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Relativistic transformations between global and local velocities of an orbiter under IAU Resolutions *

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Abstract Einstein's general relativity (GR) has become an inevitable part of deep space missions. According to the International Astronomical Union (IAU) Resolutions which are built in the framework of GR, several time scales and reference systems are recommended to be used in the solar system for control, navigation and scientific operation of a spacecraft. Under the IAU Resolutions, we derive the transformations between global and local velocities of an arbitrary orbiter. These transformations might be used in orbit determination with Doppler tracking and prediction of Doppler observables for the spacecraft. Taking the YingHuo-1 Mission as a technical example of future Chinese Mars explorations, we evaluate the significance and contributions of various components in the transformations. The largest contribution of the relativistic parts in the transformations can reach the level of $\sim 5 \times 10^{-5}$ m s^{-1}. This suggests that, for such a spacecraft like we have assumed, if the accuracy of Doppler tracking is better than $\sim 5 \times 10^{-5}$ m s^{-1} then the relativistic parts of the transformations of velocities will be required.

Key words: reference systems — method: numerical — space vehicles — techniques: radial velocities

1 INTRODUCTION

Einstein's general relativity (GR) has become an inevitable part of deep space missions. This is driven by significant increases of measurement accuracy with modern techniques. It also makes GR go far beyond the territory of theoretical astronomy and physics into the realm of practice and engineering. Relativistic effects obviously appear in the radio links of the *Cassini* spacecraft (Bertotti et al. 2003) and the *New Horizons* spacecraft (Jensen & Weaver 2007). Measurement of the frequency shift in the links connecting *Cassini* and Earth yields the most stringent test to demonstrate the validity of GR in the solar system (Bertotti et al. 2003); whereas Kopeikin et al. (2007) pointed out that this test of GR is under a restrictive condition that the Sun's gravitational field is static, and if this restriction is removed then the test becomes less stringent.

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One fundamental point associated with GR is to distinguish between proper time and coordinate times (Misner et al. 1973; Landau & Lifshitz 1975). The reading of an ideal clock is the proper time τ, which is in the reference frame of the clock. Although coordinate times *cannot* be measured directly, some of them can be taken as independent variables in the equations of motion of celestial and artificial bodies, as well as photons. Coordinate times are related to proper time through the invariant 4-dimensional space-time interval, which depends on the motion of the clock (effects of special relativity) and gravitational fields (effects of GR). This dramatically changes the method of clock synchronization and time transfer (Nelson 2011). In exploration missions to Mars and other planets, synchronization of the clock onboard a spacecraft and the clock on the ground is critical for control, navigation and scientific operation. According to the International Astronomical Union (IAU) Resolutions (Soffel et al. 2003), which are built in the framework of GR, relativistic synchronization of clocks requires several time scales. The procedures of time transfer have been widely investigated (e.g. Petit & Wolf 1994, 2005; Nelson 2011; Deng 2012; Pan & Xie 2013, 2014). These time/frequency transfer links might also be used for testing theories of gravity (e.g. Samain 2002; Wolf et al. 2009; Christophe et al. 2009, 2012; Deng & Xie 2013a,b, 2014).

In addition to the need for diverse time scales in GR, a variety of reference systems are also required. Although all reference systems are mathematically equivalent, using some specific systems can largely simplify calculations in modeling astronomical and astrophysical processes. In the solar system the description of a gravitational body's motion is not conceivable without a self-consistent theory of astronomical relativistic reference systems because the solar system has a hierarchical structure. Although the Sun is the most massive body in the system, giant planets, like Jupiter and Saturn, can still make it revolve at some distance around the solar system barycenter. Thus, a global barycentric reference system for the solar system is required to describe the orbital motion of bodies in the solar system and to model the light propagation from distant celestial objects. On the other hand, the rotational motion of a body is more natural to describe in their local reference systems associated with each of the bodies. A local reference system of a body is also adequate to describe its oblateness and the motion of its satellites. The IAU Resolutions (Soffel et al. 2003) also lay down a foundation for definitions and applications of these reference systems in the solar system.

In the fully relativistic framework of reference systems under IAU Resolutions (Soffel et al. 2003), we will investigate the transformations between global and local velocities of an orbiter around a celestial body, which is a practical issue for deep space missions. These transformations might be used in orbit determination with Doppler tracking and in the prediction of Doppler observables for the spacecraft. Doppler tracking can be used to determine its *global* velocities in the line of sight with respect to a tracking station in the solar system's barycentric reference system (Moyer & Yuen 2000; Kopeikin et al. 2011). In order to determine its orbit, such as the solution of orbital elements in the *local* reference system of the body, these *global* velocities need to be transformed to *local* velocities with respect to the body. When one predicts Doppler shifts of the spacecraft, its *local* velocities, which might be calculated based on the orbital elements, need to be transformed into *global* ones. It is certain that, if the accuracy of measurement is sufficiently high, a classical Galilean transformation will *not* be adequate and relativistic transformations will be required.

In Section 2, relativistic transformations between the global and local velocities for an orbiter around a gravitational body will be derived. Taking the YingHuo-1 Mission (Ping et al. 2010a,b) as a technical example of future Chinese Mars explorations, we will evaluate the significance and contributions of various components in the transformations given in Section 3. The conclusions and a discussion will be presented in Section 4.

2 RELATIVISTIC TRANSFORMATIONS BETWEEN GLOBAL AND LOCAL VELOCITIES

In order to derive the relativistic transformations between global and local velocities, we need a global reference system and a local reference system for body C in the solar system. The definitions of these reference systems are adopted from the IAU Resolutions (Soffel et al. 2003; Kopeikin et al. 2011), although our notations might be different from the original ones. It is worth mentioning that the formulae obtained in this section can be used on a spacecraft around an arbitrary body. These formulae might also be derived in a framework which extends the IAU Resolutions (Soffel et al. 2003), such as in the framework of the scalar-tensor theory of gravity (Kopeikin & Vlasov 2004; Xie & Kopeikin 2010; Kopeikin & Xie 2010; Kopeikin et al. 2011). Although it is beyond the scope of this paper, the approach involved in this framework is almost identical to ours.

2.1 From Global Velocity to a Local One

In the local reference system of gravitational body C, the local coordinates of a field point in the vacuum are $(c\Xi_{\rm C}, \boldsymbol{Z}_{\rm C})$, where $\Xi_{\rm C}$ is its local coordinate time and $\boldsymbol{Z}_{\rm C}$ is the position vector of the field point. For an orbiter with negligible mass surrounding body C, its position vector can also be represented by $\boldsymbol{Z}_{\rm C}$. The local coordinates $(c\Xi_{\rm C}, \boldsymbol{Z}_{\rm C})$ of the spacecraft have the following relationships with its global coordinates $(ct, \boldsymbol{x}_{\rm P})$ (Soffel et al. 2003)

$$\Xi_{\rm C} \;=\; t + \epsilon^2 \xi^0_{\rm C}\,, \tag{1}$$

$$Z^i_{\rm C} \;=\; r^i_{\rm PC} + \epsilon^2 \xi^i_{\rm C}\,, \tag{2}$$

where $\epsilon \equiv c^{-1}$ and c is the speed of light, and $r^i_{\rm PC} = x^i_{\rm P} - x^i_{\rm C}$ and $x^i_{\rm P}$ and $x^i_{\rm C}$ are respectively the positions of the spacecraft and body C in the global reference system. The scalar function $\xi^0_{\rm C}$ and the vector function $\xi^i_{\rm C}$ are, respectively,

$$\xi^0_{\rm C} \;=\; -\mathcal{A}_{\rm C} - v^k_{\rm C} r^k_{\rm PC} + \mathcal{O}(\epsilon^2), \tag{3}$$

$$\xi^i_{\rm C} \;=\; \frac{1}{2} v^i_{\rm C} v^k_{\rm C} r^k_{\rm PC} + \bar{U}_{\rm C}(\boldsymbol{x}_{\rm C}) r^i_{\rm PC} + a^k_{\rm C} r^k_{\rm PC} r^i_{\rm PC} - \frac{1}{2} a^i_{\rm C} r^2_{\rm PC} + \mathcal{O}(\epsilon^2)\,, \tag{4}$$

where $v^i_{\rm PC} = v^i_{\rm P} - v^i_{\rm C}$, $v^i_{\rm P}$ and $v^i_{\rm C}$ are, respectively, the velocities of the spacecraft and body C in the global reference system, $a^i_{\rm C}$ is the acceleration of body C in the global reference system, and

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{A}_{\rm C} = \frac{1}{2} v^2_{\rm C} + \bar{U}_{\rm C}(\boldsymbol{x}_{\rm C})\,. \tag{5}$$

Here, the Newtonian gravitational potential $\bar{U}_{\rm C}(\boldsymbol{x}_{\rm C})$ is defined as

$$\bar{U}_{\rm C}(\boldsymbol{x}_{\rm C}) = \sum_{\rm A \neq C} U_{\rm A}(\boldsymbol{x}_{\rm C})\,, \tag{6}$$

and $U_{\rm A}(\boldsymbol{x}_{\rm C})$ is the Newtonian gravitational potential of body A, which is evaluated at $\boldsymbol{x}_{\rm C}$.

With the help of

$$\begin{aligned} \frac{\mathrm{d}\xi^0_{\rm C}}{\mathrm{d}t} \;&=\; -\frac{\mathrm{d}\mathcal{A}_{\rm C}}{\mathrm{d}t} - a^k_{\rm C} r^k_{\rm PC} - v^k_{\rm C} v^k_{\rm PC} + \mathcal{O}(\epsilon^2) \\ &=\; -\frac{1}{2} v^2_{\rm C} - \bar{U}_{\rm C}(\boldsymbol{x}_{\rm C}) - a^k_{\rm C} r^k_{\rm PC} - v^k_{\rm C} v^k_{\rm PC} + \mathcal{O}(\epsilon^2)\,, \end{aligned} \tag{7}$$

and

$$\begin{aligned} \frac{\mathrm{d}\xi^i_{\rm C}}{\mathrm{d}t} \;&=\; \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{2} v^i_{\rm C} v^k_{\rm C} r^k_{\rm PC} + \bar{U}_{\rm C}(\boldsymbol{x}_{\rm C}) r^i_{\rm PC} + a^k_{\rm C} r^k_{\rm PC} r^i_{\rm PC} - \frac{1}{2} a^i_{\rm C} r^2_{\rm PC}\right] + \mathcal{O}(\epsilon^2) \\ &=\; +\frac{1}{2} a^i_{\rm C} v^k_{\rm C} r^k_{\rm PC} + \frac{1}{2} v^i_{\rm C} a^k_{\rm C} r^k_{\rm PC} + \frac{1}{2} v^i_{\rm C} v^k_{\rm C} v^k_{\rm PC} + \dot{\bar{U}}_{\rm C}(\boldsymbol{x}_{\rm C}) r^i_{\rm PC} + \bar{U}_{\rm C}(\boldsymbol{x}_{\rm C}) v^i_{\rm PC} \\ &\quad\; + \dot{a}^k_{\rm C} r^k_{\rm PC} r^i_{\rm PC} + a^k_{\rm C} v^k_{\rm PC} r^i_{\rm PC} + a^k_{\rm C} r^k_{\rm PC} v^i_{\rm PC} - \frac{1}{2} \dot{a}^i_{\rm C} r^2_{\rm PC} - a^i_{\rm C} r^k_{\rm PC} v^k_{\rm PC} + \mathcal{O}(\epsilon^2)\,, \end{aligned} \tag{8}$$

where a dot means derivative with respect to time, we can express the *local* velocity of spacecraft $V^i_{\rm C}$ in terms of the *global* quantities as

$$V^i_{\rm C} \equiv \frac{{\rm d}Z^i_{\rm C}}{{\rm d}\Xi_{\rm C}} = \frac{{\rm d}r^i_{\rm PC}}{{\rm d}\Xi_{\rm C}} + \epsilon^2 \frac{{\rm d}\xi^i_{\rm C}}{{\rm d}\Xi_{\rm C}} = \frac{{\rm d}r^i_{\rm PC}}{{\rm d}t}\frac{{\rm d}t}{{\rm d}\Xi_{\rm C}} + \epsilon^2 \frac{{\rm d}\xi^i_{\rm C}}{{\rm d}\Xi_{\rm C}} = v^i_{\rm PC} + \sum_{j=1}^{5} g^i_j + \mathcal{O}(\epsilon^4)\,, \tag{9}$$

where

$$g^i_1 = f_1 v^i_{\rm PC}, \quad g^i_2 = f_2 r^i_{\rm PC}, \quad g^i_3 = f_3 v^i_{\rm C}, \quad g^i_4 = f_4 a^i_{\rm C}, \quad g^i_5 = -f_5 \dot{a}^i_{\rm C}\,, \tag{10}$$

and

$$f_1 = \epsilon^2\left[\frac{1}{2}v^2_{\rm C} + 2\bar{U}_{\rm C}(\boldsymbol{x}_{\rm C}) + 2\boldsymbol{r}_{\rm PC}\cdot\boldsymbol{a}_{\rm C} + \boldsymbol{v}_{\rm C}\cdot\boldsymbol{v}_{\rm PC}\right], \tag{11}$$

$$f_2 = \epsilon^2\left[\dot{\bar{U}}_{\rm C}(\boldsymbol{x}_{\rm C}) + \boldsymbol{r}_{\rm PC}\cdot\dot{\boldsymbol{a}}_{\rm C} + \boldsymbol{v}_{\rm PC}\cdot\boldsymbol{a}_{\rm C}\right], \tag{12}$$

$$f_3 = \frac{1}{2}\epsilon^2(\boldsymbol{r}_{\rm PC}\cdot\boldsymbol{a}_{\rm C} + \boldsymbol{v}_{\rm C}\cdot\boldsymbol{v}_{\rm PC})\,, \tag{13}$$

$$f_4 = \epsilon^2\left(\frac{1}{2}\boldsymbol{r}_{\rm PC}\cdot\boldsymbol{v}_{\rm C} - \boldsymbol{r}_{\rm PC}\cdot\boldsymbol{v}_{\rm PC}\right), \tag{14}$$

$$f_5 = \frac{1}{2}\epsilon^2 r^2_{\rm PC}\,. \tag{15}$$

On the right-hand-side of Equation (9), there are a lot of terms. The first one, $v^i_{\rm PC}$, comes from a classical Galilean transformation, which is at the Newtonian order. All the other terms with ϵ^2 account for the relativistic contributions, which are associated with kinematics (i.e. positions and velocities) and dynamics (i.e. acceleration and its time derivative) of the spacecraft and body C, and gravitational potentials. If the gravitational potentials and terms related to acceleration in Equation (9) are negligible, then this will reduce to the special relativistic transformation of velocity (Poincaré 1906).

With Equation (9), we can transform the *global* velocity of a spacecraft into the *local* velocity with respect to body C. This might be used in orbit determination of the spacecraft with Doppler tracking.

2.2 From a Local Velocity to the Global One

In order to obtain the transform from a local velocity to the global velocity, we will apply the same procedure. We can find

$$t = \Xi_{\rm C} + \epsilon^2\zeta^0_{\rm C}\,, \tag{16}$$

$$r^i_{\rm PC} = Z^i_{\rm C} + \epsilon^2\zeta^i_{\rm C}\,, \tag{17}$$

where the scalar function $\zeta^0_{\rm C}$ and the vector function $\zeta^i_{\rm C}$ are, respectively,

$$\zeta^0_{\rm C} = \mathcal{A}_{\rm C} + v^k_{\rm C}Z^k_{\rm C} + \mathcal{O}(\epsilon^2)\,, \tag{18}$$

$$\zeta^i_{\rm C} = -\frac{1}{2}v^i_{\rm C}v^k_{\rm C}Z^k_{\rm C} - \bar{U}_{\rm C}(\boldsymbol{x}_{\rm C})Z^i_{\rm C} - a^k_{\rm C}Z^k_{\rm C}Z^i_{\rm C} + \frac{1}{2}a^i_{\rm C}Z^2_{\rm C} + \mathcal{O}(\epsilon^2)\,. \tag{19}$$

By making use of the relations defined by Equation (7) and

$$\begin{aligned}\frac{{\rm d}\zeta^i_{\rm C}}{{\rm d}t} &= \frac{\rm d}{{\rm d}t}\left[-\frac{1}{2}v^i_{\rm C}v^k_{\rm C}Z^k_{\rm C} - \bar{U}_{\rm C}(\boldsymbol{x}_{\rm C})Z^i_{\rm C} - a^k_{\rm C}Z^k_{\rm C}Z^i_{\rm C} + \frac{1}{2}a^i_{\rm C}Z^2_{\rm C}\right] + \mathcal{O}(\epsilon^2)\\ &= -\frac{1}{2}a^i_{\rm C}v^k_{\rm C}Z^k_{\rm C} - \frac{1}{2}v^i_{\rm C}a^k_{\rm C}Z^k_{\rm C} - \frac{1}{2}v^i_{\rm C}v^k_{\rm C}V^k_{\rm C} - \dot{\bar{U}}_{\rm C}(\boldsymbol{x}_{\rm C})Z^i_{\rm C} - \bar{U}_{\rm C}(\boldsymbol{x}_{\rm C})V^i_{\rm C}\\ &\quad -\dot{a}^k_{\rm C}Z^k_{\rm C}Z^i_{\rm C} - a^k_{\rm C}V^k_{\rm C}Z^i_{\rm C} - a^k_{\rm C}Z^k_{\rm C}V^i_{\rm C} + \frac{1}{2}\dot{a}^i_{\rm C}Z^2_{\rm C} + a^i_{\rm C}Z^k_{\rm C}V^k_{\rm C} + \mathcal{O}(\epsilon^2)\,,\end{aligned} \tag{20}$$

we can express the *global* velocity of spacecraft $v^i_{\rm PC}$ in terms of the *local* quantities

$$v^i_{\rm PC} = \frac{{\rm d}Z^i_{\rm C}}{{\rm d}t} + \epsilon^2\frac{{\rm d}\zeta^i_{\rm C}}{{\rm d}t} = \frac{{\rm d}Z^i_{\rm C}}{{\rm d}\Xi_{\rm C}} + \epsilon^2\frac{{\rm d}Z^i_{\rm C}}{{\rm d}\Xi_{\rm C}}\frac{{\rm d}\xi^0_{\rm C}}{{\rm d}t} + \epsilon^2\frac{{\rm d}\zeta^i_{\rm C}}{{\rm d}t} = V^i_{\rm C} - \sum_{j=1}^{5} G^i_j + \mathcal{O}(\epsilon^4)\,, \tag{21}$$

where

$$G^i_1 = F_1 V^i_{\rm C},\quad G^i_2 = F_2 Z^i_{\rm C},\quad G^i_3 = F_3 v^i_{\rm C},\quad G^i_4 = F_4 a^i_{\rm C},\quad G^i_5 = -F_5 \dot{a}^i_{\rm C}\,, \tag{22}$$

and

$$F_1 \;=\; \epsilon^2\left[\frac{1}{2}v^2_{\rm C} + 2\bar{U}_{\rm C}(\boldsymbol{x}_{\rm C}) + 2\boldsymbol{Z}_{\rm C}\cdot\boldsymbol{a}_{\rm C} + \boldsymbol{v}_{\rm C}\cdot\boldsymbol{V}_{\rm C}\right], \tag{23}$$

$$F_2 \;=\; \epsilon^2\left[\dot{\bar{U}}_{\rm C}(\boldsymbol{x}_{\rm C}) + \boldsymbol{Z}_{\rm C}\cdot\dot{\boldsymbol{a}}_{\rm C} + \boldsymbol{V}_{\rm C}\cdot\boldsymbol{a}_{\rm C}\right], \tag{24}$$

$$F_3 \;=\; \frac{1}{2}\epsilon^2\left(\boldsymbol{Z}_{\rm C}\cdot\boldsymbol{a}_{\rm C} + \boldsymbol{v}_{\rm C}\cdot\boldsymbol{V}_{\rm C}\right), \tag{25}$$

$$F_4 \;=\; \epsilon^2\left(\frac{1}{2}\boldsymbol{Z}_{\rm C}\cdot\boldsymbol{v}_{\rm C} - \boldsymbol{Z}_{\rm C}\cdot\boldsymbol{V}_{\rm C}\right), \tag{26}$$

$$F_5 \;=\; \frac{1}{2}\epsilon^2 Z^2_{\rm C}\,. \tag{27}$$

The structure of Equation (21) is quite similar to that of Equation (9). The first term $V^i_{\rm C}$ on its right hand side is a case of a classical Galilean transformation and all the other terms with ϵ^2 account for the relativistic contributions. If the gravitational potentials and terms related to acceleration in Equation (21) are negligible, it will also return to the special relativistic transformation of velocity (Poincaré 1906). With this equation, we can transform the *local* velocity of a spacecraft with respect to body C into the *global* velocity. This might be used in the prediction of Doppler shifts of the spacecraft.

Again, Equations (9) and (21) can be applied to an orbiter around an arbitrary body in the solar system. There is an important property of these two transformations in that the scalar functions of relativistic contributions $f_\cdot$ and $F_\cdot$ are correspondingly equal at the post-Newtonian order, that is,

$$\begin{aligned} f_1 \;&=\; F_1 + \mathcal{O}(\epsilon^4)\,,\quad f_2 = F_2 + \mathcal{O}(\epsilon^4)\,,\quad f_3 = F_3 + \mathcal{O}(\epsilon^4)\,,\\ f_4 \;&=\; F_4 + \mathcal{O}(\epsilon^4)\,,\quad f_5 = F_5 + \mathcal{O}(\epsilon^4)\,. \end{aligned}$$

This also means the overall relativistic corrections in Equations (9) and (21) have the same absolute values but with opposite signs. This is convenient when we numerically evaluate their contributions, so that we will only focus on Equation (9) for a Mars orbiter as a case study in the next section.

3 CASE STUDY OF A MARS ORBITER

Taking the YingHuo-1 Mission (Ping et al. 2010a,b) as a technical example of future Chinese Mars explorations, we will evaluate the significance and contributions of various components in the transformation given by Equation (9).

We assume there is a spacecraft orbiting around Mars from 2017 January 1st at the time $00^{\rm h}00^{\rm m}00.00^{\rm s}$ to 2018 January 1st at $00^{\rm h}00^{\rm m}00.00^{\rm s}$ under the time scale of the Barycentric Dynamical Time (TDB). All the time coordinates are represented by taking $00^{\rm h}00^{\rm m}00.00^{\rm s}$ on 2017 January 1st as a zero point in the other parts of this paper. The orbital inclination of the spacecraft with respect to the Martian equator is 5°. The apoapsis altitude is 80 000 km and the periapsis altitude is 800 km, with a period of about 3.2 d. In particular, the positions and velocities of celestial

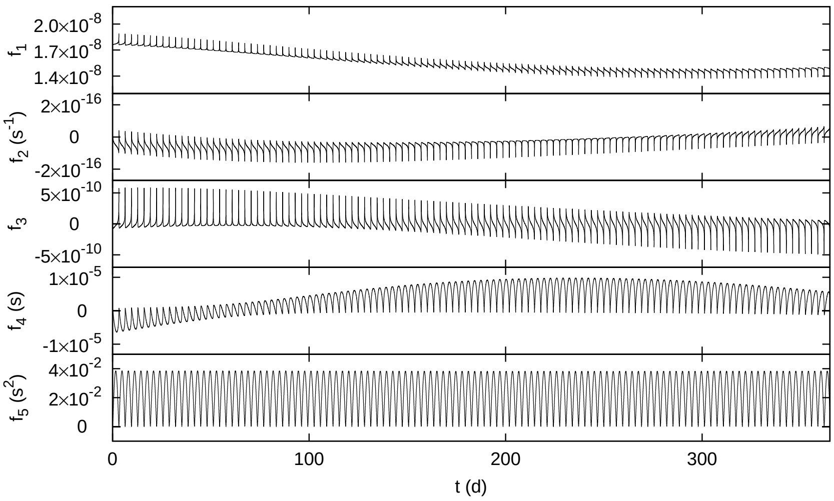

Fig. 1 The curves of f_1, f_2, f_3, f_4 and f_5 based on the technical example of the Mars orbiter we assumed.

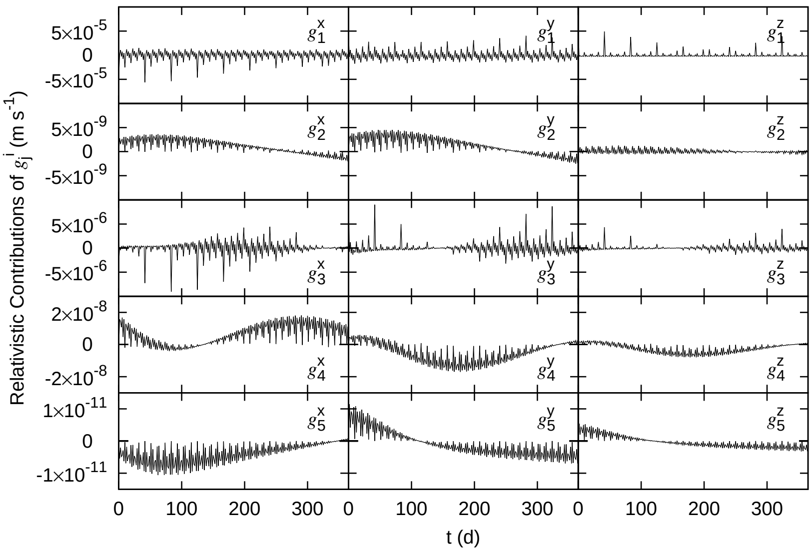

Fig. 2 The relativistic contributions of g_1^i, g_2^i, g_3^i, g_4^i and g_5^i are respectively shown in rows. Their x, y and z components are shown in columns.

bodies are taken from the ephemeris DE405 provided by NASA's JPL and the orbit of the spacecraft is solved by numerically integrating the Einstein-Infeld-Hoffmann equation (Einstein et al. 1938) with the Runge-Kutta 7 method (Stoer & Bulirsch 2002), with the step-size being one-hundredth of its Keplerian period. In the calculation, we include the gravitational contributions from the Sun, eight planets, the Moon and three large asteroids, Ceres, Pallas and Vesta.

Figure 1 shows the curves of $f_{1,\cdots,5}$ based on this technical example. Among them, the dimensionless f_1 and f_3 can, respectively, reach the levels of $\sim 10^{-8}$ and $\sim 10^{-10}$. The relativistic contributions of $g^i_{1,\cdots,5}$ are, respectively, shown in the rows of Figure 2. Their x, y and z components are shown in the columns. g^i_1 has the largest contribution in the relativistic transformation between the global and local velocity. Its greatest value can reach the level of $\sim 5 \times 10^{-5}$ m s^{-1}. The contributions of $g^i_{2,\cdots,5}$ are all less than it by at least one order of magnitude (see Fig. 2). These relativistic contributions are hardly able to affect most current Doppler tracking, except for some specific cases such as *Cassini* (Bertotti et al. 2003). In the case we investigated here, if the accuracy of Doppler tracking is better than $\sim 5 \times 10^{-5}$ m s^{-1}, then the Galilean transformations are not sufficiently adequate for practice and the relativistic parts of the transformations of velocities will be required.

4 CONCLUSIONS AND DISCUSSION

Einstein's general relativity (GR) has become an inevitable part of deep space missions. This is driven by significant increases in measurement accuracy with modern techniques, such as Doppler tracking. According to the IAU Resolutions (Soffel et al. 2003), which are built in the framework of GR, several time scales and reference systems are recommended to be used in the solar system for control, navigation and scientific operation of a spacecraft. Under the IAU Resolutions, we derive the transformations between global and local velocities for an arbitrary orbiter (see Eqs. (9) and (21)). These transformations might be used in orbit determination with Doppler tracking and in the prediction of Doppler observables for the spacecraft.

Taking the YingHuo-1 Mission as a technical example of future Chinese Mars explorations, we evaluate the significance and contributions of various components in the transformations. The largest contribution of the relativistic parts in these transformations can reach a level of $\sim 5 \times 10^{-5}$ m s^{-1}. This suggests that, for such a spacecraft like we have assumed, if the accuracy of Doppler tracking is better than $\sim 5 \times 10^{-5}$ m s^{-1} then the Galilean transformations are not sufficiently adequate for practice and the relativistic parts of the transformations of velocities will be required.

With the rapid development of optical time/frequency standards (e.g. Chou et al. 2010a,b), we might be able to access more of these subtle relativistic effects in the transformations of velocities in the future.

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