

Possibility of measuring spin precession of the nearest supermassive black hole by using S stars *

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Abstract The supermassive black hole (SMBH) with a mass of 4 million M_{\odot} inside the radio source Sgr A* in our Galactic center is the nearest SMBH. Once S stars with a shorter period are observed, relativistic precessions especially the Lense-Thirring effect can be measured by astronomical observations at the $10 \mu\text{as}$ level in the future. An interesting but so far unaddressed problem is that the SMBH not only has spin but also spin precession like similar objects. We study the effect of such spin precession on the orbital precessions of orbiting stars. Our results show that the spin precession can produce a periodic oscillation in the precession of the star's orbital plane, but has no obvious effect on the periape shift. For stars with an orbital period of $O(0.1)$ yr or less, such visible oscillations occur when the SMBH's spin-precession period ranges from about a few tens of years to hundreds of years. The period of oscillation is the same as the one of the spin precession. In principle, the precession of this oscillating orbital plane can be observed and then the spin and spin precession of the nearest SMBH can be determined.

Key words: black hole physics — Galaxy: center — relativity

1 INTRODUCTION

It is widely believed that there is a supermassive black hole (SMBH) associated with the radio source Sgr A* in the center of our Galaxy (Kormendy & Gebhardt 2001; Begelman 2003; Shen et al. 2005). The mass of the SMBH is estimated to be about 4 million M_{\odot} (Ghez et al. 2008; Gillessen et al. 2009). The distance from the SMBH to the Sun is about 8 kpc, which is 100 times closer than the SMBH in the Andromeda galaxy. For this reason, the Galactic black hole offers the best laboratory for strong gravitational field physics and testing general relativity (GR) (see reviews Alexander 2005; Genzel et al. 2010 for more details).

Using stars orbiting the Galactic SMBH, one can detect post-Newtonian effects and probe GR in the weak and strong field limits near the SMBH. Several successful experiments have been done in our solar system for detecting very weak GR effects (Fomalont & Kopeikin 2003; Bertotti et al. 2003; Lucchesi & Peron 2010; Everitt et al. 2011), but the SMBH will give us the best chance to test GR in a strong gravitational field near an SMBH. The star Source 2 (S2), one of a group of stars labeled by S at distances ranging from $10^0 - 10^2$ milliparsec (mpc) from the Galactic center has an orbital period of about 15 yr and an advance of periape of about $0.2^{\circ} \text{ yr}^{-1}$ based on GR. However,

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the Lense-Thirring effect on S2 is too small to detect with current technology. Recently, Meyer et al. observed a star named S0-102 orbiting our Galactic SMBH with a period of only 11.5 yr, which is the shortest one known (Meyer et al. 2012). As suggested by Will (2008), if we can find some stars around the Galactic center with very small semimajor axes of $a \lesssim 1$ mpc and high eccentricity, precessions of the orbital plane induced by the spin angular momentum J and quadrupole moment Q of the SMBH can be larger than $10 \mu\text{as yr}^{-1}$ as observed from Earth and can be detected by some upcoming projects, for example, GRAVITY (<http://www.mpe.mpg.de/ir/gravity>).

However, we cannot exclude this kind of possibility: the spin axis of the SMBH is precessing. For example, the Earth has a period of spin precession of about 25 800 yr, and the angle between the rotation axis and the precession axis is about 23.5° . The origin of this precession mainly comes from the coupling between the J_2 of the Earth and the gravitational force of the Sun and Moon. The sources of precession for the black hole will be the collision with a compact star, the relativistic geodetic precession induced by a mass orbiting the black hole, and the coupling between the quadrupole moment of the black hole and the Newtonian gravitational field of external bodies. Therefore, we should consider the influences of the spin precession of the SMBH on the orbital evolution of S stars. If the spin precession can produce some obvious effects on the orbits of S stars, researchers would be able to observe these effects and determine the spin precession of the SMBH. Otherwise, if we do not consider the spin precession, the observation may appear to be some ‘‘anomalous’’ phenomenon, and will influence data analysis which can obscure the test of general relativity and features of the SMBH. In this case, the assumption of spin precession and the corresponding research should be valuable.

In the present paper, we try to investigate the effects of the SMBH’s spin precession on the orbital precession of a star inside a radius of 1 mpc from the Galactic center. Without losing any scientific generality, the star is treated as a test mass, and the dynamical equation includes the first and second post-Newtonian (1PN and 2PN respectively) corrections, frame-dragging effect and quadrupole moment of the SMBH. Actually, we do not know the precession angle or rate of the SMBH’s spin axis. We select a ‘‘rational’’ range for the angle and rate, and numerically evolve the orbit of the star to extract the effects of the spin precession. This paper is organized as follows. In Section 2, we briefly introduce the relativistic precessions of a star orbiting the SMBH at the Galactic center. Then in Section 3, we calculate and analyze the orbital evolution of a star around the SMBH when including spin precession. In the last section, conclusions and discussions are made.

2 RELATIVISTIC EFFECTS IN THE GALACTIC CENTER

As mentioned in Section 1, an SMBH with mass $M = 4 \times 10^6 M_\odot$ is located in our Galactic center. The Schwarzschild radius of the SMBH is about 0.08 AU, which covers $10 \mu\text{as}$ as seen from the Earth. If a star with semimajor axis a orbits around the SMBH, the orbital period is

$$P = \frac{2\pi a^{3/2}}{\sqrt{GM}} \approx 1.48 \tilde{a}^{3/2} \text{ yr}, \quad (1)$$

where \tilde{a} is the semimajor axis in units of mpc. From Equation (1), we can see that if a star has a semimajor axis of $0.1 \sim 1$ mpc, the period will be about $0.1 \sim 1$ yr. The orbital periape and plane precessions per orbit are given as (Will 2008),

$$\Delta\bar{\omega} = A_S - 2A_J \cos i - \frac{1}{2} A_Q (1 - 3 \cos^2 i), \quad (2)$$

$$\Delta\Omega = A_J - A_Q \cos i, \quad (3)$$

where $\Delta\bar{\omega} = \Delta\omega + \cos i \Delta\Omega$ is the precession of the pericenter relative to the fixed reference direction; i , ω and Ω are the the orbital inclination, argument of periape and longitude of ascending

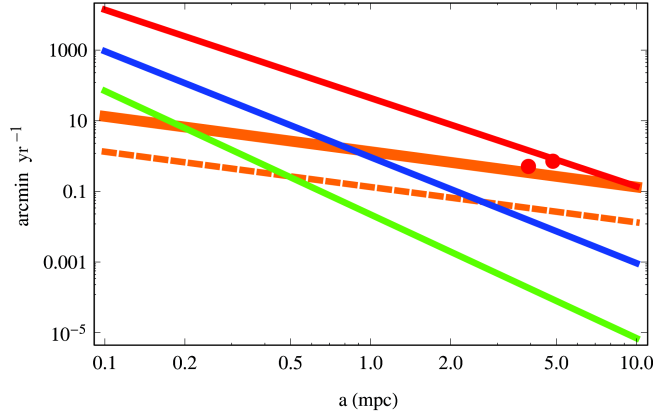


Fig. 1 The magnitudes of precession for the Schwarzschild component (A_S , *solid red line*), angular momentum component (A_J , *solid blue line*) and quadrupole moment (A_Q , *solid green line*) compared with the precessions corresponding to observed astrometric displacements of $10 \mu\text{as}$ (*solid orange line*) and $1 \mu\text{as}$ (*dashed orange line*).

node, respectively. A_S , A_J and A_Q represent the relativistic effects due to the black hole's mass, angular momentum and quadrupole moment, respectively.

$$A_S = \frac{6\pi}{c^2} \frac{GM}{(1-e^2)a} \approx 8.351(1-e^2)^{-1} \tilde{a}^{-1} \text{ arcmin yr}^{-1}, \quad (4)$$

$$A_J = \frac{4\pi\chi}{c^3} \left[\frac{GM}{(1-e^2)a} \right]^{3/2} \approx 0.0769(1-e^2)^{-3/2} \chi \tilde{a}^{-3/2} \text{ arcmin yr}^{-1}, \quad (5)$$

$$A_Q = \frac{3\pi\chi^2}{c^4} \left[\frac{GM}{(1-e^2)a} \right]^2 \approx 7.979 \times 10^{-4} (1-e^2)^{-2} \chi^2 \tilde{a}^{-2} \text{ arcmin yr}^{-1}. \quad (6)$$

We can easily find that the Schwarzschild precession (A_S) is much larger than the other two.

Figure 1 plots the magnitudes of relativistic effects (4)–(6) (assuming $e = 0.9$ and $\chi = 1$) with 10 and $1 \mu\text{as}$. The solid and dashed orange bands denote the values of precession seen from the SMBH corresponding to astrometric precession rates of 10 and $1 \mu\text{as}$ per year as seen from the Earth respectively. We can see that for detecting the Schwarzschild part, the semimajor is required to be less than 10 mpc. Up to now, two S stars have been found that satisfy this requirement; one is S0-2, and the other is S0-102, which has the shortest period. The eccentricity of S0-102 is smaller than that of S0-2, so perhaps the relativistic effects of the latter one can be observed more easily. However, for both stars, their orbital periods are longer than 10 yr, which means that one needs tens of years of observations to measure orbital precessions. For the frame-dragging effect, it can only be observed in stars that have an orbital period of $\lesssim 1$ yr. S-stars with such a short period are also more convenient for assessing the Schwarzschild precession. An S-star with an orbital radius of $O(0.1)$ mpc is far from the region where gravitational radiation is significant and also outside of the tidal radius of the black hole (Alexander 2005), therefore we do not need to consider these effects in the calculation.

3 EFFECT OF SPIN PRECESSION FROM THE SMBH ON S-STARS' ORBITS

For completeness, the dynamical equation we used to calculate the motion of stars includes 1PN, 1.5PN (frame-dragging effect), 2PN and 2.5PN (quadrupole moment) terms beyond Newtonian

gravitation. Considering that the mass of an SMBH is much larger than stars that orbit around it, the acceleration of the star can be written as,

$$\ddot{\mathbf{x}} = \mathbf{a}_N + \mathbf{a}_{1\text{PN}} + \mathbf{a}_{2\text{PN}} + \mathbf{a}_{\text{F-D}} + \mathbf{a}_Q. \quad (7)$$

We can see that in addition to the Newtonian part (\mathbf{a}_N), and the 1PN and 2PN Schwarzschild contributions, the equation also includes the terms for spin: the frame-dragging effect ($\mathbf{a}_{\text{F-D}}$) and the quadrupole moment (\mathbf{a}_Q). From Kidder (1995); Will (2008), we get the detailed expressions

$$\mathbf{a}_{1\text{PN}} = \frac{M\mathbf{x}}{r^3} \left(4\frac{M}{r} - v^2 \right) + 4\frac{M\dot{r}}{r^2}\mathbf{v}, \quad (8)$$

$$\mathbf{a}_{2\text{PN}} = -\frac{M\mathbf{x}}{r^3} \left(9\frac{M^2}{r^2} - 2\frac{M}{r}\dot{r}^2 \right) - 2\frac{M^2}{r^3}\dot{r}\mathbf{v}, \quad (9)$$

$$\mathbf{a}_{\text{F-D}} = -\frac{2\mathbf{J}}{r^3} \left[2\mathbf{v} \times \hat{\mathbf{J}} - 3\dot{r}\mathbf{n} \times \hat{\mathbf{J}} - 2\mathbf{n}(\mathbf{h} \cdot \hat{\mathbf{J}})/r \right], \quad (10)$$

$$\mathbf{a}_Q = \frac{3}{2}\frac{Q_2}{r^4} \left[5\mathbf{n}(\mathbf{n} \cdot \hat{\mathbf{J}})^2 - 2(\mathbf{n} \cdot \hat{\mathbf{J}})\hat{\mathbf{J}} - \mathbf{n} \right], \quad (11)$$

where \mathbf{x} is the position vector of the star, \mathbf{v} the velocity vector, r the distance to the central black hole, \mathbf{J} the angular momentum of the black hole and $Q_2 = -J^2/M$ the quadrupole moment based on the no-hair theorem.

Now for the purpose of this work, we need to add a term describing precession to the SMBH's spin axis. The precession equation is

$$\dot{\mathbf{J}} = \boldsymbol{\Omega} \times \mathbf{J}, \quad (12)$$

where $\boldsymbol{\Omega}$ is the precession vector. As is well known, the precession described by Equation (12) makes the spin axis rotate around the precession axis but without changing the value of spin, that is, $|\mathbf{J}| = \text{const}$. The spin precession defined by Equation (12) and the equation of motion (7) are used to calculate trajectories of the S stars in this paper.

Now we estimate the possible value of the spin-precession frequency. Firstly, for the relativistic geodetic precession induced by a mass orbiting the SMBH, we have

$$\Omega_{\text{geodetic}} \sim (Gm_*/r^2c^2)(GM/r) \sim 10^{-9} \text{ yr}^{-1} (m_*/M_\odot)(r/\text{mpc})^{-5/2}, \quad (13)$$

where m_* and r are the mass and distance of the orbiting body. This looks extremely small. For example, a one solar mass star orbiting the SMBH from 1 mpc will induce a precession period of 10^9 yr, which is almost the age of the Universe. The precession caused by the coupling between the quadrupole moment of the SMBH and the Newtonian gravitational field of external bodies is

$$\Omega_{\text{Quad}} \sim (3/2)(Q/J)(Gm/r^3) \sim 10^{-11} \text{ yr}^{-1} (\chi/1)(m_*/M_\odot)(r/\text{mpc})^{-3}, \quad (14)$$

which is even less than Equation (13) by several orders. Here Q and χ are the quadrupole moment and the Kerr parameter of the SMBH respectively.

Obviously, a 10^9 yr spin precession is meaningless for research and observation. However, we cannot exclude that there is a star orbiting the SMBH at a distance of ~ 0.1 mpc, so the precession period could be 10^6 yr. Inside the 0.01 mpc radius (still far from the tidal radius of the SMBH $\sim 3 \times 10^{-3}$ mpc), we can see the period reduces to ten thousand years. Furthermore, a compact object (neutron star or black hole with stellar mass) orbiting the SMBH at its innermost stable circular orbit (ISCO) (this system is called an extreme-mass-ratio inspiral, a very important source of gravitational wave radiation for the eLISA project), the radius is $\lesssim 10^{-3}$ mpc, which will make the central SMBH precess with a period of ~ 10 yr. Therefore, in this paper, we can assume a very large range of values

for the spin-precession frequency (or period) of the SMBH. We think that all these values could be realistic for this SMBH.

Let us first see if the spin precession of the SMBH can affect the periaapse shifts of known S stars S0-2 and S0-102. We set the precession angle to be $\pi/3$ and change the precession frequency ϖ (unit $1/M$) to have values 10^{-10} , 10^{-9} , 10^{-8} , 10^{-7} , 10^{-6} , \dots , 10^{-1} and 0.5 (with corresponding precession periods of $\approx 4 \times 10^4$, 4×10^3 , 4×10^2 , 40, 4 yr, \dots , 21 min and 4.2 min.). The results are unfortunately no.

In Figure 2, we can only see two lines that represent the periaapse shifts for S0-102 and S0-2. At this distance, the spin precession of the SMBH has no obvious influence on the periaapse shifts and the lines denoting different spin precessions overlap each other. This does not mean that the spin precession has no coupling with the periaapse shift, but it is quite small compared to the magnitude of the periaapse shift. In fact, the advance of pericenter is dominated by the Schwarzschild part of the geometry, and the effect due to frame dragging is already a tiny correction; any effect due to precession is a tiny correction to that term. This is the reason that the curves in Figure 2 do not show any effect from spin precession.

However, the effect of spin precession on the precession of the orbital planes of S0-102 and S0-2 is obvious for several certain spin-precession periods. The SMBH's spin precession can make the orbital planes precess with a periodic oscillation behavior, and the oscillation period matches with the spin precession one (see Fig. 3 for details). However, from Figure 1, we know that such frame-dragging effects of both stars cannot be observed even with a precision of $1 \mu\text{as}$. Also, the time scales are too long to observe these oscillations. Even for the periaapse shift, because the orbital period is longer than 10 yr, it is also hard to observe.

Let us now focus on S stars with shorter orbital periods ($\sim O(0.1)$ yr), because as we have seen, the relativistic effects of such stars can be measured at a level of precision of $10 \mu\text{as}$. The calculation finds that the spin precession still has almost no obvious effect on the periaapse shift. But this time, the shift in orbital plane can be observed, and for some certain values of spin precession, obvious oscillation behavior can be found with the same period as the spin precession. We show our results of two stars with $a = 0.3$ mpc, $e = 0.8$ and $a = 0.5$ mpc, $e = 0.8$ and the spin-precession angle is 30° ; the response frequency of spin precession on evolution of the orbital plane is around 10^{-6} to 10^{-8} , which means, the period of oscillation for the shift in orbital plane is around 10 yr to 400 yr. Observations can find the oscillating behavior if the SMBH has a spin precession period of a few tens of years. See Figures 4 and 5 for details.

The response of the spin-precession period on evolution of the orbital plane reduces as the orbital period reduces. When the orbital period is ten years (like the cases of S0-102 and S0-2), the spin precession with a period of thousands of years can have an obvious effect on the shift in orbital plane; if the orbital period is $\sim O(1)$ yr, the corresponding spin-precession period is about a hundred years; when the orbital period is $\sim O(0.1)$ yr, the corresponding spin-precession period is a few tens of years. We also calculate cases of antiparallel spin precession in the opposite direction for these two stars, and find that the results are almost the same but the phases of oscillations are opposite to the ones displayed in Figures 4 and 5.

4 CONCLUSIONS AND DISCUSSION

Our calculations demonstrate that a precession in the SMBH's spin axis can lead to an observable oscillation in the precession of the orbital plane for an S star. This responding oscillation has the same period as spin precession. As is already known, for an astrometrical precision of $10 \mu\text{as}$, observation of an S star with a $O(0.1)$ yr orbital period can reveal the frame-dragging effect of the central black hole, and then the SMBH's spin. Furthermore, in this paper, we show that if the SMBH has a spin precession with a period of a few tens of years, the observation can also confirm such a spin precession. In principle, the spin of the SMBH interacts and couples with the orbit of the star, and a

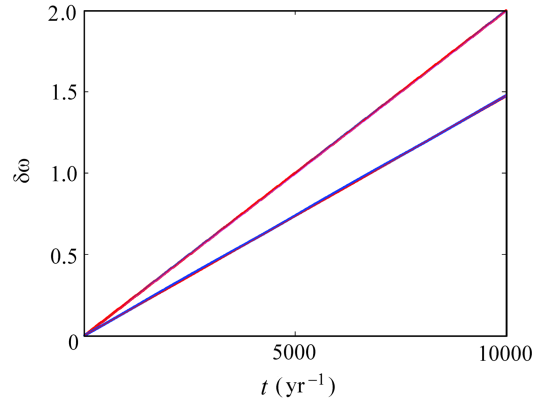


Fig. 2 The periastron shifts of S0-102 (*lower line*) and S0-2 (*upper one*) for the spin-precessing SMBH. The precession angle is 60° , and precession frequencies are $0, 10^{-10}, 10^{-9}, \dots, 10^{-1}$ and 0.5 for each star.

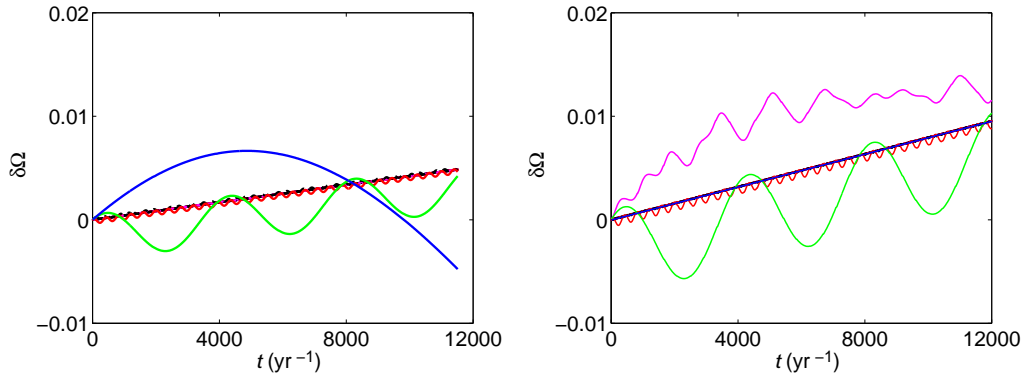


Fig. 3 The orbital precessions of S0-102 (*left panel*) and S0-2 (*right panel*) for a spin-precessing SMBH. The spin-precession parameters are the same as Fig. 2. The blue line denotes $\varpi = 10^{-10}$, the green line $\varpi = 10^{-9}$ and the red line $\varpi = 10^{-8}$. The purple line in the right panel shows an anomalous effect for $\varpi = 10^{-5}$. The other lines overlap and cannot be discerned.

change in the spin may trigger a variation in the orbit, and then the advance of periastron. However, for the distance scale we consider here, the periastron shift is mainly caused by the mass part of the SMBH, and is at least two orders of magnitude larger than the frame-dragging effect. As a result, the process of spin-precession has no obvious effect on observation of advance of the periastron.

Such a precession period is possible for the SMBH. For example, the Earth has a spin period of 24 hours and a spin-precession period of 25 800 yr. The ratio between the two periods is 10^{-7} . The spin period of the SMBH is about a few tens of seconds (which depends on the Kerr parameter χ), thus the ratio is also about 10^{-7} . However, the period of the SMBH's precession is also possibly longer or shorter. However, a shorter or longer precession may not produce observable phenomena related to the star's orbital evolution. For the former one, though it can also cause the oscillation from the shift of orbital plane with the same period as itself, the amplitude of oscillation is too small

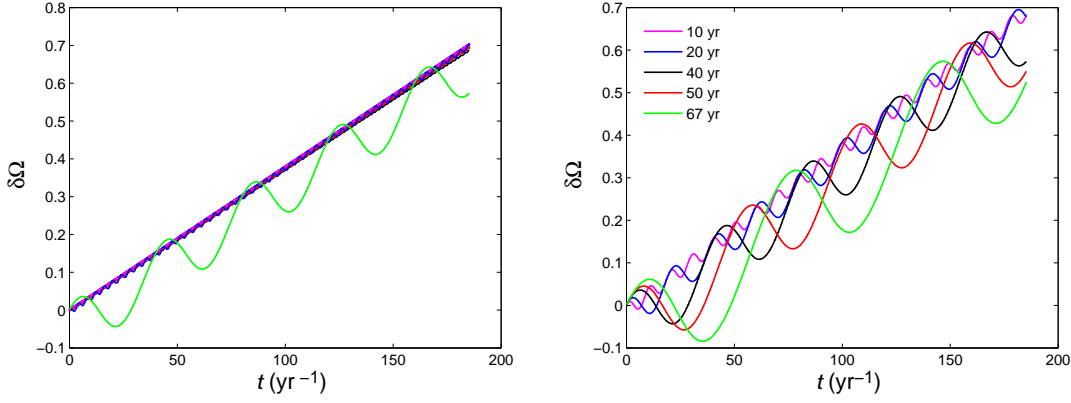


Fig. 4 The orbital precessions of a star with $a = 0.3$ mpc and $e = 0.8$. *Left panel*: the spin-precession frequency from 10^{-7} , 10^{-6} , \dots , 10^{-1} and 0.5. The precession with frequency 10^{-7} (green line) has an obvious response to the shift in orbital plane, and the one of 10^{-6} also can be seen (blue line). *Right panel*: several obvious oscillating phenomena.

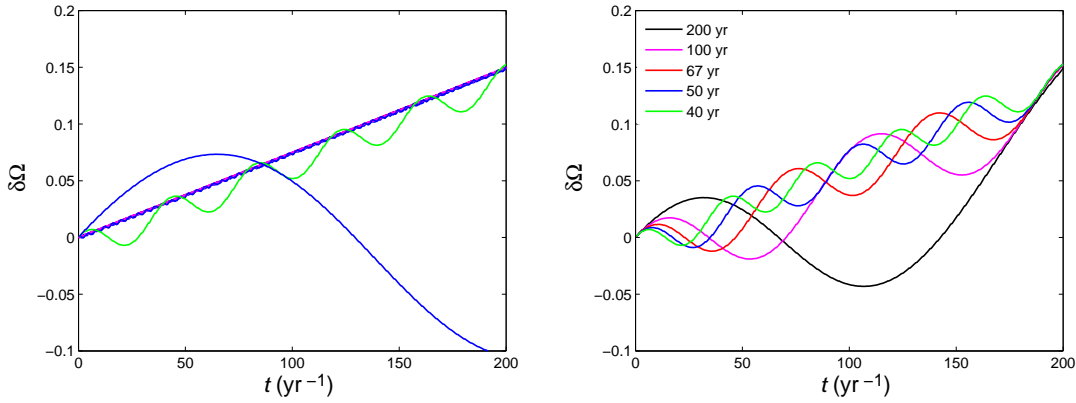


Fig. 5 The orbital precessions of a star with $a = 0.5$ mpc and $e = 0.8$. *Left panel*: the spin-precession frequency from 10^{-8} , 10^{-7} , \dots , 10^{-1} and 0.5. The precession with frequency 10^{-8} (blue line) and 10^{-7} (green line) has obvious responses on the shift in orbital plane. *Right panel*: several obvious oscillating phenomena.

to be observed. For a longer one, the time scale is too long to observe the oscillation behavior in a decade.

For determining a value of spin-precession period in this paper, we assume a compact star out of the ISCO of the SMBH to make the spin-axis precession. One may ask if this compact star with solar mass will perturb the target star and pollute the observation of orbital precession. We have calculated the perturbing effect of a close star on the target one in a previous paper (Han 2012), and we found that when the distance between the two stars is only 0.01 mpc, the effect of a solar mass perturbation is still much less (only 1%) than the frame-dragging one. In this paper, the distance between the two stars is much larger (the target star's semimajor axis is 0.3–0.5 mpc, but the perturbing star's one is only about 0.001 mpc). In this sense, we can absolutely omit the perturbation of such a compact star.

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References

- Alexander, T. 2005, *Phys. Rep.*, 419, 65
- Begelman, M. C. 2003, *Science*, 300, 1898
- Bertotti, B., Iess, L., & Tortora, P. 2003, *Nature*, 425, 374
- Everitt, C. W. F., Debra, D. B., Parkinson, B. W., et al. 2011, *Physical Review Letters*, 106, 221101
- Fomalont, E. B., & Kopeikin, S. M. 2003, *ApJ*, 598, 704
- Genzel, R., Eisenhauer, F., & Gillessen, S. 2010, *Reviews of Modern Physics*, 82, 3121
- Ghez, A. M., Salim, S., Weinberg, N. N., et al. 2008, *ApJ*, 689, 1044
- Gillessen, S., Eisenhauer, F., Trippe, S., et al. 2009, *ApJ*, 692, 1075
- Han, W.-B. 2012, *Annals of Shanghai Observatory Academia Sinica*, 33, 1 (http://202.127.29.4/twtmk/n33/2-tkmb_han.pdf)
- Kidder, L. E. 1995, *Phys. Rev. D*, 52, 821
- Kormendy, J., & Gebhardt, K. 2001, in *American Institute of Physics Conference Series*, 586, 20th Texas Symposium on Relativistic Astrophysics, eds. J. C. Wheeler, & H. Martel (New York: AIP), 363
- Lucchesi, D. M., & Peron, R. 2010, *Physical Review Letters*, 105, 231103
- Meyer, L., Ghez, A. M., Schödel, R., et al. 2012, *Science*, 338, 84
- Shen, Z.-Q., Lo, K. Y., Liang, M.-C., Ho, P. T. P., & Zhao, J.-H. 2005, *Nature*, 438, 62
- Will, C. M. 2008, *ApJ*, 674, L25