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Preliminary limits of a logarithmic correction to the Newtonian gravitational potential in binary pulsars *

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Abstract We obtain preliminary limits on a logarithmic correction to the Newtonian gravitational potential by using five binary pulsars: PSR J0737–3039, PSR B1534+12, PSR J1756–2251, PSR B1913+16 and PSR B2127+11C. This kind of correction may originate from fundamental frameworks, like string theories, effective models of gravity due to quantum effects and the non-local gravity scheme. We estimate the upper limit of the Tohline-Kuhn-Kruglyak parameter λ and the lower limit of the Fabris-Campos parameter α, which parameterize the correction and are connected to each other by $\alpha\lambda = -1$. By analyzing the advances of periastron of these binary pulsars, we find that the preliminary upper limit of α is 0.19 ± 0.14 kpc^{-1} and the preliminary lower limit of λ is -5.2 ± 3.8 kpc. They are compatible with the bounds based on dynamics of spiral galaxies but quite different from those given by solar system dynamics. These results indicate that this logarithmic correction might be more observable in current timings of binary pulsars than in motions of the solar system.

Key words: gravitation — relativity — pulsars: individual (PSR 0737–3039, PSR B1534+12, PSR J1756-2251, PSR B1913+16, PSR B2127+11C)

1 INTRODUCTION

Today, Newton's inverse-square law of gravity and Einstein's general relativity (GR) can explain and describe most astronomical and astrophysical observations and phenomena quite well. However, this success ceases when faced with the flat rotation curves of spiral galaxies (e.g. Rubin & Ford 1970; Roberts & Whitehurst 1975; Sofue & Rubin 2001) without introducing dark matter and the present acceleration of the Universe (e.g. Riess et al. 1998; Perlmutter et al. 1999) without dark energy (see Lämmerzahl 2009, for a review about some of the open problems in gravitational physics). Nevertheless, the physical nature of dark matter and dark energy still remains unknown. Another way to solve these problems is to modify the theory of gravity. These modified theories can generate interesting astrophysical and cosmological consequences (for a recent review see Clifton et al. 2012, and references therein). Among these modifications, one case is a logarithmic correction to the Newtonian gravitational potential, which may originate from fundamental frameworks, like string theories, effective models of gravity due to quantum effects (e.g. Soleng 1995; Shapiro et al. 2005;

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Capozziello et al. 2006) and the non-local gravity scheme (e.g. Hehl & Mashhoon 2009; Blome et al. 2010).

Mücket & Treder (1977) first considered a logarithmic correction to the Newtonian gravitational potential and calculated its resulting perihelion advance on a planet. The dynamics under this logarithmic correction were investigated as well (e.g. Mioc & Blaga 1991; Diacu 1992; Mioc 2004). Recently, Ragos et al. (2013) studied its effects on the anomalistic period of celestial bodies due to such a correction. The effects of modified gravity on the orbital periods of revolving test particles were also studied (e.g. Iorio 2005, 2007; Li 2010, 2012; Sampson et al. 2013).

One interesting aspect of the logarithmic correction is that it can simulate dark matter in astrophysics. Tohline (1983) showed that cold stellar disks can be dynamically stable under the Newtonian gravitational potential with a logarithmic correction and this phenomenological approach was extended by Kuhn & Kruglyak (1987). This possibility was also investigated by Kinney & Brisudova (2001), Kirillov (2006) and Fabris & Campos (2009). In the Tohline-Kuhn-Kruglyak approach, the Newtonian gravitational potential $\Phi_{\mathrm{N}}(\boldsymbol{r})$ for a point mass M is replaced by

$$\Phi(\boldsymbol{r}) = \Phi_{\mathrm{N}}(\boldsymbol{r}) + \Phi_{\ln}(\boldsymbol{r}), \tag{1}$$

where

$$\Phi_{\mathrm{N}}(\boldsymbol{r}) \ = \ -\frac{GM}{|\boldsymbol{r}|}, \tag{2}$$

$$\Phi_{\ln}(\boldsymbol{r}) \ = \ \frac{GM}{\lambda}\ln\left(\frac{|\boldsymbol{r}|}{r_0}\right). \tag{3}$$

Here λ is a constant length that we call the Tohline-Kuhn-Kruglyak parameter. r_0 is a constant length as well and it does not affect motions of objects. Meanwhile, in the work of Fabris & Campos (2009), this logarithmic correction is parametrized as

$$\Phi_{\ln}(\boldsymbol{r}) = -\alpha GM \ln\left(\frac{|\boldsymbol{r}|}{r_0}\right). \tag{4}$$

Here, α has the dimension of $[L]^{-1}$ and we call it the Fabris-Campos parameter. From Equations (3) and (4), we can have

$$\alpha\lambda = -1. \tag{5}$$

By analyzing the rotation curves of 10 spiral galaxies, Fabris & Campos (2009) suggested α is on the order of -0.1 kpc^{-1}, which means $\lambda \sim 10$ kpc.

Iorio & Ruggiero (2008) studied the *additional* perihelion advances $\dot{\omega}_{\ln}$ of the solar system's planets caused by the logarithmic correction to the Newtonian potential of (1) and found that this correction with the condition of $\alpha \sim -0.1$ kpc^{-1} (or $\lambda \sim 10$ kpc) does not match the observations of planetary motions (see Iorio et al. 2011, for a review on the perihelion precessions in the solar system for gravitational experiments). Deng & Xie (2014) found quantitatively that the upper bound of α in the solar system is at the level of -10^{-4} kpc^{-1}, which is equivalent to the lower bound of $\lambda \sim 10^4$ kpc.

In order to understand this logarithmic correction more deeply, it is necessary to test it in some places with stronger gravitational fields. For this purpose, binary pulsars provide us with good opportunities. The relativistic periastron advances in some binary pulsars can exceed the corresponding value for Mercury by a factor of $\sim 10^5$ so that these systems are taken as an ideal and clean laboratory for testing GR, alternative relativistic theories of gravity and modified gravity (e.g. Bell et al. 1996; Damour & Esposito-Farèse 1996; Kramer et al. 2006; Iorio 2007; Iorio & Ruggiero 2007; Deng et al. 2009; Iorio 2009; Li 2010; Deng 2011; Li 2011; De Laurentis et al. 2012; Shao & Wex 2012; Ragos et al. 2013; Xie 2013; Shao et al. 2013; Shao & Wex 2013; Yagi et al.

2013a,b; Shao 2014). In this work, we will adopt five well-observed binary pulsars: PSR J0737–3039, PSR B1534+12, PSR J1756-2251, PSR B1913+16 and PSR B2127+11C.

In Section 2, the orbital dynamics of binary pulsars under the logarithmic correction to the Newtonian gravitational potential will be studied. Observational data will be used to estimate the Tohline-Kuhn-Kruglyak parameter λ and the Fabris-Campos parameter α in Section 3. Conclusions and discussion will be presented in Section 4.

2 TWO-BODY PROBLEM WITH A LOGARITHMIC CORRECTION

Following the scheme used by Diacu (1992), we consider two particles with masses m_i $(i = 1, 2)$ in the Euclidean space having coordinates $\boldsymbol{y}_i = (y_i^1, y_i^2, y_i^3)$ $(i = 1, 2)$. We define a square matrix as

$$\mathbf{M} = \mathrm{diag}(m_1, m_1, m_1, m_2, m_2, m_2), \tag{6}$$

which has the above elements on the diagonal and 0 in the other entries. The 6-dimensional vector $\boldsymbol{y} = (\boldsymbol{y}_1, \boldsymbol{y}_2)$ represents the configuration of the system and is regarded as a column vector. The equation of motion of the two-body problem is given by

$$\mathbf{M}\ddot{\boldsymbol{y}} = \nabla W(\boldsymbol{y}), \tag{7}$$

where the double dots on $\boldsymbol{y}$ mean taking the second time derivative, ∇ is the gradient and $W(\boldsymbol{y}) = U(\boldsymbol{y}) + V(\boldsymbol{y})$. Here $U(\boldsymbol{y})$ is the Newtonian potential defined as

$$U(\boldsymbol{y}) = \frac{Gm_1m_2}{r_{12}}, \tag{8}$$

where $r_{12} = |\boldsymbol{r}_{12}|$ denotes the Euclidian distance between the particles and $\boldsymbol{r}_{12} = \boldsymbol{y}_1 - \boldsymbol{y}_2$; $V(\boldsymbol{y})$ denotes the logarithmic correction term expressed as

$$V(\boldsymbol{y}) = \alpha Gm_1m_2 \ln\left(\frac{r_{12}}{r_0}\right), \tag{9}$$

where α is the Fabris-Campos parameter and r_0 is a constant. Equation (7) can be written explicitly as

$$m_1\ddot{\boldsymbol{y}}_1 = -\frac{Gm_1m_2}{r_{12}^3}\boldsymbol{r}_{12} - \alpha\frac{Gm_1m_2}{r_{12}^2}\boldsymbol{r}_{12}, \tag{10}$$

$$m_2\ddot{\boldsymbol{y}}_2 = -\frac{Gm_1m_2}{r_{12}^3}\boldsymbol{r}_{21} - \alpha\frac{Gm_1m_2}{r_{12}^2}\boldsymbol{r}_{21}, \tag{11}$$

which lead to the equations of relative motion given as

$$\ddot{\boldsymbol{r}} = -\frac{\mu}{r^3}\boldsymbol{r} - \alpha\frac{\mu}{r^2}\boldsymbol{r}, \tag{12}$$

where $\mu = Gm$, $m = m_1 + m_2$ and $\boldsymbol{r} = \boldsymbol{r}_{12}$.

The second term in the above equation, which comes from the logarithmic correction, can introduce an *additional* periastron advance (Iorio & Ruggiero 2008)

$$\dot{\omega}_{\ln} = \alpha\sqrt{\frac{\mu(1-e^2)}{a}}\left(\frac{1-\sqrt{1-e^2}}{e^2}\right), \tag{13}$$

where e is the eccentricity and a is the semi-major axis. Together with the leading term in the general relativistic periastron advance (Landau & Lifshitz 1975), the whole periastron advance can be written as

$$\dot{\omega} = 3\left(\frac{P_b}{2\pi}\right)^{-5/3}\left(\frac{\mu}{c^3}\right)^{2/3}(1-e^2)^{-1} + \alpha\mu^{1/3}\left(\frac{P_b}{2\pi}\right)^{-1/3}e^{-2}\sqrt{1-e^2}(1-\sqrt{1-e^2}), \tag{14}$$

where c is the speed of light and P_b is the period of the binary.

Equation (14) will be used in the next section to confront the observational results of five binary pulsars. These observational results were obtained by fitting the "standard model" of dynamics of binary pulsars to measured timing data, where "standard model" means Newton's law of gravity and Einstein's GR (e.g. Doroshenko & Kopeikin 1990; Hobbs et al. 2006; Edwards et al. 2006). Therefore, the effects of the logarithmic correction were *not* modeled in the data processing of timing observations on binary pulsars and the parameters α or λ were not determined in these fittings. In this sense, the results we obtain in the next section may not be considered to be genuine "constraints" (they would be if one solved for them in a covariance analysis by re-analyzing the data with modified software including these effects) but as preliminary indications of acceptable values to the best of contemporary knowledge from timing observations on binary pulsars so that we call them "preliminary limits" (see Iorio 2014a, for a further discussion). In fact, several authors of other tests used this approach as well (e.g. Shao & Wex 2012; Shao et al. 2013; Shao & Wex 2013; Iorio 2014b; Shao 2014) and they did not actually re-process the pulsar(s) timing data by modifying the dynamic models in an ad-hoc way to include the effects they were interested in, which were not addressed in covariance analyses. Instead, they confronted existing data with theoretically calculated expressions to search for some effects which were applicable to other fields.

3 PRELIMINARY LIMIT ON α (OR λ)

Long-term timing observations can determine the geometrical and physical parameters of binary pulsars very well. Among them, PSR J0737–3039 (Kramer et al. 2006), PSR B1534+12 (Stairs et al. 2002), PSR J1756–2251 (Faulkner et al. 2005), PSR B1913+16 (Weisberg et al. 2010) and PSR B2127+11C (Jacoby et al. 2006) are good samples for gravitational tests. Some of their timing parameters are listed in Table 1. The estimated uncertainties for $\dot{\omega}$ are given in parentheses.

By using the method of weighted least squares and considering all the binary pulsars in Table 1, we estimate the preliminary upper limit on the Fabris-Campos parameter to be $\alpha = 0.19 \pm 0.14$ kpc^{-1} or the lower limit of the Tohline-Kuhn-Kruglyak parameter to be $\lambda = -5.2 \pm 3.8$ kpc, according to the periastron advances of these systems with Equation (14). These limits are compatible with the bounds based on dynamics of spiral galaxies (Fabris & Campos 2009). However, the upper limit of $|\alpha|$ is much larger than those given by previous works based on solar system dynamics (Deng & Xie 2014) and the lower limit of $|\lambda|$ is much less than previous estimates according to planetary motions (Deng & Xie 2014) (see Table 2 for a summary).

The estimated values of both α and λ strongly rely on the difference between the observed periastron advance $\dot{\omega}_{\rm obs}$ and the predicted one by the "standard model" $\dot{\omega}_{\rm mod}$ [see the first term in Equation (14)], i.e. $\delta\dot{\omega} \equiv \dot{\omega}_{\rm obs} - \dot{\omega}_{\rm mod}$. It might also possibly represent mismodeled or unmodeled parts of periastron advances according to Newton's laws and GR. $\delta\dot{\omega}$ of some planets in the solar system ranges from $\sim 10^{-2}$ to ~ 40 milliarcseconds per century (mas cy^{-1}) (Fienga et al. 2011; Pitjeva 2013), but $\delta\dot{\omega}$ of the five binary pulsars we adopted range from $\sim 10^2$ to $\sim 10^4$ mas cy^{-1}. This leads to the fact that our estimated upper (or lower) limit for α (or λ) is larger (or less) than those given by planetary motions in the solar system (Deng & Xie 2014) by about three orders of

Table 1 Timing Parameters of Five Binary Pulsars

PSR	P_b (d)	m ($M_\odot$)	e	$\dot{\omega}$ (° yr^{-1})	Reference
J0737–3039	0.10225156248	2.58708	0.0877775	16.89947(68)	Kramer et al. (2006)
B1534+12	0.420737299122	2.678428	0.2736775	1.755789(9)	Stairs et al. (2002)
J1756–2251	0.319633898	2.574	0.180567	2.585(2)	Faulkner et al. (2005)
B1913+16	0.322997448911	2.828378	0.6171334	4.226598(5)	Weisberg et al. (2010)
B2127+11C	0.33528204828	2.71279	0.681395	4.4644(1)	Jacoby et al. (2006)

Table 2 Summary of Limits on α and λ

	α (kpc^{-1})	λ (kpc)	Adopted data
This work	0.19 ± 0.14	-5.2 ± 3.8	Five binary pulsars in Table 1
Fabris & Campos (2009)	~ -0.1	~ 10	Rotation curves of 10 spiral galaxies
Deng & Xie (2014)	$\sim -10^{-4}$	$\sim 10^4$	Planetary motions in the solar system

magnitude, which indicates that this logarithmic correction might be more observable in current timings of binary pulsars than in motions of the solar system.

4 CONCLUSIONS AND DISCUSSION

With their stronger gravitational fields, binary pulsars are taken as a unique opportunity for testing gravitational theories. In this work, we consider a logarithmic correction to the Newtonian gravitational potential and estimate its preliminary limits by using five well-observed systems of binary pulsars. After analyzing the advances of periastron of these binary pulsars, we find that the preliminary upper limit of the Fabris-Campos parameter α (Fabris & Campos 2009) is 0.19 ± 0.14 kpc^{-1} and the preliminary lower limit of the Tohline-Kuhn-Kruglyak parameter λ (Tohline 1983; Kuhn & Kruglyak 1987) is -5.2 ± 3.8 kpc. They are compatible with the bounds based on dynamics of spiral galaxies (Fabris & Campos 2009) but quite different from those given by solar system dynamics (Deng & Xie 2014), which indicates that this logarithmic correction might be more observable in current timings of binary pulsars than in motions of the solar system. More sophisticated and accurate timing observations in the future might significantly improve the timing parameters of binary pulsars and thus provide more stringent constraints on such a logarithmic correction.

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