For practical convenience, the relation between τ and TCG is converted to directly depend on quantities which can be read from ephemerides. We find that the difference between τ and TCG can reach the level of about 0.2 s in a year. If the threshold of 1 μs is adopted, this transformation must include the effects due to the Sun, Venus, the Moon, Mars, Jupiter, Saturn and the velocities of the spacecraft and Earth.

In subsequent works, we will focus on establishing a relativistic algorithm for time synchronization of missions to Mars by including geometric and relativistic time delays. Moreover, the clock offset will also be taken into account. We hope they may be helpful for tests of fundamental physics in the solar system with onboard clocks and radio/laser links (e.g. Deng & Xie 2013a,b).

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Appendix A: $\tau - T$: DEPENDENCE ON TDB

As a time variable of ephemerides, TDB is related to TCB as (Petit & Luzum 2010)

$$\mathrm{TDB} = \mathrm{TCB} - L_B \times (\mathrm{JD}_{\mathrm{TCB}} - T_0) \times 86400\,\mathrm{s} + \mathrm{TDB}_0, \tag{A.1}$$

where $\mathrm{JD}_{\mathrm{TCB}}$ is the TCB Julian date and $T_0 = 2443144.5003725$, and where $L_B = 1.550519768 \times 10^{-8}$ and $\mathrm{TDB}_0 = -6.55 \times 10^{-5}$ s are defining constants. In the form of a Julian date which is adopted as the time variable of ephemerides by Jet Propulsion Laboratory, we can have

$$\mathrm{JD}_{\mathrm{TDB}} = \mathrm{JD}_{\mathrm{TCB}} - L_B \times (\mathrm{JD}_{\mathrm{TCB}} - T_0) + \mathrm{JD}_{\mathrm{TDB}_0}, \tag{A.2}$$

where $\mathrm{JD}_{\mathrm{TDB}_0} = \mathrm{TDB}_0/(86400\,\mathrm{s}) = -7.5810185185 \times 10^{-10}$. It can yield

$$\mathrm{JD}_{\mathrm{TCB}} = \mathrm{JD}_{\mathrm{TDB}} + g_{\mathrm{JD}}(\mathrm{JD}_{\mathrm{TDB}}) + \mathcal{O}(L_B^2), \tag{A.3}$$

where $g_{\mathrm{JD}}(\mathrm{JD}_{\mathrm{TDB}}) \equiv L_B \times (\mathrm{JD}_{\mathrm{TDB}} - T_0) - (1 + L_B)\mathrm{JD}_{\mathrm{TDB}_0}$.

If there is a function of $\mathrm{JD}_{\mathrm{TCB}}$, it can be expanded as

$$\begin{aligned} f(\mathrm{JD}_{\mathrm{TCB}}) &= f[\mathrm{JD}_{\mathrm{TDB}} + g_{\mathrm{JD}}(\mathrm{JD}_{\mathrm{TDB}}) + \mathcal{O}(L_B^2)] \\ &= f(\mathrm{JD}_{\mathrm{TDB}}) + g_{\mathrm{s}}(\mathrm{JD}_{\mathrm{TDB}}) \left.\frac{\mathrm{d}f}{\mathrm{d}t}\right|_{t=\mathrm{JD}_{\mathrm{TDB}}} + \mathcal{O}(L_B^2), \end{aligned} \tag{A.4}$$

where $g_{\rm s}(\mathrm{JD}_{\rm TDB}) = g_{\rm JD}(\mathrm{JD}_{\rm TDB}) \times 86400$ s. By making use of it and setting $\tilde{T} \equiv$TDB so that $\mathrm{d}\tilde{T} = (1 - L_B)\mathrm{d}t$, we can obtain

$$\begin{aligned}
\tau - t \;=&\; -\epsilon^2 \int_{t_0}^{t} \left(\sum_A \frac{Gm_A}{r_{sA}} + \frac{v_s^2}{2} \right) \mathrm{d}t + \mathcal{O}(\epsilon^2 J_n^{(A)}, \epsilon^4) \\
=&\; -\epsilon^2 (1 + L_B) \int_{\tilde{T}_0}^{\tilde{T}} \left(\sum_A \frac{Gm_A}{\tilde{r}_{sA}} + \frac{\tilde{v}_s^2}{2} \right) \mathrm{d}\tilde{T} \\
&\; -\epsilon^2 \int_{\tilde{T}_0}^{\tilde{T}} \left[-\sum_A \frac{Gm_A}{\tilde{r}_{sA}^3} (\tilde{\boldsymbol{r}}_{sA} \cdot \tilde{\boldsymbol{v}}_{sA}) + \tilde{\boldsymbol{v}}_s \cdot \tilde{\boldsymbol{a}}_s \right] [g_{\rm s}(\mathrm{JD}_{\tilde{T}}) - L_B \mathrm{TDB}_0] \mathrm{d}\tilde{T} \\
&\; + \mathcal{O}(\epsilon^2 J_n^{(A)}, \epsilon^4, L_B^2),
\end{aligned} \tag{A.5}$$

where $\tilde{r}_{sA} = |\boldsymbol{x}_s(\tilde{T}) - \boldsymbol{x}_A(\tilde{T})|$ and $\tilde{\boldsymbol{v}}_s = \boldsymbol{v}_s(\tilde{T})$. Hereafter, a quantity with a tilde means it takes a value at $\tilde{T}$. Similarly, the relation between t and T can be expanded as

$$\begin{aligned}
t - T \;=&\; \epsilon^2 \int_{t_0}^{t} \left[\sum_{A \neq E} \frac{Gm_A}{r_{EA}} + \frac{v_E^2}{2} \right] \mathrm{d}t + \epsilon^2 \boldsymbol{v}_E \cdot (\boldsymbol{x} - \boldsymbol{x}_E) + \mathcal{O}(\epsilon^2 J_n^{(A)}, \epsilon^4) \\
=&\; +\epsilon^2 (1 + L_B) \int_{\tilde{T}_0}^{\tilde{T}} \left(\sum_{A \neq E} \frac{Gm_A}{\tilde{r}_{EA}} + \frac{\tilde{v}_E^2}{2} \right) \mathrm{d}\tilde{T} \\
&\; +\epsilon^2 \int_{\tilde{T}_0}^{\tilde{T}} \left[-\sum_{A \neq E} \frac{Gm_A}{\tilde{r}_{EA}^3} (\tilde{\boldsymbol{r}}_{EA} \cdot \tilde{\boldsymbol{v}}_{EA}) + \tilde{\boldsymbol{v}}_E \cdot \tilde{\boldsymbol{a}}_E \right] [g_{\rm s}(\mathrm{JD}_{\tilde{T}}) - L_B \mathrm{TDB}_0] \mathrm{d}\tilde{T} \\
&\; +\epsilon^2 \tilde{\boldsymbol{v}}_E \cdot (\boldsymbol{x} - \tilde{\boldsymbol{x}}_E) \\
&\; -\epsilon^2 g_s(\mathrm{JD}_{\tilde{T}}) \tilde{v}_E^2 + \epsilon^2 g_s(\mathrm{JD}_{\tilde{T}}) \tilde{\boldsymbol{a}}_E \cdot (\boldsymbol{x} - \tilde{\boldsymbol{x}}_E) - \epsilon^2 g_s^2(\mathrm{JD}_{\tilde{T}}) \tilde{\boldsymbol{a}}_E \cdot \tilde{\boldsymbol{v}}_E \\
&\; + \mathcal{O}(\epsilon^2 J_n^{(A)}, \epsilon^4, L_B^2).
\end{aligned} \tag{A.6}$$

After combining Equations (A.5) and (A.6), we can finally obtain

$$\tau - T = \sum_{i=1}^{3} \Delta T_i + \sum_{j=1}^{7} \Delta T'_j + \mathcal{O}(\epsilon^2 J_n^{(A)}, \epsilon^4, L_B^2), \tag{A.7}$$

where

$$\Delta T_1 \;=\; -\epsilon^2 \int_{\tilde{T}_0}^{\tilde{T}} \left(\sum_A \frac{Gm_A}{\tilde{r}_{sA}} + \frac{\tilde{v}_s^2}{2} \right) \mathrm{d}\tilde{T}, \tag{A.8}$$

$$\Delta T_2 \;=\; \epsilon^2 \int_{\tilde{T}_0}^{\tilde{T}} \left(\sum_{A \neq E} \frac{Gm_A}{\tilde{r}_{EA}} + \frac{\tilde{v}_E^2}{2} \right) \mathrm{d}\tilde{T}, \tag{A.9}$$

$$\Delta T_3 \;=\; \epsilon^2 \tilde{\boldsymbol{v}}_E \cdot (\boldsymbol{x} - \tilde{\boldsymbol{x}}_E), \tag{A.10}$$

and

$$
\begin{aligned}
\Delta T_1' &= -\epsilon^2 L_B \int_{\tilde{T}_0}^{\tilde{T}} \left(\sum_A \frac{Gm_A}{\tilde{r}_{sA}} + \frac{\tilde{v}_s^2}{2} \right) \mathrm{d}\tilde{T}, & \text{(A.11)} \\
\Delta T_2' &= +\epsilon^2 L_B \int_{\tilde{T}_0}^{\tilde{T}} \left(\sum_{A \neq E} \frac{Gm_A}{\tilde{r}_{EA}} + \frac{\tilde{v}_E^2}{2} \right) \mathrm{d}\tilde{T}, & \text{(A.12)} \\
\Delta T_3' &= -\epsilon^2 \int_{\tilde{T}_0}^{\tilde{T}} \left[-\sum_A \frac{Gm_A}{\tilde{r}_{sA}^3} (\tilde{\boldsymbol{r}}_{sA} \cdot \tilde{\boldsymbol{v}}_{sA}) + \tilde{\boldsymbol{v}}_s \cdot \tilde{\boldsymbol{a}}_s \right] [g_{\mathrm{s}}(\mathrm{JD}_{\tilde{T}}) - L_B \mathrm{TDB}_0] \mathrm{d}\tilde{T}, & \text{(A.13)} \\
\Delta T_4' &= +\epsilon^2 \int_{\tilde{T}_0}^{\tilde{T}} \left[-\sum_{A \neq E} \frac{Gm_A}{\tilde{r}_{EA}^3} (\tilde{\boldsymbol{r}}_{EA} \cdot \tilde{\boldsymbol{v}}_{EA}) + \tilde{\boldsymbol{v}}_E \cdot \tilde{\boldsymbol{a}}_E \right] [g_{\mathrm{s}}(\mathrm{JD}_{\tilde{T}}) - L_B \mathrm{TDB}_0] \mathrm{d}\tilde{T}, & \text{(A.14)} \\
\Delta T_5' &= -\epsilon^2 g_s(\mathrm{JD}_{\tilde{T}}) \tilde{v}_E^2, & \text{(A.15)} \\
\Delta T_6' &= +\epsilon^2 g_s(\mathrm{JD}_{\tilde{T}}) \tilde{\boldsymbol{a}}_E \cdot (\boldsymbol{x} - \tilde{\boldsymbol{x}}_E), & \text{(A.16)} \\
\Delta T_7' &= -\epsilon^2 g_s^2(\mathrm{JD}_{\tilde{T}}) \tilde{\boldsymbol{a}}_E \cdot \tilde{\boldsymbol{v}}_E. & \text{(A.17)}
\end{aligned}
$$

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