

An interacting scenario for dark energy in a Bianchi type-I universe

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Abstract We study the interaction between dark energy (DE) and dark matter in the scope of anisotropic Bianchi type-I space-time. First we derive the general form of the DE equation of state (EoS) parameter in both non-interacting and interacting cases and then we examine its future by applying a hyperbolic scale factor. It is shown that in the non-interacting case, depending on the value of the anisotropy parameter K , the DE EoS parameter varies from phantom to quintessence whereas in the interacting case the EoS parameter varies in the quintessence region. However, in both cases, the DE EoS parameter ω^{de} ultimately (i.e. at $z = -1$) tends to the cosmological constant ($\omega^{\text{de}} = -1$). Moreover, we fix the cosmological bound on the anisotropy parameter K by using recent observational data about the Hubble parameter.

Key words: cosmology: Bianchi type-I model — dark energy — dark matter

1 INTRODUCTION

The direct observations and evidence collected by the High- z Supernova Search Teams (Riess et al. 1998; Perlmutter et al. 1999) in 1998 and 1999 indicated that the rate of expansion of our universe is positive, i.e. we live in an accelerating expanding universe. The above fact has also been confirmed by astrophysical observations such as measurements of the cosmic microwave background (CMB, de Bernardis et al. 2000; Benoît et al. 2003; Spergel et al. 2003) and the power spectrum of galaxies with high redshift (Tegmark et al. 2004; Page et al. 2003). These observations show that the geometry of the present day universe is almost flat. However, the most surprising and counterintuitive result coming from these observations is the fact that only $\sim 4.6\%$ of the total energy density in the universe is in the form of baryonic (non-relativistic) matter, $\sim 24\%$ is non-baryonic (relativistic) matter called dark matter (DM), and almost $\sim 71.4\%$ is a completely unknown component with negative pressure called dark energy (DE). Despite the gravitational attraction of matter, DE produces a repulsive force which gives rise to the current accelerating expansion. Since 1998, many theoretical and observational attempts have been made in order to investigate the real nature of DE. The most important problem in the study of DE is the fact that this mysterious component does not interact with baryonic matter and hence we do not have any way to detect it. Although some current observations (Bertolami et al. 2007; Le Delliou et al. 2007) show that there is an interaction between DE and DM, the amount of this interaction is very small and it is not detectable by today's technology.

To date, we only know that DE is non-clustering and spatially homogeneous; although it dominates the present universe, its effect was small in early times.

From a theoretical point of view, the study of the nature of DE is possible through its equation of state parameter ω^{de} which is the ratio of the pressure to the energy density of DE. However, the exact value of the DE equation of state (EoS) parameter at the present time is not yet clear. Our lack of knowledge allows us to suggest different theoretical candidates for DE. The first natural candidate is a cosmological constant Λ with $\omega = -1$ (Weinberg 1989; Carroll 2001). But, this model cannot explain why the present amount of DE is so small compared with the fundamental scale (fine-tuning problem) and why it is comparable with the critical density today (coincidence problem). To solve such fundamental problems associated with the cosmological constant scenario, different forms of dynamically changing DE with an effective EoS, including quintessence ($-1 < \omega^{\text{de}} < -\frac{1}{3}$) (Wetterich 1988; Ratra & Peebles 1988), phantom ($\omega^{\text{de}} < -1$) (Caldwell 2002), quintom ($\omega^{\text{de}} < -\frac{1}{3}$) (Feng et al. 2005), Chaplygin gas models (Srivastava 2005; Bertolami et al. 2004), etc, have been proposed.

A simple and straightforward way to solve the coincidence problem in cosmology is to consider an energy flow from DE to DM (Chimento et al. 2003; Dalal et al. 2001). Such an energy transfer could easily explain why, at the present time, the energy densities of DE and DM are almost equal. Theoretical models of interacting and non-interacting DE have been widely studied in the literature (Zhang 2005; Zimdahl & Pavón 2004; Setare 2007a,b; Setare et al. 2009; Sheykhi & Setare 2011; Pradhan et al. 2011a,b; Amirhashchi et al. 2013; Amirhashchi 2013). Recently, Saha et al. (2012), Saha (2013a,b), Pradhan (2013), Yadav (2012) and Yadav & Sharma (2013) have investigated DE in different contexts.

In this paper, we study the interaction between DE and DM on the basis of anisotropic Bianchi type-I space-time. To our knowledge, this work is the first study of interacting DE in an anisotropic space-time in its general form. The outline of the paper is as follows: In Section 2, the metric and field equations as well as the Friedmann-like equation are described. Section 3 deals with the non-interacting two fluid DE case. The interaction between DE and DM will be studied in Section 4. In Section 5, we constrain the anisotropy parameter K using a direct fitting procedure involving the Hubble rate $H(z)$. Finally, conclusions are summarized in the final section.

2 THE METRIC AND FIELD EQUATIONS

In an orthogonal form, a Bianchi type-I line-element is given by

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2, \quad (1)$$

where $A(t)$, $B(t)$ and $C(t)$ are only functions of time.

Einstein's field equations (in gravitational units $8\pi G = c = 1$) read as

$$R_j^i - \frac{1}{2}Rg_j^i = T_j^{(m)i} + T_j^{(\text{de})i}, \quad (2)$$

where $T_j^{(m)i}$ and $T_j^{(\text{de})i}$ are the energy momentum tensors of DM and viscous DE, respectively. They are given by

$$\begin{aligned} T_j^{(m)i} &= \text{diag}[-\rho^m, p^m, p^m, p^m], \\ &= \text{diag}[-1, \omega^m, \omega^m, \omega^m]\rho^m, \end{aligned} \quad (3)$$

and

$$\begin{aligned} T_j^{(\text{de})i} &= \text{diag}[-\rho^{\text{de}}, p^{\text{de}}, p^{\text{de}}, p^{\text{de}}], \\ &= \text{diag}[-1, \omega^{\text{de}}, \omega^{\text{de}}, \omega^{\text{de}}]\rho^{\text{de}}, \end{aligned} \quad (4)$$

where ρ^m and p^m are the energy density and pressure of the perfect fluid component while $\omega^m = p^m/\rho^m$ is its EoS parameter. Similarly, ρ^{de} and p^{de} are, respectively, the energy density and pressure of the viscous DE component while $\omega^{de} = p^{de}/\rho^{de}$ is the corresponding EoS parameter.

In a co-moving coordinate system ($u^i = \delta_0^i$), Einstein’s field Equations (2) with (3) and (4) for a Bianchi type-I metric (1) subsequently lead to the following system of equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\omega^m \rho^m - \omega^{de} \rho^{de}, \tag{5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -\omega^m \rho^m - \omega^{de} \rho^{de}, \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -\omega^m \rho^m - \omega^{de} \rho^{de}, \tag{7}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} = \rho^m + \rho^{de}. \tag{8}$$

A solution to the above set of differential equations (Eqs. (5)–(8)) has already been given in Saha (2005) as

$$A(t) = a_1 a \exp(b_1 \int a^{-3} dt), \tag{9}$$

$$B(t) = a_2 a \exp(b_2 \int a^{-3} dt), \tag{10}$$

and

$$C(t) = a_3 a \exp(b_3 \int a^{-3} dt), \tag{11}$$

where

$$a_1 a_2 a_3 = 1, \quad b_1 + b_2 + b_3 = 0.$$

Here $a = (ABC)^{\frac{1}{3}}$ is the average scale factor of the Bianchi type-I model. Using Equations (9)–(11) in Equation (8) we obtain

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho^m + \rho^{de}}{3} + K a^{-6}, \tag{12}$$

which is the analog of the Friedmann equation and $K = b_1 b_2 + b_1 b_3 + b_2 b_3$, which is a constant. Note that K denotes the deviation from isotropy, e.g. $K = 0$ represents a flat Friedmann-Lemaître-Robertson-Walker universe.

3 NON-INTERACTING DARK ENERGY

In this section we assume that there is no interaction between DE and DM. In this case, we can simply rewrite the equation describing energy conservation ($T^i_j = 0$) which yields

$$\dot{\rho}^m + 3\frac{\dot{a}}{a}(1 + \omega^m)\rho^m + \dot{\rho}^{de} + 3\frac{\dot{a}}{a}(1 + \omega^{de})\rho^{de} = 0, \tag{13}$$

for these two dark components separately as

$$\dot{\rho}^m + 3\frac{\dot{a}}{a}(1 + \omega^m)\rho^m = 0, \tag{14}$$

$$\dot{\rho}^{de} + 3\frac{\dot{a}}{a}(1 + \omega^{de})\rho^{de} = 0. \tag{15}$$

Integrating Equation (14), we find

$$\rho^m = \rho_0^m a^{-3(1+\omega^m)}, \tag{16}$$

where ρ_0^m is a constant of integration.

Using Equation (16) in Equation (12), we can find the energy density of the DE in terms of the average scale factor a as

$$\rho^{\text{de}} = 3H^2(1 - \Omega_0^m a^{-3(1+\omega^m)}) - 3Ka^{-6}, \tag{17}$$

where $\Omega^m = \frac{\rho^m}{3H^2}$ is the energy density of the dark matter and the subscript 0 shows the present value of Ω^m .

Now, using Equations (10), (11), (16) and (17) in Equation (5), we finally obtain the EoS parameter of the DE as

$$\omega^{\text{de}} = \frac{2H^2(q - 1/2) + Ka^{-6}}{3H^2(1 - \Omega_0^m a^{-3(1+\omega^m)}) - 3Ka^{-6}}, \tag{18}$$

where $q = -\frac{\ddot{a}}{aH^2}$ is the deceleration parameter. This is the general form of the EoS parameter of DE in the non-interacting scenario. To get more results about the behavior of the EoS parameter given by Equation (18), especially at late time, we consider the following hyperbolic scale factor

$$a = (1 + z)^{-1} = \sinh(t), \tag{19}$$

where z is the redshift. Using Equation (19) in (18), we obtain the EoS parameter in terms of redshift as

$$\omega^{\text{de}} = -\frac{1}{3} \left(\frac{1 + \frac{2}{1+(1+z)^2} + K \frac{(1+z)^6}{1+(1+z)^2}}{1 + K \frac{(1+z)^6}{1+(1+z)^2} - \Omega_0^m (1+z)^{-3(1+\omega^m)}} \right), \tag{20}$$

The behavior of EoS in term of redshift z is shown in Figure 1 for different values of the anisotropy parameter K . It is observed that for small values of K , the EoS parameter varies in the quintessence region whereas for bigger values of K it varies in the phantom region. At the later stage of evolution it tends to the same constant value, namely cosmological constant $\omega^{\text{de}} = -1$ independent of the parameter K . It is worth mentioning that while the current cosmological data from type Ia supernovae (SNIa) (Riess et al. 2004; Astier et al. 2006), CMB (Komatsu et al. 2009; MacTavish et al. 2006) and large scale structure studies from the Sloan Digital Sky Survey (SDSS) (Eisenstein et al. 2005) rule out $\omega^{\text{de}} \geq 1$, they mildly favor dynamically evolving DE crossing the phantom divide line (PDL) (see Zhao et al. 2007; Copeland et al. 2006 for the theoretical and observational status of crossing the PDL). Thus our DE model is in good agreement with recent well established theoretical results and recent observations as well.

In this case, the expressions for the matter-energy density Ω^m and dark-energy density Ω^{de} are given by

$$\Omega^m = \frac{\rho^m}{3H^2} = \frac{\rho_0(1+z)^{3(1+\omega^m)}}{3(1+(1+z)^2)}, \tag{21}$$

and

$$\Omega^{\text{de}} = \frac{\rho^{\text{de}}}{3H^2} = 1 + \frac{K(1+z)^6 - \rho_0(1+z)^{3(1+\omega^m)}}{3(1+(1+z)^2)} \tag{22}$$

respectively. Hence the total energy density is given by

$$\Omega = \Omega^m + \Omega^{\text{de}} = 1 + \frac{K(1+z)^6}{3(1+(1+z)^2)}. \tag{23}$$

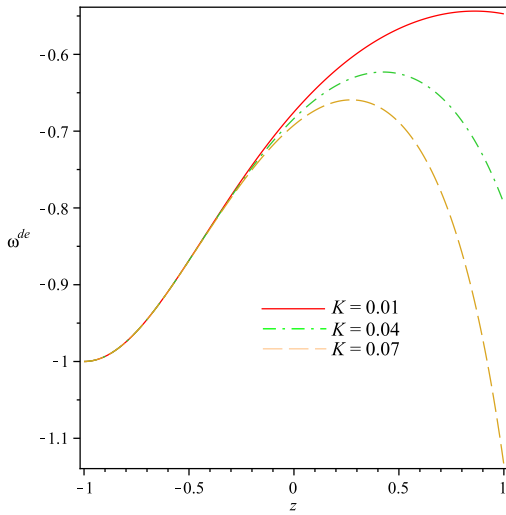


Fig. 1 The EoS parameter ω^{de} vs. z for $K = 0.01, 0.04, 0.07$ and $\Omega_0^m = 0.24$.

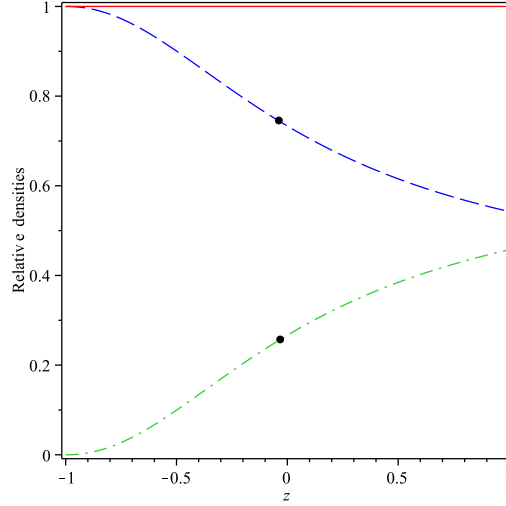


Fig. 2 The plot of the DM, DE and total energy densities ($\Omega^m, \Omega^{\text{de}}, \Omega$) vs. z for $K = 0.09$. The dots show the present values of Ω^m and Ω^{de} .

Figure 2 shows the permitted values of Ω^m and Ω^{de} in our model. The dots denote the current values of these two parameters. From this figure we observe that the predicted values of these two dark components are in good agreement with those obtained through recent observations.

As usual we can examine our DE models through energy conditions. (For a recent review see Zhang et al. 2013.) The plot of weak, dominant and strong energy conditions is shown in Figure 3. From this figure we see that in the non-interacting case

$$(i) \rho^{\text{de}} \geq 0, \quad (ii) \rho^{\text{de}} + p^{\text{de}} \leq 0, \quad (iii) \rho^{\text{de}} + p^{\text{de}} \leq 0. \quad (24)$$

Thus, from the above expressions, we observe that the phantom model, which violates both the strong and weak energy conditions, is a possible scenario in this case. It is worth mentioning that recent observational data indicate the phantom model of the universe with $\omega^{\text{de}} \leq -1$ is allowed at the 68% confidence level (C.L.).

4 INTERACTING DARK ENERGY

In this case we consider an energy transfer from DE to DM. Therefore, the interaction between the two dark components, which is represented by the quantity Q , should be a positive function of time or equivalently redshift (see Eqs. (25) and (26)). A positive Q ensures that the second law of thermodynamics is fulfilled (Pavon & Wang 2009). Here, the energy conservation equation, Equation (19), may be written as

$$\dot{\rho}^m + 3\frac{\dot{a}}{a}(1 + \omega^m)\rho^m = Q, \quad (25)$$

$$\dot{\rho}^{\text{de}} + 3\frac{\dot{a}}{a}(1 + \omega^{\text{de}})\rho^{\text{de}} = -Q. \quad (26)$$

Since the nature of DE is still unknown to us, we have the freedom to choose different but appropriate functions for Q . The most important forms of Q are: (i) $Q \propto H\rho^X$ and (ii) $Q \propto H(\rho^m + \rho^X)$. In our study we assume

$$Q = 3H\sigma\rho^m, \quad (27)$$

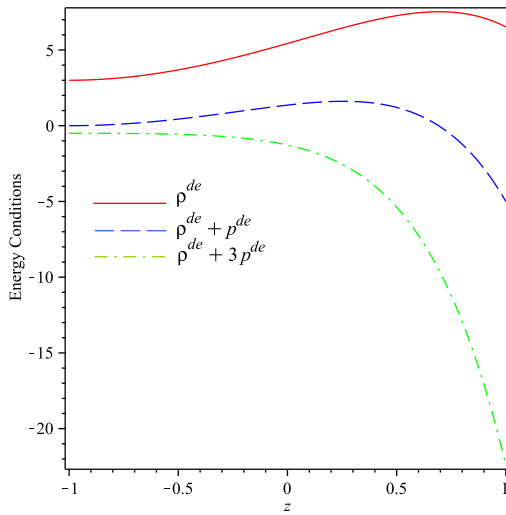


Fig. 3 The plot of the weak $\rho^{de} \geq 0$, dominant $\rho^{de} + p^{de} \geq 0$ and strong $\rho^{de} + 3p^{de} \geq 0$ energy conditions vs. z .

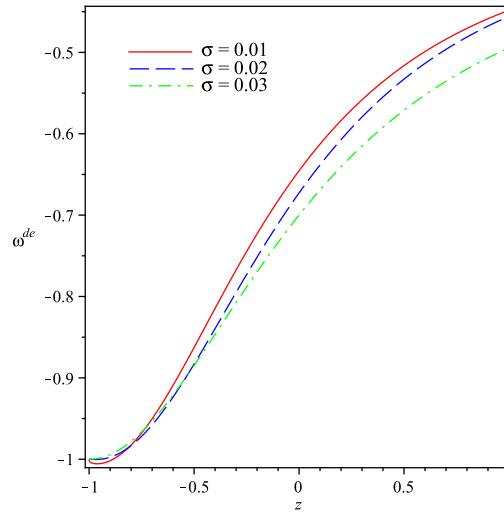


Fig. 4 The plot of the DE EoS parameter ω^{de} vs. z . Here, we fix the parameter $K = 0.01$ and vary σ as 0.01, 0.02 and 0.03.

where σ is a coupling constant. Recent astrophysical observations (Guo et al. 2007) show that in the constant coupling models, $-0.08 < \sigma < 0.03$ (95% C.L.).

Putting Equation (27) into Equation (25) and after integrating, we obtain

$$\rho^m = \rho_0^m a^{-3(1+\omega^m-\sigma)}, \tag{28}$$

where ρ_0^m is a constant of integration. Substituting Equation (28) in Equation (12), we obtain

$$\rho^{de} = 3H^2(1 - \Omega_0^m a^{-3(1+\omega^m-\sigma)}) - 3Ka^{-6}. \tag{29}$$

Using Equations (10), (11), (28) and (29) in Equation (5), the general form of the DE EoS parameter is obtained as

$$\omega^{de} = \frac{2H^2(q - 1/2) + Ka^{-6}}{3H^2(1 - \Omega_0^m a^{-3(1+\omega^m-\sigma)}) - 3Ka^{-6}}. \tag{30}$$

Using the scale factor (19), we can rewrite Equation (30) in terms of redshift as below

$$\omega^{de} = -\frac{1}{3} \left(\frac{1 + \frac{2}{1+(1+z)^2} + K \frac{(1+z)^6}{1+(1+z)^2}}{1 + K \frac{(1+z)^6}{1+(1+z)^2} - \Omega_0^m (1+z)^{-3(1+\omega^m-\sigma)}} \right). \tag{31}$$

The variation of the EoS parameter for DE in terms of redshift z is shown in Figure 4. As the late time evolution of DE is interesting for us, we assume $\omega^m = 0$. In Figure 4 we fix the parameter $K = 0.01$ and vary σ as 0.01, 0.02 and 0.03. The plot shows that the evolution of ω^{de} depends on the parameters σ but at the present time the DE EoS parameter does not cross the PDL for any value of σ . In summary, the EoS parameter only varies in the quintessence region and ultimately tends to the cosmological constant region $\omega^{de} = -1$. However, the figure shows that for $\sigma = 0.01$, at late time, the DE EoS parameter could jump to the phantom region temporarily. As already mentioned, the current SNIa, CMB and SDSS cosmological data mildly favor a dynamically evolving DE crossing the PDL.

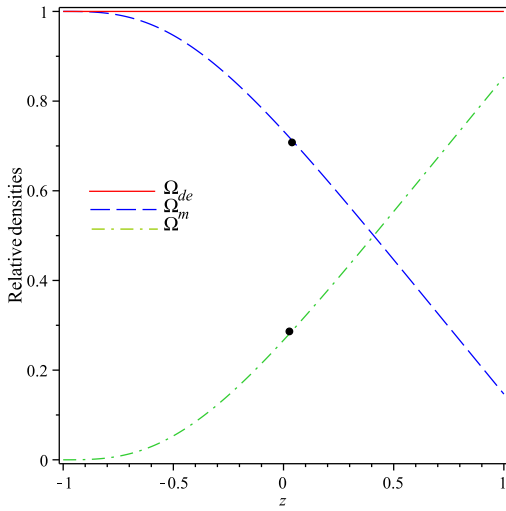


Fig. 5 The plot of the DM, DE and total energy densities ($\Omega^m, \Omega^{de}, \Omega$) vs. z for $K = 0.11$. The dots show the present values of Ω^m and Ω^{de} .

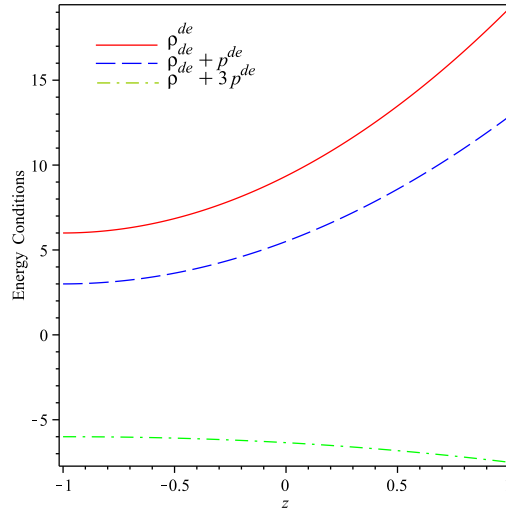


Fig. 6 The plot of the weak $\rho^{de} \geq 0$, dominant $\rho^{de} + p^{de} \geq 0$ and strong $\rho^{de} + 3p^{de} \geq 0$ energy conditions vs. z .

The expressions for the matter-energy density Ω^m and dark-energy density Ω^{de} are given by

$$\Omega^m = \frac{\rho^m}{3H^2} = \frac{\rho_0(1+z)^{3(1+\omega^m-\sigma)}}{3(1+(1+z)^2)}, \tag{32}$$

and

$$\Omega^{de} = \frac{\rho^{de}}{3H^2} = 1 + \frac{K(1+z)^6 - \rho_0(1+z)^{3(1+\omega^m-\sigma)}}{3(1+(1+z)^2)}, \tag{33}$$

respectively. Therefore, the total energy density is given by

$$\Omega = \Omega^m + \Omega^{de} = 1 + \frac{K(1+z)^6}{3(1+(1+z)^2)}, \tag{34}$$

which is the same as Equation (23) in the non-interacting case as expected.

The permitted values of Ω^m and Ω^{de} for the interacting case are shown in Figure 5. From this figure we observe that the current values of Ω^m and Ω^{de} predicted by our model are in good agreement with those obtained by recent observations. In Figure 5, the present values of DE and DM energy densities are indicated by dots.

Figure 6 shows the plot of weak, dominant and strong energy conditions. In this case, the energy conditions obey the following restrictions:

$$(i) \rho^{de} \geq 0, \quad (ii) \rho^{de} + p^{de} \geq 0, \quad (iii) \rho^{de} + p^{de} \leq 0 \text{ only for } \sigma > 0.01. \tag{35}$$

From the above expressions and Figure 6, we see that in the interacting case only the strong energy condition is violated. Hence, in this case, the only possible scenario at the present time is quintessence.

Table 1 The Cosmological Data

z	$H(z)$ (km s ⁻¹ Mpc ⁻¹)	1 σ error	Reference
0.090	69	± 12	[1]
0.170	83	± 8	[1]
0.270	77	± 14	[1]
0.400	95	± 17	[1]
0.900	117	± 23	[1]
1.300	168	± 17	[1]
1.430	177	± 18	[1]
1.530	140	± 14	[1]
1.750	202	± 40	[1]
0.480	97	± 62	[2]
0.880	90	± 40	[2]
0.179	75	± 4	[3]
0.199	75	± 5	[3]
0.352	83	± 14	[3]
0.593	104	± 13	[3]
0.680	92	± 8	[3]
0.781	105	± 12	[3]
0.875	125	± 17	[3]
1.037	154	± 20	[3]
0.24	79.69	± 3.32	[4]
0.43	86.45	± 3.27	[4]
0.07	69.0	± 19.6	[5]
0.12	68.6	± 26.2	[5]
0.20	72.9	± 29.6	[5]
0.28	88.8	± 36.6	[5]

References: [1] Simon et al. (2005); [2] Stern et al. (2010); [3] Moresco et al. (2012); [4] Gaztañaga et al. (2009); [5] Zhang et al. (2012).

5 THE EXPERIMENTAL $H(z)$ TEST

As the anisotropy parameter K plays a very significant role in our study, in this section we try to fix the cosmological bound on it by using a direct fitting procedure involving the Hubble rate $H(z)$. Here we use the so called “differential age” method proposed by Jimenez et al. (2003) and Simon et al. (2005). Later on, this method was widely used by others to put constraints on the cosmological parameters (for example see Zhang et al. 2012; Zhang et al. 2010; Ma & Zhang 2011; Luongo 2011).

First of all, we note that in our study the scaled Hubble parameter is given by

$$E(z) = \frac{H(z)}{H_0} = \left[\Omega^m (1+z)^2 + \Omega^{\text{de}} \frac{\rho^{\text{de}}(z)}{\rho^{\text{de}}(0)} \right]^{\frac{1}{2}}, \quad (36)$$

where $\rho^X(z)$, Ω^m and Ω^{de} for the non-interacting case are given by Equations (16), (21) and (22) respectively and for the interacting case are given by Equations (28), (32) and (33) respectively.

To constrain the model parameter K , we try to minimize the following reduced χ^2 .

$$\chi_{\text{Hub}}^2 = \sum_{i=1}^N \frac{[H^{\text{th}}(z_i) - H^{\text{obs}}(z_i)]^2}{\sigma_{\text{obs}}^2(z_i)}, \quad (37)$$

where H^{obs} are the values from Table 1, H^{th} and H^{obs} refer to the theoretical and observational values for the Hubble parameter respectively and the sum is taken over the cosmological dataset.

Since we are interested in the present value of the anisotropy parameter K , we fix all other parameters as follows: $\Omega_0^m = 0.24$, $\Omega_0^{\text{de}} = 0.71$, $\omega^m = 0$, $H_0 = 71$ and $\sigma = 0.03$. Our results for non-interacting and interacting cases are given in Table 2.

Table 2 The Best Fit Parameter with 1σ Error in the Non-interacting Case

Case of Study	H_0 (km s ⁻¹ Mpc ⁻¹)	Ω_0^m	Ω_0^{de}	σ	K
Non-Interacting Case	71	0.24	0.71	0	0.09
Interacting Case	71	0.24	0.71	0.03	0.11

6 CONCLUSIONS

Non-interacting and interacting DE with DM have been investigated in the scope of anisotropic Bianchi type-I space-time. In both cases, first the general form of the DE EoS parameter was derived. Then we examined our general results for the case when the scale factor of the universe behaves as a hyperbolic function of time or redshift. It is shown that in the non-interacting case, depending on the value of the anisotropy parameter K , the DE EoS parameter varies from phantom to quintessence whereas in the interacting case the EoS parameter varies in the quintessence region. However, in both cases, the DE EoS parameter ω^{de} ultimately (i.e. at $z = -1$) tends to the cosmological constant ($\omega^{\text{de}} = -1$). It is worth mentioning that in both cases, the phantom phase is a temporary state. Carroll et al. (2003) have already mentioned that any phantom model with $\omega < -1$ should decay to the cosmological constant model with $\omega^{\text{de}} = -1$ at late time. Finally, the chi-squared statistical method has been used in order to constrain the model parameter K with the observational data for the Hubble parameter. In this case, we observed that the value of the anisotropy parameter in the interacting case is greater than its value in the non-interacting case. Considering the fact that at the present time our universe is almost flat (i.e. $K \sim 0$), the above result indicates that the amount of interaction between DE and DM should decrease as the universe expands (or as time proceeds)¹.

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¹ One can also conclude that space-time is much more curved in the interacting case.

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