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# Testing X-ray measurements of galaxy cluster gas mass fraction using the cosmic distance-duality relation *

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**Abstract** We propose a consistency test for some recent X-ray gas mass fraction ($f_{\rm gas}$) measurements in galaxy clusters, using the cosmic distance-duality relation, $\eta_{\rm theory} = D_{\rm L}(1+z)^{-2}/D_{\rm A}$, with luminosity distance ($D_{\rm L}$) data from the Union2 compilation of type Ia supernovae. We set $\eta_{\rm theory} \equiv 1$, instead of assigning any redshift parameterizations to it, and constrain the cosmological information preferred by $f_{\rm gas}$ data along with supernova observations. We adopt a new binning method in the reduction of the Union2 data, in order to minimize the statistical errors. Four data sets of X-ray gas mass fraction, which are reported by Allen et al. (two samples), LaRoque et al. and Ettori et al., are analyzed in detail in the context of two theoretical models of $f_{\rm gas}$. The results from the analysis of Allen et al.'s samples demonstrate the feasibility of our method. It is found that the preferred cosmology by LaRoque et al.'s sample is consistent with its reference cosmology within the $1\sigma$ confidence level. However, for Ettori et al.'s $f_{\rm gas}$ sample, the inconsistency can reach more than a $3\sigma$ confidence level and this dataset shows special preference to an $\Omega_\Lambda = 0$ cosmology.

**Key words:** X-rays: galaxies: clusters — cosmology: distance scale — galaxies: clusters: general — cosmology: observations — supernovae: general

## 1 INTRODUCTION

As the largest virialized objects, clusters of galaxies play a critical role in enhancing our knowledge about matter distributions in the distant universe as well as the formation and evolution of large-scale structures (Voit 2005; Allen et al. 2011). Using galaxy clusters, there have been accumulated works to obtain the Hubble constant (Mason et al. 2001; Cunha et al. 2007), to put constraints on the matter/energy content of the universe (Lima et al. 2003; Vikhlinin et al. 2009), to study the evolution of underlying massive halos via N-body and hydrodynamical simulations (Eke et al. 1998; Kravtsov et al. 2005), and to measure distance scales independent of cosmological models using clusters as standard rulers (De Filippis et al. 2005; Bonamente et al. 2006). In practice, the observed abundance

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of massive clusters at high redshift ($z \sim 1$) provides strong indirect evidence for the existence of dark energy (Bahcall & Fan 1998), which was first introduced to explain the cosmic acceleration based on observations of type Ia supernovae (SNe Ia) (Riess et al. 1998; Perlmutter et al. 1999).

The cluster gas mass fraction measured from X-ray observations, $f_{\rm gas} = M_{\rm gas}/M_{\rm tot}$, i.e. the ratio between the mass of gas in the intracluster medium (ICM) and the total mass of the cluster, serves as a powerful cosmological probe. According to White et al. (1993), the baryon budget of rich clusters should reflect the cosmic value of $\Omega_{\rm b}/\Omega_{\rm m}$, where $\Omega_{\rm b}$ and $\Omega_{\rm m}$ are the mean baryonic and total matter densities of the universe, in units of the critical density, $\rho_{\rm c}(z) = 3H(z)^2/(8\pi G)$. Moreover, the estimates from Fukugita et al. (1998) indicate that the constituent of baryon mass in clusters is dominated by hot intracluster gas, with the contribution from the optically luminous stellar component being less than twenty percent, and that from other sources is negligible. In a series of works, using $f_{\rm gas}$ as a proxy of the cosmic baryon budget, Allen et al. (2002, 2004, 2008) improved the analysis method, enlarged the cluster sample (from 7 to 26, then to 42 data points), and tightened the constraints on cosmological parameters. The idea of determining the dark energy equation of state was also explored in Mantz et al. (2010), via the combination of $f_{\rm gas}$ measurements and other observations. Allen et al. (2003) made use of $f_{\rm gas}$ to constrain the relation between the normalization of the power spectrum of mass fluctuations, i.e. $\sigma_8$, and $\Omega_{\rm m}$. Ettori et al. (2006) investigated how miscellaneous physical processes in clusters, e.g. radiative cooling, star formation activities and galactic wind feedback, affect the measurements of baryon fraction, through hydrodynamical simulations.

In calculating $f_{\rm gas}$, a general duality between two distance scales,

$$\eta_{\rm theory} = \frac{D_{\rm L}}{D_{\rm A}}(1+z)^{-2} = 1\,, \tag{1}$$

is assumed in almost all previous studies (e.g. see Allen et al. 2008, footnote 1), where $D_{\rm L}$ and $D_{\rm A}$ represent luminosity and angular diameter distances, respectively. This distance duality was first proposed by Etherington (1933), and is usually termed Etherington's reciprocity relation or the cosmic distance-duality relation (CDDR). The CDDR is vital for observational cosmology, since any marked intrinsic violation of the CDDR may give rise to exotic physics (Bassett & Kunz 2004). The validity of the CDDR only depends on photon conservation on cosmic scales and the condition that the effect of gravitational lensing should be negligible, regardless of which metric for gravitation is used. Several research groups have used various observational data to test the validity of the CDDR (Uzan et al. 2004; de Bernardis et al. 2006; Avgoustidis et al. 2010; Liang et al. 2011). In particular, using galaxy clusters' $D_{\rm A}$ from the joint analysis of X-ray surface brightness, the Sunyaev-Zel'dovich technique and SNe Ia's $D_{\rm L}$ from the Union compilation, Holanda et al. (2010) performed a cosmologically independent test on the CDDR. Following this route, Li et al. (2011) tested the CDDR using the latest compilation comprised of 557 SNe Ia (Union2 compilation, Amanullah et al. 2010). Both Holanda et al. (2010) and Li et al. (2011) employed a moderate redshift criterion, $\Delta z = |z_{\rm cluster} - z_{\rm SN}| < 0.005$, to select the nearest SN Ia for every galaxy cluster. Meng et al. (2012) improved this analysis by developing two sophisticated methods to guarantee all appropriate SN Ia data selected, so as to reduce statistical errors. They found that the CDDR is compatible with the sample of galaxy clusters that are modeled as having an elliptical shape (De Filippis et al. 2005) at the $1\sigma$ confidence level (CL). However for some parameterizations, the CDDR cannot be accommodated even at a $3\sigma$ CL for the sample of clusters that are modeled as having a spherical shape, described by the $\beta$-model (Bonamente et al. 2006). Therefore their results support the conclusion that the marked triaxial ellipsoidal model better describes the structure of the galaxy cluster than the spherical $\beta$ model, if the CDDR is valid in cosmological observations. Holanda et al. (2012) arrived at similar conclusions.

More recently, Gonçalves et al. (2012) proposed the idea of testing the validity of the CDDR using X-ray $f_{\rm gas}$ data. In obtaining a sample for $f_{\rm gas}$, one has to assume some reference cosmology to solve the dependence of $f_{\rm gas}$ on metric distances. As a consequence, this test for the CDDR is not independent of cosmology. Moreover, because the CDDR is already assumed to be valid in the measurements of $f_{\rm gas}$, this test is not observationally robust. In this paper, we reverse the procedure of Gonçalves et al. (2012), via fixing $\eta_{\rm theory} \equiv 1$ instead of assigning any redshift parameterizations to $\eta_{\rm theory}$ (see Gonçalves et al. 2012, eq. (15)), and then constrain the preferred cosmological information by a given set of $f_{\rm gas}$ data. Thus a straightforward comparison between the cosmology preferred by the $f_{\rm gas}$ sample and its reported reference model is allowed. This may be a viable approach to present a consistency test for current measurements of $f_{\rm gas}$.

This paper is organized as follows. In Section 2, we briefly review the theoretical basis for formulating $f_{\rm gas}$ as a function of redshift and metric distances. The data samples and analysis method are then described in Section 3. Section 4 presents the main results, and Section 5 gives the conclusions and discussion.

## 2 THEORY: INCORPORATING THE CDDR INTO GAS FRACTION

The possibility of deriving cosmological constraints through the apparent redshift dependence of baryon mass fraction of a cluster was first discussed by Sasaki (1996) and Pen (1997). Supposing X-ray emission from ICM gas is mainly due to thermal bremsstrahlung (Sarazin 1988), the gas mass enclosed within a measurement radius $R$ can be derived as,

$$\begin{aligned} M_{\rm gas}(<R) \;=\; & \left[\frac{3\pi\hbar m_{\rm e}c^2}{2(1+X)e^6}\right]^{1/2}\left(\frac{3m_{\rm e}c^2}{2\pi k_{\rm B}T_{\rm e}}\right)^{1/4} m_{\rm H} \\ & \times\frac{1}{[\overline{g_{\rm B}}(T_{\rm e})]^{1/2}} r_{\rm c}^{3/2}\left[\frac{I_{\rm M}(R/r_{\rm c},\beta)}{I_{\rm L}^{1/2}(R/r_{\rm c},\beta)}\right][L_{\rm X}(<R)]^{1/2}\,, \end{aligned} \tag{2}$$

where $L_{\rm X}(<R)$ is the X-ray bolometric luminosity, $r_{\rm c}$ denotes the core radius, and the other symbols have their usual meanings. Furthermore, under the assumption of hydrostatic equilibrium and isothermality ($T_{\rm e} = {\rm const.}$) for the ICM, the total mass in a cluster of galaxies within $R$ is given by

$$M_{\rm tot}(<R) = -\frac{k_{\rm B}T_{\rm e}R}{G\mu m_{\rm H}}\frac{{\rm d}\ln n_{\rm e}(r)}{{\rm d}\ln r}\bigg|_{r=R}\,. \tag{3}$$

In the above estimations, the measurement radius is determined by fixing a certain value for the cluster overdensity ($\Delta = 3M_{\rm tot}(<R_\Delta)/(4\pi\rho_{\rm c}(z_{\rm cluster})R_\Delta^3)$), where $z_{\rm cluster}$ represents the cluster's redshift. Usually $\Delta$ is adopted as 2500 (Allen et al. 2004; LaRoque et al. 2006) or 500 (Ettori et al. 2009). Discussion has been raised regarding which value is more trustworthy in measuring $f_{\rm gas}$ (Vikhlinin et al. 2006; Allen et al. 2011). We also study this problem by analyzing two groups of $f_{\rm gas}$ datasets, assuming different values for $\Delta$.

The reference cosmology enters these relations via

$$L_{\rm X}(<R) \;=\; 4\pi D_{\rm L}^2 f_{\rm X}(<\theta), \tag{4}$$

$$r_{\rm c} \;=\; \theta_{\rm c} D_{\rm A}, \tag{5}$$

$$R \;=\; \theta D_{\rm A}\,. \tag{6}$$

Equations (2) and (3) then indicate

$$M_{\rm gas} \;\propto\; D_{\rm L} D_{\rm A}^{3/2}, \tag{7}$$

$$M_{\rm tot} \;\propto\; D_{\rm A}\,. \tag{8}$$

Thus it is straightforward to derive

$$f_{\rm gas} = M_{\rm gas}/M_{\rm tot} \propto D_{\rm L} D_{\rm A}^{1/2}\,. \tag{9}$$

Note that in all previous measurements of $f_{\rm gas}$, Equation (9) is readily reduced to $f_{\rm gas} \propto D_{\rm A}^{3/2}$, which already assumes the CDDR in the first place, and therefore strongly biases the test for the validity of the CDDR with data describing $f_{\rm gas}$. Aiming at using the CDDR to constrain $f_{\rm gas}$ samples, we employ more original forms for $f_{\rm gas}$ in subsequent analyses.

We model $f_{\rm gas}$ using the popular expression proposed by Allen et al. (2004),

$$f_{\rm gas}(z) = \frac{b}{\left(1+0.19\sqrt{h}\right)} \cdot \frac{\Omega_{\rm b}}{\Omega_{\rm m}} \cdot \left(\frac{D_{\rm L}^*(z) D_{\rm A}^*(z)^{1/2}}{D_{\rm L}(z) D_{\rm A}(z)^{1/2}}\right) , \tag{10}$$

with the dependence on metric distances modified according to Equation (9). A more generalized form recently proposed by Allen et al. (2008) is also considered,

$$f_{\rm gas}(z) = \frac{K\gamma(b_0+b_1 z)}{1+s_0(1+\alpha_{\rm s} z)} \cdot \frac{\Omega_{\rm b}}{\Omega_{\rm m}} \cdot \left(\frac{H(z) D_{\rm A}(z)}{H^*(z) D_{\rm A}^*(z)}\right)^{\xi} \cdot \left(\frac{D_{\rm L}^*(z) D_{\rm A}^*(z)^{1/2}}{D_{\rm L}(z) D_{\rm A}(z)^{1/2}}\right) , \tag{11}$$

which has also been revised due to the aforementioned intrinsic dependence on distance. In Equations (10) and (11), $\Omega_{\rm b}$ represents the baryonic matter density, which can be inferred from the big bang nucleosynthesis. $b$ (or $b(z) = b_0 + b_1 z$) represents the baryonic depletion being independent (or dependent) on redshift, which is a consequence of the thermodynamical evolution of clusters. $h$ depicts the Hubble constant via $H_0 = 100\,h$ km s$^{-1}$ Mpc$^{-1}$, and is adopted from the final results of the *Hubble Space Telescope* Key Project (Freedman et al. 2001). $s(z) = s_0\,(1+\alpha_{\rm s} z)$ models the baryonic matter fraction contributed from the stellar component. $\gamma$ considers the non-thermal pressure contributing to the hydrostatic equilibrium and lowering $M_{\rm tot}$. $K$ represents instrument calibration and $\xi$ corresponds to the relationship between the characteristic radius and the angular aperture used for measurement. Table 1 summarizes two sets of a priori knowledge about these nuisance parameters, for different $f_{\rm gas}$ samples measured under different $\Delta$.

Two sets of metric distances appear in Equations (10) and (11). The distances that are marked with a star correspond to the distances calculated from a certain reference cosmological model, which in the context of the $\Lambda$CDM cosmology are given by

$$\begin{aligned} D_{\rm L}^*(z) = D_{\rm A}^*(z)(1+z)^2 \; &= \; \frac{c(1+z)}{H_0} \frac{S(\omega)}{\sqrt{|\Omega_{\rm K}|}}, \\ \omega \; &= \; \sqrt{|\Omega_{\rm K}|} \int_0^z \frac{{\rm d}\zeta}{E(\zeta)} , \end{aligned} \tag{12}$$

where $S(\omega)$ is $\sinh(\omega)$, $\omega$ or $\sin(\omega)$ for $\Omega_{\rm K}$ larger than, equal to or smaller than zero, respectively. $E(z) = \frac{H(z)}{H_0} = \left[\Omega_{\rm m}(1+z)^3 + \Omega_{\rm K}(1+z)^2 + \Omega_\Lambda\right]^{1/2}$ represents the $\Lambda$CDM expansion history. Usually, it is safe to write $\Omega_{\rm m} + \Omega_{\rm K} + \Omega_\Lambda = 1$, with $\Omega_\Lambda$ and $\Omega_{\rm K}$ accounting for the constant dark energy density and the curvature of space. The distances that are not marked with a star can be connected through the CDDR, $\eta_{\rm obs}(z) = \frac{D_{\rm L}(z)}{D_{\rm A}(z)(1+z)^2}$. Then we obtain

$$\eta_{\rm obs}(z) = \left(\frac{1+0.19\sqrt{h}}{b\Omega_{\rm b}}\right)^2 \cdot \frac{\Omega_{\rm m}^2}{(1+z)^6} \cdot \left(\frac{D_{\rm L}(z)}{D_{\rm A}^*(z)}\right)^3 (f_{\rm gas}(z))^2 \tag{13}$$

for the expression of $f_{\rm gas}(z)$ given by Equation (10), and

$$\begin{aligned} \eta_{\rm obs}(z) \; = \; & \left(\frac{K\gamma(b_0+b_1 z)\Omega_{\rm b}}{[1+s_0(1+\alpha_{\rm s} z)]\,\Omega_{\rm m}}\right)^{\frac{2}{2\xi-1}} \cdot (1+z)^{\frac{-4\xi+6}{2\xi-1}} \\ & \cdot \left(\frac{H(z)}{H^*(z)}\right)^{\frac{2\xi}{2\xi-1}} \cdot \left(\frac{D_{\rm A}^*(z)}{D_{\rm L}(z)}\right)^{\frac{-2\xi+3}{2\xi-1}} (f_{\rm gas}(z))^{\frac{2}{-2\xi+1}} \end{aligned} \tag{14}$$

**Table 1** Summary of the Priors and Systematic Allowances for Nuisance Parameters Present in the Two Expressions for $\eta_{\rm obs}(z)$

| $\eta_{\rm obs}(z)$ Expression in Equation (13) | | |
|---|---|---|
| | Allowance | |
| Nuisance Parameter | A04 ΛCDM, A04 SCDM, L06 | E09, L06 ($\Delta = 500$) |
| $\Omega_{\rm b}h^2$ | $0.0214 \pm 0.0020$ | – |
| $\Omega_{\rm b}$ | – | $0.0462 \pm 0.0012$ |
| $h$ | $0.72 \pm 0.08$ | $0.72 \pm 0.08$ |
| $b$ | $0.824 \pm 0.089$ | $0.874 \pm 0.023$ |

| $\eta_{\rm obs}(z)$ Expression in Equation (14) | | |
|---|---|---|
| | Allowance | |
| Nuisance Parameter | L06 | E09, L06 ($\Delta = 500$) |
| $K$ | $1.0 \pm 0.1$ | $1.0 \pm 0.1$ |
| $\gamma$ | $(1.0, 1.1)$ | $(1.0, 1.1)$ |
| $\Omega_{\rm b}h^2$ | $0.0214 \pm 0.0020$ | – |
| $\Omega_{\rm b}$ | – | $0.0462 \pm 0.0012$ |
| $h$ | $0.72 \pm 0.08$ | $0.72 \pm 0.08$ |
| $b_0$ | $(0.65, 1.0)$ | $0.923 \pm 0.006$ |
| $b_1$ | – | $0.032 \pm 0.010$ |
| $\alpha_{\rm b}$ [a] | $(-0.1, 0.1)$ | – |
| $s_0$ | $0.16 \pm 0.048$ | $0.18 \pm 0.05$ |
| $\alpha_{\rm s}$ | $(-0.2, 0.2)$ | $(-0.2, 0.2)$ |
| $\xi$ | $0.214 \pm 0.022$ | 0.2 [b] |

Notes: ($a$) Allen et al. (2008) used $b(z) = b_0(1 + \alpha_{\rm b} z)$ to denote the depletion factor, where $b_0 \times \alpha_{\rm b}$ is equivalent to $b_1$ in our definition (Eq. (11)). ($b$) This value is determined from eq. (4) in Ettori et al. (2003).

for $f_{\rm gas}(z)$ given by Equation (11). In Equations (13) and (14), $D_{\rm A}^*(z)$ can be calculated according to Equation (12). In order to obtain $\eta_{\rm obs}(z)$, we still need the observational results of $f_{\rm gas}(z)$ and $D_{\rm L}(z)$, which are introduced in the next section.

## 3 DATA SETS AND ANALYSIS METHOD

Here we first describe the $f_{\rm gas}$ samples analyzed following the aforementioned idea and the SNe Ia data that furnish $D_{\rm L}(z)$. Then we describe the key procedures in our method as a whole.

### 3.1 Galaxy Cluster Samples and the SNe Ia Union2 Data

Allen et al. (2004) analyzed a sample of 26 luminous, dynamically relaxed galaxy clusters observed with *Chandra* at redshift $0.07 < z < 0.9$. They used the NFW model (Navarro et al. 1997) to parameterize total mass profiles of the clusters. Assuming different reference cosmological models, i.e. $\Lambda$CDM ($h = 0.7, \Omega_{\rm m} = 0.3, \Omega_\Lambda = 0.7$) and SCDM ($h = 0.5, \Omega_{\rm m} = 1, \Omega_\Lambda = 0$) (see Allen et al. 2004, table 2), they provided two samples of $f_{\rm gas}$, which are referred to as A04 $\Lambda$CDM and A04 SCDM, respectively. For consistency, we only use Equation (13) as $\eta_{\rm obs}(z)$ for these two samples, since they come from the same paper. The priors and systematic allowances of nuisance parameters associated with these two samples can be found in Table 1. $\Delta = 2500$ is chosen by measuring $f_{\rm gas}$.

As a follow-up study of Ettori et al. (2003), the paper by Ettori et al. (2009) focused on 52 clusters with *Chandra* measurements, spanning the range of $0.3 < z < 1.3$. The electron density profiles are fit with a functional form adapted from Vikhlinin et al. (2006). We choose the dataset assuming a constant temperature given by spectral analysis for each individual cluster (see Ettori et al. 2009, table 1), and quote this sample as E09 hereafter. Three clusters with spectral temperatures below 4 keV are excluded. The reported reference cosmology is $\Lambda$CDM ($h = 0.7, \Omega_{\rm m} = 0.3, \Omega_\Lambda = 0.7$). The priors/allowances for nuisance parameters are obtained mainly from the original paper by Ettori et al. (2009), which fixes $\Delta = 500$.

Combining *Chandra* X-ray observations and measurements of the Sunyaev-Zel'dovich effect from *OVRO* /*BIMA* interferometric arrays, LaRoque et al. (2006) obtained $f_{\rm gas}$ results of 38 massive galaxy clusters, in the redshift range $0.142 - 0.89$. We use their X-ray $f_{\rm gas}$ dataset assuming the gas distribution is described by the isothermal $\beta$-model (Cavaliere & Fusco-Femiano 1976) with the central 100 kpc excised (see LaRoque et al. 2006, table 4). This sample also employs a reference cosmology of $\Lambda$CDM ($h = 0.7, \Omega_{\rm m} = 0.3, \Omega_\Lambda = 0.7$). The original sample (referred to as L06) assumes $\Delta = 2500$, and thus can be analyzed using the priors/allowances proposed by Allen et al. (2004, 2008), since they adopt the same $\Delta$. Furthermore, we use the correlation obtained by Vikhlinin et al. (2006), $f_{{\rm gas},\Delta=2500}/f_{{\rm gas},\Delta=500} = 0.84$, to derive a new $f_{\rm gas}$ sample at $R_{\Delta=500}$, which is quoted as L06($\Delta = 500$). Besides the errors contributed from the original L06 data, a 10% uncertainty is also added to the errors of the L06($\Delta = 500$) data. For this sample, the priors/allowances are chosen to be exactly the same as those for E09. Note that L06($\Delta = 500$) is not directly measured at $R_{\Delta=500}$. Its analysis result reflects the accuracy of $f_{\rm gas}$ measurement at $R_{\Delta=2500}$.

To calculate the luminosity distances, we choose the Union2 compilation comprised of 557 SNe Ia (Amanullah et al. 2010)[1]. The uncertainty in the absolute magnitude of SNe Ia (Riess et al. 2011), i.e. a systematic error of 0.05 magnitude, is also considered as an additive covariance, and combined in quadrature among all distance moduli, provided by the *Supernova Cosmology Project*[2]. Gonçalves et al. (2012) used the criterion, $\Delta z = |z_{\rm cluster} - z_{\rm SN}| < 0.006$, to select the nearest SN Ia for each cluster for the sake of a direct test. However selecting merely one SN Ia within a certain redshift range will definitely lead to larger statistical errors (see Meng et al. 2012, footnote 7). Instead, we take an average of all the selected data that is weighted by the inverse variance,

$$\begin{aligned} \bar{D_{\rm L}} &= \frac{\sum\left(D_{{\rm L}i}/\sigma^2_{D_{{\rm L}i}}\right)}{\sum 1/\sigma^2_{D_{{\rm L}i}}}, \\ \sigma^2_{\bar{D_{\rm L}}} &= \frac{1}{\sum 1/\sigma^2_{D_{{\rm L}i}}}, \end{aligned} \tag{15}$$

where $D_{{\rm L}i}$ represents the $i$th selected luminosity distance within $\Delta z < 0.005$ and $\sigma_{D_{{\rm L}i}}$ denotes its observational uncertainty. What we ultimately utilize is $\bar{D_{\rm L}}$, the weighted mean luminosity distance at the corresponding $z_{\rm cluster}$, with $\sigma_{\bar{D_{\rm L}}}$ being its uncertainty. This binning method can significantly decrease statistical errors. Additionally, in all the five $f_{\rm gas}$ samples, if a cluster is not associated with any SN Ia within $\Delta z < 0.005$, then it is excluded to avoid large systematic uncertainties.

### 3.2 Statistics

Since it is assumed that $\eta_{\rm theory} \equiv 1$, we can calculate $\chi^2$ as,

$$\chi^2 = \sum_z \frac{(\eta_{\rm obs}(z) - 1)^2}{\sigma^2_{\eta_{\rm obs}(z)}}, \tag{16}$$

where $\eta_{\rm obs}(z)$ is given by Equation (13) or (14), while $\sigma_{\eta_{\rm obs}(z)}$ is obtained through error propagations from $\sigma_{D_{\rm L}(z)}$ and $\sigma_{f_{\rm gas}(z)}$. The asymmetric uncertainties in L06 data are handled using the technique proposed by D'Agostini (2004). The likelihood function, $\mathbb{L} \propto {\rm e}^{-\chi^2/2}$, is calculated over

[1] Using the more updated Union2.1 compilation (Suzuki et al. 2012) does not influence our results. Compared with the Union2 sample, this dataset includes 23 new events over the high redshift range ($0.6 < z < 1.4$), which has little overlap with the redshift ranges of the cluster samples.

[2] *http://supernova.lbl.gov/Union/*

**Table 2** Summary of the best-fit values and $1\sigma$ uncertainties for the preferred cosmological parameters of each $f_{\rm gas}$ sample, given the CDDR and Union2 SNe Ia data.

| Sample | Flat $\Lambda$CDM | General $\Lambda$CDM | |
|---|---|---|---|
| | $\Omega_{\rm m}$ | $\Omega_{\rm m}$ | $\Omega_\Lambda$ |
| $\eta_{\rm obs}(z)$ Expression in Equation (13) | | | |
| A04 $\Lambda$CDM | $0.282^{+0.072}_{-0.060}$ | $0.25^{+0.14}_{-0.08}$ | $0.52^{+0.42}_{-0.52}$ |
| A04 SCDM | $0.545^{+0.154}_{-0.112}$ | $0.30^{+0.17}_{-0.10}$ | $0.00^{+0.20}_{-0.00}$ |
| E09 | $0.439^{+0.026}_{-0.023}$ | $0.38^{+0.09}_{-0.03}$ | $0.00^{+0.78}_{-0.00}$ |
| L06 | $0.284^{+0.069}_{-0.052}$ | $0.28^{+0.15}_{-0.09}$ | $0.69^{+0.32}_{-0.44}$ |
| L06($\Delta = 500$) | $0.295^{+0.013}_{-0.012}$ | $0.31^{+0.03}_{-0.04}$ | $0.92^{+0.32}_{-0.57}$ |
| $\eta_{\rm obs}(z)$ Expression in Equation (14) | | | |
| E09 | $0.395^{+0.069}_{-0.055}$ | $0.34^{+0.30}_{-0.08}$ | $0.00^{+1.48}_{-0.00}$ |
| L06 | $0.286^{+0.077}_{-0.064}$ | $0.29^{+0.18}_{-0.11}$ | $0.77^{+0.67}_{-0.77}$ |
| L06($\Delta = 500$) | $0.310^{+0.041}_{-0.038}$ | $0.33^{+0.12}_{-0.10}$ | $0.95^{+0.53}_{-0.95}$ |

Notes: The quoted constraints are obtained after marginalization of all nuisance parameters.

a certain range of grid values for cosmological parameters, $\Omega_{\rm m}$ and $\Omega_\Lambda$. Then, after marginalizing over nuisance parameters in Equation (13) or (14), we can obtain the posterior probability of each reference cosmological model.

For each $f_{\rm gas}$ sample, the marginalization process requires specific a priori knowledge about all nuisance parameters. In our analysis, all the systematic allowances and priors, listed in Table 1, are carefully chosen according to previous studies (Allen et al. 2004, 2008; Ettori et al. 2003, 2009). The best-fit values are defined as the marginalized probability reaching its maximum. For 1-dimensional analysis giving a constraint on the flat $\Lambda$CDM reference cosmology (with only one parameter, $\Omega_{\rm m}$), the $1\sigma$, $2\sigma$ and $3\sigma$ CLs are defined with the marginalized probability equivalent to $e^{-1.0/2}$, $e^{-4.0/2}$ and $e^{-9.0/2}$ of the maximum respectively, whereas for the 2-dimensional constraint on $(\Omega_{\rm m}, \Omega_\Lambda)$, i.e. on usual $\Lambda$CDM cosmology, the ratios are taken to be $e^{-2.30/2}$, $e^{-6.17/2}$ and $e^{-11.8/2}$ respectively.

## 4 RESULTS

Using the method described above, we have constrained the cosmological information preferred by the five $f_{\rm gas}$ samples. The best-fit parameter values at $1\sigma$ CL using corresponding $\eta_{\rm obs}(z)$ expressions are summarized in Table 2.

In Figures 1–3, we plot the marginalized posterior probabilities of the reference cosmology for each sample, taking $\Omega_{\rm K} = 0$ in the left panels, and $\Omega_{\rm K} = 1 - \Omega_{\rm m} - \Omega_\Lambda$ in the right panels.

Note that the Union2 compilation of SNe Ia suggests a $\Omega_{\rm m} = 0.270 \pm 0.021$ flat $\Lambda$CDM universe. This is a relatively strong constraint from direct observations. For the $f_{\rm gas}$ sample measured under a certain reference cosmology, the constrained results should reflect both this reference model as well as the cosmology indicated by the observations of SNe Ia. This is actually what our consistency test is designed for.

In Figure 1, for A04 $\Lambda$CDM, its reference cosmology $(\Omega_{\rm m} = 0.3, \Omega_\Lambda = 0.7)$ is close to the SN Ia cosmology $(\Omega_{\rm m} = 0.27, \Omega_\Lambda = 0.73)$. They are all very consistent with the constrained cosmology within $1\sigma$ CL (Panel (a)). However, the result of the 1-dimensional analysis with A04 SCDM is not so good. The best-fit parameter is $\Omega_{\rm m} = 0.545$, which deviates from both the reference cosmology $(\Omega_{\rm m} = 1, \Omega_\Lambda = 0)$ and the SN Ia cosmology. Such a result is reasonable, because the

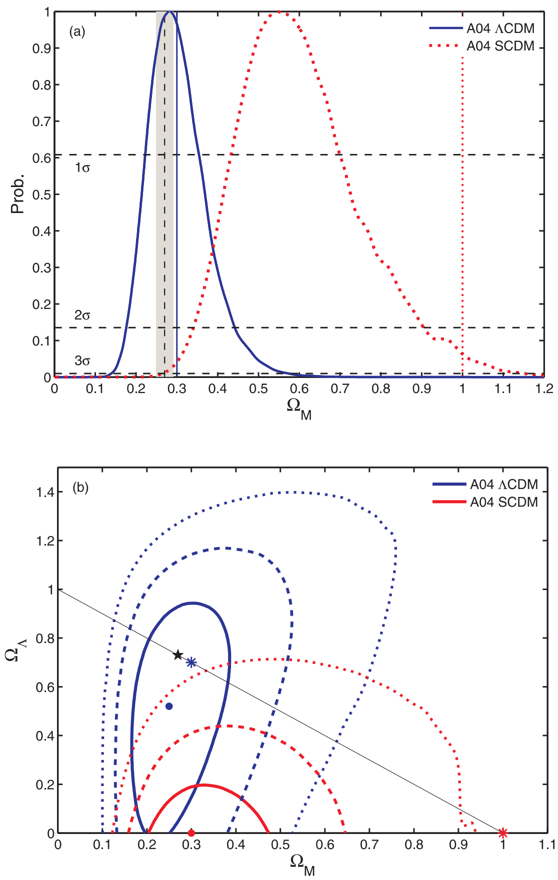


**Fig. 1** Marginalized constraints on the preferred cosmological models by A04 ΛCDM and A04 SCDM, given the CDDR and Union2 SNe Ia data. Panel (a) shows the constraints on $\Omega_{\rm m}$, taking $\Omega_{\rm K} = 0$. The horizontal *dashed* lines correspond to $1\sigma$, $2\sigma$ and $3\sigma$ CLs respectively. The cosmological information from Union2 SNe Ia $(\Omega_{\rm m} = 0.270 \pm 0.021)$ is marked by the vertical *dashed* line with the shaded region. The reported reference cosmological models are indicated by the vertical *solid* and *dotted* lines, respectively. Panel (b) shows the constraints in the $(\Omega_{\rm m}, \Omega_\Lambda)$ plane for a ΛCDM cosmology with $\Omega_{\rm K}$ included as a free parameter. The $1\sigma$, $2\sigma$ and $3\sigma$ CLs are plotted by solid, dashed and dotted lines, respectively. The best-fit values and reference cosmologies for these samples are represented by big dots and stars in corresponding colors. The straight thin line indicates a flat geometry. SN Ia cosmology is marked by the pentagram.

reference cosmology and the SN Ia cosmology themselves are quite different. Nevertheless from the 2-dimensional analysis, the correct reference cosmological information ($\Omega_\Lambda = 0$) is unambiguously revealed by the best-fit parameter value (Fig. 1b), which is convincing evidence that our method can shed light upon the intrinsic reference cosmology of the $f_{\rm gas}$ measurement. Generally speaking, using the datasets reported by Allen et al. (2004), we proved the validity of our method.

The analysis results of L06 and L06($\Delta = 500$) are displayed in Figures 2 and 3. For both L06 (using priors/allowances proposed by Allen et al.) and L06($\Delta = 500$) (using priors/allowances proposed by Ettori et al.), the constrained cosmological parameters are always consistent with their reported reference cosmology $(\Omega_{\rm m} = 0.3, \Omega_\Lambda = 0.7)$ within $1\sigma$ CL. The priors on nuisance pa-

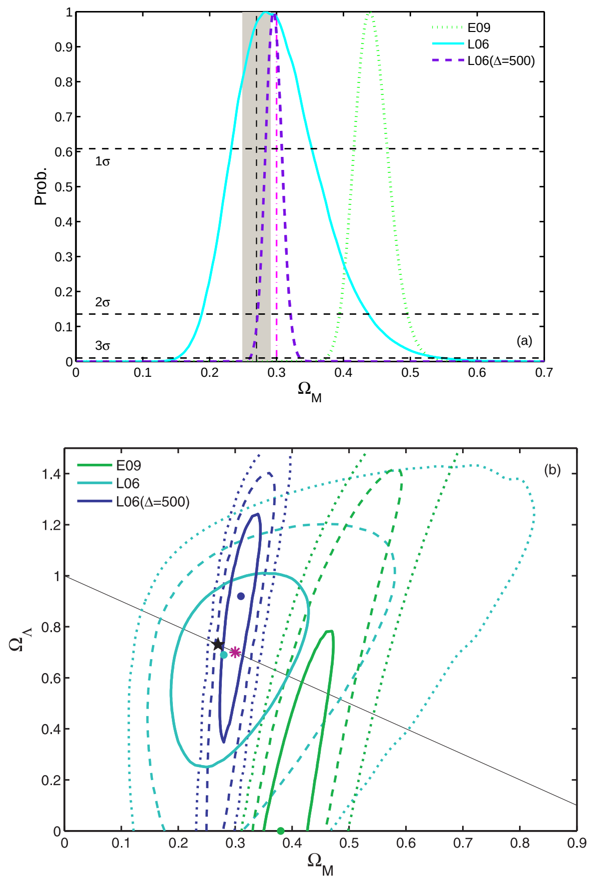


**Fig. 2** Marginalized constraints on the preferred cosmological models by E09, L06 and L06($\Delta = 500$), given the CDDR and Union2 SNe Ia data. The $\eta_{\rm obs}(z)$ expression is given by Eq. (13). Panel (a) shows the constraints on $\Omega_{\rm m}$, under the assumption of a flat universe. The horizontal *dashed* lines correspond to $1\sigma$, $2\sigma$ and $3\sigma$ CLs respectively. The cosmological information from Union2 SNe Ia $(\Omega_{\rm m} = 0.270 \pm 0.021)$ is marked by the vertical *dashed* line with the shaded region. These three samples' reference cosmological model is represented by the vertical *dash-dotted* line. Panel (b) shows the constraints in the $(\Omega_{\rm m}, \Omega_\Lambda)$ plane for a usual $\Lambda$CDM cosmology model, with curvature kept free. The $1\sigma$, $2\sigma$ and $3\sigma$ CLs are plotted by straight, dashed and dotted lines, respectively. The straight thin line indicates a flat geometry. SN Ia cosmology is marked by the pentagram. The best-fit values and reference cosmology for these samples are represented by big dots in corresponding colors and the star in magenta respectively. Note that these samples have the same reference cosmology in their original papers, which report these samples.

rameters proposed by Ettori et al. in modeling $f_{\rm gas}$ are rather strong compared with those proposed by Allen et al. Therefore one must be exceedingly careful when trying to derive cosmological constraints via $f_{\rm gas}$ results using those stringent assumptions. Comparing Figure 2 with Figure 3, it is also clear that the CLs are enlarged owing to the change of $\eta_{\rm obs}(z)$ from Equation (13) to Equation (14). This is reasonable since Equation (14) includes more nuisance parameters, which are capable of reflecting more physical effects and systematic uncertainties, and thus is a more generalized expression.

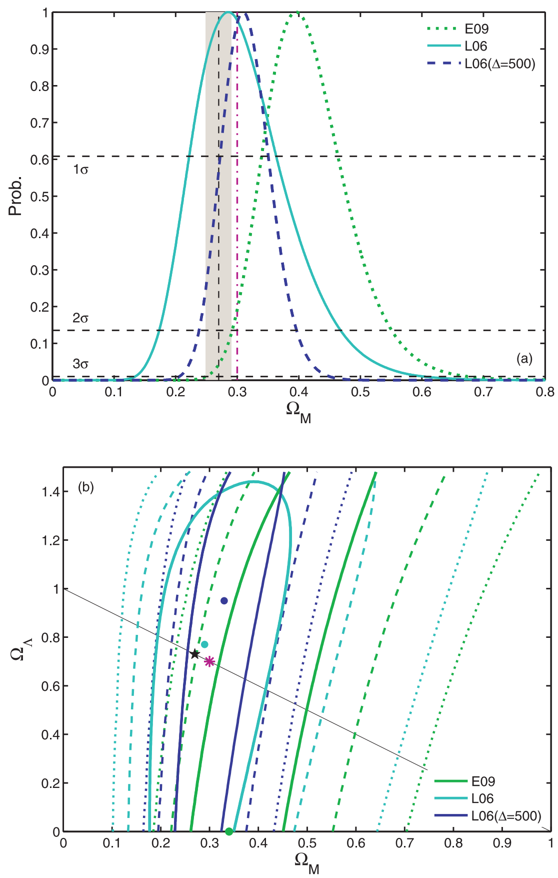


**Fig. 3** Same as Fig. 2, except that the $\eta_{\rm obs}(z)$ expression is given by Eq. (14).

However, as shown in Figures 2 and 3, the cosmological parameters preferred by E09 greatly deviate from their reported reference cosmology ($\Omega_{\rm m} = 0.3, \Omega_\Lambda = 0.7$), which can never be accommodated within $1\sigma$ CL, regardless of 1- or 2-dimensional constraints. The inconsistency can deviate by as much as a $6\sigma$ CL, with $\Omega_{\rm m} = 0.439^{+0.026}_{-0.023}$ for the flat $\Lambda$CDM cosmology under the $\eta_{\rm obs}(z)$ expression of Equation (13). If more nuisance parameters are considered in modeling $f_{\rm gas}$, i.e. $\eta_{\rm obs}(z)$ is altered from Equation (13) to Equation (14), the confidence regions are enlarged as expected, yet E09's inconsistency still exists at least at the $1.7\sigma$ CL ($\Omega_{\rm m} = 0.395^{+0.069}_{-0.055}$). We also note that no matter which $\eta_{\rm obs}(z)$ expression is adopted, the best-fit values from 2-dimensional analyses always read as $\Omega_\Lambda = 0$ (see Table 2). In light of the result from A04 SCDM, we argue that the characteristics of E09 $f_{\rm gas}$ data prefer a cosmology without a dark energy component[3], which can lead to biased cosmological parameter constraints when this dataset is combined with probes that support concordance cosmology.

## 5 CONCLUSIONS AND DISCUSSION

In this paper, we proposed a consistency test to reveal the cosmological information preferred by X-ray $f_{\rm gas}$ measurements, using the CDDR and Union2 SNe Ia. We applied this test to the $f_{\rm gas}$

[3] It is necessary to point out that the original study by Ettori et al. (2003), which was followed and updated by Ettori et al. (2009), did employ a reference cosmology of the Einstein-de Sitter ($\Omega_{\rm m} = 1, \Omega_\Lambda = 0$) universe.

samples provided by Allen et al. (A04 $\Lambda$CDM, A04 SCDM), LaRoque et al. (L06, L06 ($\Delta = 500$)) and Ettori et al. (E09). It is found that the samples of A04 $\Lambda$CDM, L06 and L06($\Delta = 500$) show a high level of consistency in the context of our test. Despite the great discrepancy between the A04 SCDM's reference cosmology and the SN Ia cosmology, our 2-dimensional analysis is still capable of probing its intrinsic cosmological information ($\Omega_\Lambda = 0$) through the best-fit result.

However, our method reveals an inconsistency of more than $3\sigma$ CL compared with E09, the $f_{\rm gas}$ dataset estimated by Ettori et al. (2009) assuming isothermal ICM. Although endowed with an $\Omega_{\rm m} = 0.3, \Omega_\Lambda = 0.7$ $\Lambda$CDM reference cosmology as reported, E09 shows special preference to an $\Omega_\Lambda = 0$ cosmology. This result offers a reasonable explanation for a recent CDDR test by Gonçalves et al. (2012), who found a significant conflict when using the Ettori et al. (2009) sample, and this highly significant violation was only spotted in the $f_{\rm gas}$ sample from Ettori et al. (2009)[4].

The strength of nuisance parameters' priors proposed by Allen et al. and Ettori et al. is also vividly demonstrated. The major differences between these two sets of priors exist in the allowances on the depletion factor ($b$ or $b(z)$) and the baryonic matter density ($\Omega_{\rm b}$)[5]. The comparison between the results of L06 and L06($\Delta = 500$) shows that the priors on these parameters given by Ettori et al. (at $\Delta = 500$) are much more stringent than those given by Allen et al. (at $\Delta = 2500$). However, since the X-ray background and the impact of ICM clumpiness can become a concern for $\Delta \leq 500$ (Allen et al. 2011), more reliable a priori knowledge on some influencing factors (baryon depletion, background contamination, cluster substructure, etc.) is still lacking for $f_{\rm gas}$ measurements and modeling at $\Delta = 500$.

Furthermore, there are many physical processes affecting the measurements of $f_{\rm gas}$ as well, particularly whether the cluster is in hydrostatic equilibrium or undergoes a major merger. Deviation from the equilibrium may give rise to large errors in $f_{\rm gas}$ results (Nagai et al. 2007; David et al. 2012), potentially leading to the inconsistency presented by our analysis compared to the E09 sample.

In Ettori et al. (2009)'s study, the total baryon budget of clusters includes the contribution from the ICM gas and the cold baryons. The cold baryons themselves are composed of a stellar component and an intracluster light component. Additionally, their studies unexpectedly infer that there is still another baryonic matter component ($f_{\rm ob}$), whose percentage is non-negligible and can be as high as 25%. This will bring significant systematic uncertainties to the measurement of the total baryon mass (Ettori et al. 2006).

Moreover, another concern is the morphology hypothesis in modeling the gas distribution of the ICM. Although our test demonstrates the high consistency of the L06 sample, we should still bear in mind that the galaxy clusters in this sample are modeled under the assumption of spherical symmetry. Recently, the work by several groups (Meng et al. 2012; Holanda et al. 2012) infers that compared with the spherical geometry, the ellipsoidal morphology for the gas distribution is more preferable.

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[4] Note that the specific datasets adopted by Gonçalves et al. and us from Ettori et al. (2009) are different. Unlike the sample used by Gonçalves et al., our choice for investigation (E09) obeys the assumption of isothermality, which plays a critical role in the determination of $M_{\rm tot}$. Besides, we consider all available cluster data for the purpose of minimizing systematic uncertainties.

[5] The difference between allowances on $\xi$ is negligible, since it affects $f_{\rm gas}$ values by less than ten percent (Ettori et al. 2003). Originally, the factor $A$ ($A = (\frac{H(z)D_{\rm A}(z)}{H^*(z)D^*_{\rm A}(z)})^\xi$) was introduced by Allen et al. (2008) to account for the change in angle subtended by $R_{\Delta=2500}$ as the underlying reference cosmology varies.

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