Horizon area spectrum and entropy spectrum of a noncommutative geometry inspired regular black hole in three dimensions

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Abstract By employing an adiabatic invariant and implementing the Bohr-Sommerfield quantization rule, I study the quantization of a regular black hole inspired by noncommutative geometry in AdS_3 spacetime. The entropy spectrum as well as the horizon area spectrum of the black hole is obtained. It is shown that the spectra are discrete, and the spacing of the entropy spectrum is equidistant; in the limit $\frac{r_h^2}{4\theta} \gg 1$, the area spectrum depends on the noncommutative parameter and the cosmological constant, but the spacing of the area spectrum is equidistant up to leading order $\sqrt{\theta}e^{-\frac{2Ml^2}{\theta}}$ in θ, and is independent of the noncommutative parameter and the cosmological constant.

Key words: black holes physics — quantization

1 INTRODUCTION

The quantization of black holes is one of the important issues in black hole physics ('t Hooft 1985), and it has not yet been satisfactorily solved. In early 1973, Bekenstein supposed that the horizon area of a black hole is an adiabatic invariant, and proposed that the horizon area of a black hole in equilibrium has a discrete and equally spaced spectrum of the form (Bekenstein 1973, 1974; Bekenstein & Mukhanov 1995; Bekenstein 1997, 1998)

$$A_n = 8\pi n l_p^2 , \qquad n = 0, 1, 2, ... , \tag{1}$$

where l_p denotes the Planck length. Subsequently, Hod (1998) made an important step. He suggested that in the semiclassical limit, the quantum of the black hole area can be determined from the asymptotic value of the real part of complex quasinormal frequencies. This proposal is usually known as Hod's conjecture. Based on Hod's conjecture, Kunstatter (2003) further pointed out that for a system with energy E and vibrational frequency $\omega(E)$, $I = \int \frac{dE}{\omega(E)}$ is an adiabatic invariant, and has an equally spaced spectrum, i.e., $I \approx n\hbar$. Applying the Bohr-Sommerfeld quantization rule in the limit of large n, Kunstatter derived an equally spaced entropy spectrum for a $d(\geq 4)$-dimensional Schwarzschild black hole by implementing this approach. However, Maggiore (2008) suggested that the quasinormal modes of a black hole can be described as a set of damped harmonic oscillators, and proposed that in the semiclassical limit the quantum of black hole horizon area is determined by the physical frequency $\omega_p = \sqrt{|\omega_R|^2 + |\omega_I|^2}$, where ω_R and ω_I stand for the real and imaginary parts of the complex quasinormal frequencies respectively; especially in the case of high damping

or in the limit of large n, $\omega_\mathrm{R} \ll \omega_\mathrm{I}$, the physical frequency becomes $\omega_\mathrm{p} \approx |\omega_\mathrm{I}|$. Using this proposal Maggiore found that the area quantum of the Schwarzschild black hole is equally spaced. There have been many works to study area spectra and entropy spectra in various types of black holes by using quasinormal frequencies (Dreyer 2003; Polychronakos 2004; Setare 2004a,b; Setare & Vagenas 2005; Vagenas 2008; Medved 2008; López-Ortega 2011, 2009; Daghigh & Green 2009; Fernando 2009; Kwon & Nam 2010; Wei et al. 2009; Wei & Liu 2009; Li et al. 2009).

Recently, Majhi & Vagenas (2011) proposed a new approach to derive the entropy spectrum and the horizon area spectrum of spherically symmetric static black holes. It is especially interesting that they derived the entropy spectrum and the horizon area spectrum without employing the quasinormal frequencies at all. They utilized an adiabatic invariant $\int p_i dq_i$ and employed the Bohr-Sommerfeld quantization rule $\int p_i dq_i = 2\pi n\hbar$, where p_i is the conjugate momentum of the coordinate q_i with $i = 0, 1$, for which $q_0 = \tau$ and $q_1 = r$. Note here τ is the Euclidean time. In addition, the Einstein summation convention is adopted. The extensions of their work to other black holes were developed in (Chen & Yang 2012a,b; Zeng et al. 2012; Zeng & Liu 2012; Jiang & Han 2012).

On the other hand, in recent years, black holes inspired by noncommutative geometry have aroused a lot of interest among researchers because noncommutativity is supposed to remove the so-called paradox of black hole information loss (Hawking 2005). Nicolini et al. first found a noncommutative solution inspired by a Schwarzschild black hole in four dimensions (Nicolini et al. 2006a,b; Nicolini 2005). In their papers, the effect of noncommutativity is incorporated in the mass term of the gravitational source, i.e., instead of being represented by a Dirac delta function, the mass density of the black hole is replaced by a Gaussian distribution. Subsequently, many works inspired by black holes that use noncommutative geometry were done. Kim et al. (2008) investigated thermodynamical similarity between the noncommutative Schwarzschild black hole and the Reissner-Nordström black hole; Nozari & Mehdipour investigated Parikh-Wilczek tunneling of noncommutative black holes (Nozari & Mehdipour 2008, 2009; Mehdipour 2010a,b); Mann & Nicolini (2011) investigated the pair creation of noncommutative black holes in a background with a positive cosmological constant; Giri (2007) found and calculated the asymptotic quasinormal modes of a Schwarzschild black hole inspired by noncommutative geometry; Ding & Jing (2011) studied the influence of a noncommutative spacetime parameter on strong field gravitational lensing in the Reissner-Nordström black hole spacetime; Mureika & Nicolini (2011) comprehensively analyzed a noncommutative $(1 + 1)$-dimensional black hole; Myung et al. (Myung et al. 2007; Myung & Yoon 2009), Tejeiro & Larranaga (2012) and Liang & Liu (2012) studied black hole thermodynamics in noncommutative spaces (for a comprehensive review of black holes inspired by noncommutative geometry, see Nicolini 2009).

Inspired by the work of Majhi and Vagenas, in the present paper, I study the entropy spectrum and horizon area spectrum of a regular black hole inspired by noncommutative geometry (Myung & Yoon 2009) by utilizing an adiabatic invariant and implementing the Bohr-Sommerfield quantization rule to explore the quantization of the noncommutative black hole.

Strictly speaking, one should use $\oint p_i dq_i$ to study the quantization of the black hole since only this quantity, not $\int p_i dq_i$, is invariant under canonical transformation, and thus only $\oint p_i dq_i$ is a proper observable (Akhmedov et al. 2008; Jiang & Han 2012). However, a freely falling particle which crosses the horizon does not have any barrier in the Painlevé coordinates, and this leads to

$$\oint p_i dq_i = \int p_i^\mathrm{out} dq_i - \int p_i^\mathrm{in} dq_i = \int p_i^\mathrm{out} dq_i - 0 = \int p_i^\mathrm{out} dq_i,$$

where p_i^out (p_i^in) are for outgoing (incoming) particles. Therefore, $\int p_i^\mathrm{out} dq_i$ can be treated as an adiabatic invariant in the Painlevé coordinates.

In this paper, I have improved the Majhi-Vagenas method. First, I Euclideanize the metric in order to quantize the noncommutative black hole via the adiabatic invariance and the Bohr-Sommerfeld quantization rule, then introduce Painlevé coordinates. Since $\int p_i^\mathrm{out} dq_i$ can be treated as an adiabatic

invariant in the Painlevé coordinates, we can use it to discuss the quantization of the noncommutative black hole. I get the entropy spectrum as well as the horizon area spectrum of the black hole, and it is found that the spectra are discrete, and the spacing of the entropy spectrum is equidistant. In the limit $\frac{r_{\rm h}^2}{4\theta} \gg 1$, the area spectrum depends on the noncommutative parameter and the cosmological constant, but the spacing of the area spectrum is equidistant up to leading order $\sqrt{\theta}\mathrm{e}^{-\frac{2Ml^2}{\theta}}$ in θ, and is independent of the noncommutative parameter and the cosmological constant.

The paper is organized as follows. In Section 2, we review some results related to a regular noncommutative black hole in three dimensions. In Section 3, we will study the entropy spectrum and horizon area spectrum of this noncommutative black hole. Section 4 is for conclusion and discussion.

2 REVIEW OF RESULTS RELATED TO A REGULAR NONCOMMUTATIVE BLACK HOLE IN THREE DIMENSIONS

We take the units $G = c = k_B = 1$, in which $l_{\rm p}^2 = M_{\rm p}^2 = \hbar$, where $M_{\rm p}$ is Planck mass and $l_{\rm p} =$ is Planck length which has been mentioned above.

Myung & Yoon (2009) constructed a regular black hole in a three dimensional anti-de Sitter space by introducing an anisotropic perfect fluid inspired by the four dimensional noncommutative black hole. In this black hole model, a new mass density of a cylindrically symmetric, smeared gravitational source, i.e. a Rayleigh smeared mass density distribution, is introduced as

$$\rho_\theta(r) = \frac{Mr}{4(\pi\theta)^{\frac{3}{2}}} \mathrm{e}^{-\frac{r^2}{4\theta}} \,, \tag{2}$$

where θ is the noncommutative parameter. The corresponding mass distribution is given by the integral of the density over the black hole volume

$$m_\theta(r) = \int_0^r 2\pi r' \rho(r') dr' = \frac{2M}{\sqrt{\pi}} \gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right) \,. \tag{3}$$

In the limit $\frac{r}{\sqrt{\theta}} \to \infty$, one gets $m_\theta(r) \to M$.

The lower incomplete Gamma function in the above equation is defined to be

$$\gamma(a, x) = \int_0^x t^{a-1} \mathrm{e}^{-t} dt \,. \tag{4}$$

The line element is obtained by solving the Einstein equation as (Myung & Yoon 2009)

$$ds^2 = -f_\theta(r) dt^2 + f_\theta^{-1}(r) dr^2 + r^2 d\phi^2 \,, \tag{5}$$

where

$$f_\theta(r) = \frac{r^2}{l^2} - \frac{16M}{\sqrt{\pi}} \gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right) = \frac{r^2}{l^2} - 8m_\theta(r) \,, \tag{6}$$

where l is related to the cosmological constant by $\Lambda = -\frac{1}{l^2}$. In the limit of $\frac{r}{\sqrt{\theta}} \to \infty$, this solution reduces to the well known BTZ black hole solution with mass M,

$$ds^2 = -\left(8M + \frac{r^2}{l^2}\right) dt^2 + \left(8M + \frac{r^2}{l^2}\right)^{-1} dr^2 + r^2 d\phi^2 \,. \tag{7}$$

For given θ, the horizon of the black hole described by element (5) is

$$r_{\rm h} = \frac{4\sqrt{M}l}{\pi^{\frac{1}{4}}} \gamma^{\frac{1}{2}}\left(\frac{3}{2}, \frac{r_{\rm h}^2}{4\theta}\right) \,. \tag{8}$$

In the commutative limit, $\theta \to 0$, one retrieves the horizon of the BTZ black hole as

$$r_{\text{h}}^{\text{BTZ}} = 2\sqrt{2M}l \,. \tag{9}$$

Using the mathematical formula

$$\frac{d}{dx}\gamma\left(\frac{3}{2}, x^2\right) = 2x^2 \text{e}^{-x^2} \,, \tag{10}$$

the Hawking temperature of the noncommutative black hole is calculated to be

$$T_{\text{h}} = \frac{\hbar}{4\pi}\partial_r f_\theta|_{r_{\text{h}}} = \frac{\hbar r_{\text{h}}}{2\pi l^2}\left[1 - \frac{(\frac{r_{\text{h}}}{2\sqrt{\theta}})^3 \text{e}^{-\frac{r_{\text{h}}^2}{4\theta}}}{\gamma\left(\frac{3}{2}, \frac{r_{\text{h}}^2}{4\theta}\right)}\right] \,. \tag{11}$$

Finally, it should be emphasized that there exists an extreme mass M_0 below which no noncommutative black hole can be found. For $M > M_0$, there are two horizons, i.e., the cosmological horizon r_{C} and event horizon r_{h}. For the extreme case of $M = M_0$, there is a unique degenerate horizon $r_{\text{h}} = r_0$, at which the Hawking temperature of the noncommutative black hole drops to absolute zero. The horizon radius of the black hole in the extreme case is determined from the condition $f_\theta(r_0) = 0$ and $\partial_r f_\theta(r_0) = 0$ as

$$\gamma\left(\frac{3}{2}, \frac{r_0^2}{4\theta}\right) = \left(\frac{r_0^2}{4\theta}\right)^{\frac{3}{2}} \text{e}^{-\frac{r_0^2}{4\theta}} \,. \tag{12}$$

The numerical solution is $r_0 \simeq 2\sqrt{\theta}$. Then the mass of the extreme black hole can be obtained from the condition $f_\theta(r_0) = 0$ as

$$M_0 = \frac{\theta\sqrt{\pi\theta}\text{e}^{\frac{r_0^2}{4\theta}}}{2r_0 l^2} \simeq \frac{e\sqrt{\pi\theta}}{4l^2} \,. \tag{13}$$

3 QUANTIZATION OF THE NONCOMMUTATIVE BLACK HOLE

3.1 Entropy Spectrum of the Black Hole

In order to quantize a noncommutative black hole via adiabatic invariance and the Bohr-Sommerfeld quantization rule, let us first Euclideanize the metric given in Equation (5) by using the transformation $t \to -i\tau$ as done in Majhi & Vagenas (2011). This leads to

$$ds^2 = f_\theta(r)d\tau^2 + f_\theta^{-1}(r)dr^2 + r^2 d\phi^2 \,. \tag{14}$$

Then we do a Painlevé coordinate transformation (Painlevé 1921),

$$d\tau \to d\tau + \frac{\sqrt{f_\theta(r) - 1}}{f_\theta(r)}dr \,. \tag{15}$$

The element (14) becomes

$$ds^2 = f_\theta(r)d\tau^2 + 2\sqrt{f_\theta(r) - 1}d\tau dr + dr^2 + r^2 d\phi^2 \,. \tag{16}$$

The outgoing radial null path $(ds^2 = d\phi = 0)$ is given by

$$\dot{r} = \frac{dr}{d\tau} = i(1 - \sqrt{1 - f_\theta(r)}) \simeq i\kappa(r - r_{\text{h}}) \,, \tag{17}$$

where κ is the surface gravity of the black hole defined by

$$\kappa = \frac{1}{2} \frac{\partial f_\theta(r)}{\partial r}\bigg|_{r_{\rm h}} \, . \tag{18}$$

Now, we consider an adiabatically invariant quantity of the form

$$\begin{aligned}
\int p_i dq_i &= \int \int_0^{p_i} dp_i' dq_i = \int \int_{H_0}^{H} \frac{dH'}{\dot{q}_i} dq_i \\
&= \int \int_{H_0}^{H} \frac{dH'}{\dot{r}} dr + \int \int_{H_0}^{H} dH' d\tau \\
&= 2 \int_{r_0}^{r} \int_{H_0}^{H} \frac{dH'}{\dot{r}'} dr' \, .
\end{aligned} \tag{19}$$

Here, for convenience, I have omitted the "out" of $p_i^{\rm out}$. For the second step of Equation (19), I have utilized Hamilton's canonical equation $\dot{q}_i = \frac{dH}{dp_i}$, where Hamiltonian H is the total energy of the black hole. In addition, note that the lower integration limit should be set to H_0 (it equals the mass of the black hole in the extreme case M_0). For the last step, I have used $\dot{r} = \frac{dr}{d\tau}$ and here the lower integration limit r_0 for r' is the horizon radius of the black hole in the extreme case.

Employing Equation (17), we can rewrite the adiabatically invariant quantity as

$$\int p_i dq_i = 2 \int_{H_0}^{H} \frac{dH'}{i\kappa} \int_{r_0}^{r} \frac{dr'}{r' - r_{\rm h}} = 2 \int_{H_0}^{H} \frac{\pi}{\kappa} dH' \, . \tag{20}$$

Using the relation between Hawking temperature and surface gravity

$$T_{\rm h} = \frac{\hbar\kappa}{2\pi} \, , \tag{21}$$

the adiabatically invariant quantity in Equation (20) becomes

$$\int p_i dq_i = \hbar \int_{H_0}^{H} \frac{dH'}{T_{\rm h}} = \hbar \int_0^{S} dS' = \hbar S \, , \tag{22}$$

where in the second step, I have utilized the first law of black hole thermodynamics $dH = T_{\rm h} dS$, and chosen $S = 0$ for the black hole in the extreme case according to the third law of thermodynamics. Finally, implementing the Bohr-Sommerfield quantization rule

$$\int p_i dq_i = 2\pi n\hbar \tag{23}$$

in Equation (22), we derive the entropy spectrum as

$$S = 2\pi n, \quad n = 0, 1, 2, 3, \dots \, . \tag{24}$$

Therefore, the spacing in the entropy spectrum is

$$\Delta S = S_{n+1} - S_n = 2\pi \, . \tag{25}$$

This shows that the entropy spectrum is quantized and equidistant for this noncommutative black hole.

3.2 Horizon Area Spectrum of the Black Hole

Equation (8) cannot be solved in a closed form, so we take the large radius regime $\frac{r_h^2}{4\theta} \gg 1$, using (9) and the following mathematical formula

$$\gamma\left(\frac{3}{2}, \frac{r_h^2}{4\theta}\right)\bigg|_{\frac{r_h^2}{4\theta} \gg 1} \simeq \frac{\sqrt{\pi}}{2} - \left(\frac{r_h}{2\sqrt{\theta}} + \frac{\sqrt{\theta}}{r_h}\right)e^{-\frac{r_h^2}{4\theta}} . \tag{26}$$

We can solve r_h by iteration. Keeping up to the leading order $\sqrt{\theta}e^{-\frac{2Ml^2}{\theta}}$, we get

$$r_h \simeq 2\sqrt{2M}l - \left(\frac{4Ml^2}{\sqrt{\pi\theta}} + \sqrt{\frac{\theta}{\pi}}\right)e^{-\frac{2Ml^2}{\theta}} . \tag{27}$$

Similarly, Equation (11) can be approximately written as

$$T_h \simeq \frac{\hbar\sqrt{2M}}{\pi l}\left[1 - \frac{4M\sqrt{2M}l^3}{\theta\sqrt{\pi\theta}}\left(1 - \frac{\theta}{4Ml^2} - \frac{\theta^2}{16M^2l^4}\right)e^{-\frac{2Ml^2}{\theta}}\right] . \tag{28}$$

From (22), the entropy of the noncommutative black hole can be directly calculated as

$$\begin{aligned}
S &= \int_{M_0}^{M} \frac{dM}{T_h} \\
&= \int_{M_0}^{M} \frac{\pi l}{\hbar\sqrt{2M}}\left[1 + \frac{4M\sqrt{2M}l^3}{\theta\sqrt{\pi\theta}}\left(1 - \frac{\theta}{4Ml^2} - \frac{\theta^2}{16M^2l^4}\right)e^{-\frac{2Ml^2}{\theta}}\right]dM \\
&\simeq \frac{1}{\hbar}\left[\pi\sqrt{2M}l - \left(2Ml^2\sqrt{\frac{\pi}{\theta}} + \frac{1}{2}\sqrt{\pi\theta}\right)e^{-\frac{2Ml^2}{\theta}}\right] \\
&\quad - \frac{1}{\hbar}\left[\pi\sqrt{2M_0}l - \left(2M_0l^2\sqrt{\frac{\pi}{\theta}} + \frac{1}{2}\sqrt{\pi\theta}\right)e^{-\frac{2M_0l^2}{\theta}}\right] .
\end{aligned} \tag{29}$$

On the other hand, using (27), we obtain the horizon area of the noncommutative black hole as

$$A = 2\pi r_h \simeq 4\pi\sqrt{2M}l - \left(8Ml^2\sqrt{\frac{\pi}{\theta}} + 2\sqrt{\pi\theta}\right)e^{-\frac{2Ml^2}{\theta}} . \tag{30}$$

Comparing (29) and (30), we get

$$A \simeq 4\hbar S + 4\pi\sqrt{2M_0}l - \left(8M_0l^2\sqrt{\frac{\pi}{\theta}} + 2\sqrt{\pi\theta}\right)e^{-\frac{2M_0l^2}{\theta}} . \tag{31}$$

Inserting (24) into (31), we get the area spectrum of the noncommutative black hole as

$$A \simeq 8\pi n\hbar + 4\pi\sqrt{2M_0}l - \left(8M_0l^2\sqrt{\frac{\pi}{\theta}} + 2\sqrt{\pi\theta}\right)e^{-\frac{2M_0l^2}{\theta}} . \tag{32}$$

This implies that the area spectrum of a noncommutative black hole is quantized, but it depends on the noncommutative parameter and the cosmological constant (since M_0 depends on these two quantities).

The spacing in the area spectrum is

$$\Delta A = 8\pi\hbar = 8\pi l_p^2 . \tag{33}$$

This shows that the area spectrum is equidistant and is independent of both the noncommutative parameter and the cosmological constant for a noncommutative black hole.

4 CONCLUSIONS AND DISCUSSION

I study the quantization of a regular black hole in AdS_3 spacetime by introducing an anisotropic perfect fluid inspired by the four dimensional noncommutative black hole. The entropy spectrum as well as the horizon area spectrum of the black hole is obtained. It is found that the spectra are discrete, and the spacing of the entropy spectrum is equidistant. In the limit $\frac{r_{\rm h}^2}{4\theta} \gg 1$, the area spectrum depends on the noncommutative parameter θ and the cosmological constant Λ, but the spacing of the area spectrum is equidistant up to leading order $\sqrt{\theta}{\rm e}^{-\frac{2Ml^2}{\theta}}$ in θ, and is independent of the noncommutative parameter and the cosmological constant.

In addition, I would like to emphasize that, in this paper, the entropy spectrum of the noncommutative regular black hole (24) is obtained by implementing the Bohr-Sommerfield quantization rule (23). Nevertheless, it is worthwhile to study how to obtain the entropy spectrum (24) from the result of the straightforward calculation (29) in the limit of a large horizon radius $\frac{r_{\rm h}^2}{4\theta} \gg 1$ rather than using the Bohr-Sommerfield quantization rule. I hope this problem can be solved in the future.

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