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A new study of neutrino energy loss of nuclides $^{53-60}$Cr by electron capture in magnetars *

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Abstract Following the theory of relativity, in the presence of an ultrastrong magnetic field (UMF) and utilizing a nuclear shell model, we carry out an estimation of the neutrino energy loss (NEL) rates of nuclides $^{53-60}$Cr, which occur by electron capture in magnetars. The results show that the NEL rates greatly increase when a UMF is present, and can even exceed nine orders of magnitude at relatively lower density and temperature (e.g. $\rho_7 = 5.86, Y_{\rm e} = 0.47, T_9 = 7.33$) in the range from 10^{13} G to 10^{18} G. However, the increase in rates was no more than six orders of magnitude at relatively higher density and temperature (e.g. $\rho_7 = 4.86 \times 10^8, Y_{\rm e} = 0.39, T_9 = 14.35$).

Key words: physical data and processes: neutrinos, nuclear reactions — stars: supernovae — stars: evolution

1 INTRODUCTION

Neutrino processes play a crucial role in magnetars and some neutron stars through electron capture (EC) and beta decay. A great deal of energy can be released when a neutrino escapes. Thus, works on neutrinos and neutrino energy loss (NEL) rates have been a hot topic, along with the former-frontier issue of magnetars and some neutron stars. The magnetic fields can be as large as 10^{13}–10^{15} G (Peng & Tong 2007; Peng et al. 2012). How would an ultrastrong magnetic field (UMF) effect NEL? How would the UMF effect the cooling system in magnetars? How would the UMF effect the chemical potential of electron gas in magnetars? These are very interesting problems in the study of magnetars. Some authors (Fuller et al. 1980, 1982; Nabi 2010; Liu et al. 2011; Dai et al. 1993; Sofiah Ahmad et al. 2011; Liu & Luo 2007a,b, 2008a; Liu 2010a, b) have investigated many results related to calculation of NEL rates and some related weak interaction rates. However, they did not consider the NEL rates in a UMF.

Previous research (Liu et al. 2007a,b; Liu & Luo 2008b,c) has shown that the effect of a UMF on EC rates and NEL rates varies greatly, and with an increase in the magnetic field strength, the EC rates and NEL rates strongly decrease. Recent studies (Peng & Tong 2007; Peng et al. 2012) found that strengthening the magnetic field would make the Fermi surface elongate from a spherical surface to a Landau surface along the direction of the magnetic field and its surface would be perpendicular to the direction of the magnetic field and quantized. Thus, we should revise the theory of relativistic Landau levels.

Following the theory of relativity in the presence of a UMF, and including a nuclear shell model (Peng & Tong 2007; Peng et al. 2012), in this paper we will study the NEL rates of nuclides $^{53-60}$Cr by EC. We will also discuss the rates of change for electron abundance (RCEA) in the EC reactions.

2 THE STUDY OF NEL RATES IN A UMF

In astrophysical cataclysms like the coalescence of neutron stars, UMFs which could be generated can actively influence quantum processes. However, the magnetic field only significantly influences quantum processes in the case when it is ultrastrong. A UMF is considered along the z-axis and the Dirac equation can be solved exactly. The positive energy levels of an electron in a UMF are given by (Peng & Tong 2007; Peng et al. 2012)

$$\frac{\epsilon_n}{m_e c^2} = \left[\left(\frac{p_z}{m_e c}\right) + 1 + 2\left(n + \frac{1}{2} + \sigma\right)b\right]^{1/2} = (p_z^2 + \Theta)^{1/2}\,, \tag{1}$$

where $\Theta = 1 + 2(n + \frac{1}{2} + \sigma)b$, $n = 0, 1, 2, 3....$, $b = \frac{B}{B_{\rm cr}} = 0.02266B_{12}$, $B = 10^{12}B_{12}$, B_{12} is the magnetic field strength in units of 10^{12} G; $B_{\rm cr} = \frac{m_e^2 c^3}{e\hbar} = 4.414 \times 10^3$ G; p_z is the electron momentum along the field, and σ is the spin quantum number of an electron; $\sigma = 1/2$, when $n = 0$; $\sigma = \pm 1/2$, when $n \geq 1$.

In an extremely strong magnetic field $(B \gg B_{\rm cr})$, the Landau column becomes a very long and very narrow cylinder parallel to the magnetic field, and the electron chemical potential is found by inverting the expression for lepton number density (Peng et al. 2012)

$$\begin{aligned} n_e &= \frac{3\pi}{bN_A}\left(\frac{m_{ec}}{h}\right)^3 \varepsilon_n^4 \int_0^1 \left(1 - \frac{1}{\varepsilon_n^2} - x^2\right)dx - \frac{2\pi\varepsilon_n}{N_A}\left(\frac{m_{ec}}{h}\right)^3 \sqrt{2b} \\ &= \frac{b}{2\pi^2\lambda_e^3}\sum_0^\infty q_{n0}\int_0^\infty (f_{-e} - f_{+e})dp_z\,, \end{aligned} \tag{2}$$

where $x = \frac{p_z c}{\varepsilon_n}$; $Y_e = \frac{Z}{A}$ is the electron fraction; N_A is the Avogadro constant; m_e is the electron mass and c is the speed of light. $\lambda_e = \frac{h}{m_e c}$ is the Compton wavelength, $q_{n0} = 2 - \delta_{n0}$ is the electron degeneracy number,

$$f_{-e} = \left[1 + \exp\left(\frac{\varepsilon_n - U_F - 1}{kT}\right)\right]^{-1}$$

and

$$f_{+e} = \left[1 + \exp\left(\frac{\varepsilon_n + U_F + 1}{kT}\right)\right]^{-1}$$

are the electron and positron distribution functions respectively, k is the Boltzmann constant, T is the electron's temperature and U_F is the electron's chemical potential.

The NEL rates due to EC for the kth nucleus (Z, A) in thermal equilibrium at temperature T are given by a sum over the initial parent states i and the final daughter states f (Fuller et al. 1980, 1982)

$$\lambda_k^\nu = \lambda_{ec} = \sum_i \frac{(2J_i + 1)e^{\frac{-E_i}{kT}}}{G(Z, A, T)} \sum_f \lambda_{if}^\nu\,, \tag{3}$$

where J_i and E_i are the spin and excitation energies of the parent states respectively, and $G(Z, A, T)$ is the nuclear partition function. The NEL rates by EC from one of the initial states to all possible final states is λ_{if}^ν; $\lambda_{if}^\nu = \frac{\ln 2}{(ft)_{if}} f_{if}$ with the relation $\frac{1}{(ft)_{if}} = \frac{1}{(ft)_{if}^{\rm F}} + \frac{1}{(ft)_{if}^{\rm GT}}$. The ft values and the corresponding Gamow-Teller (GT) and Fermi transition matrix elements are related by the following expression

$$\frac{1}{(ft)_{if}} = \frac{1}{(ft)^{\rm F}_{if}} + \frac{1}{(ft)^{\rm GT}_{if}} = \frac{10^{3.79}}{|M_{\rm F}|^2_{if}} + \frac{10^{3.596}}{|M_{\rm GT}|^2_{if}}\,, \tag{4}$$

where $|M_{\rm F}|^2$ and $|M_{\rm GT}|^2$ are the squares of the absolute value of Fermi and GT matrix elements, respectively.

The Fermi matrix element and the GT matrix element are given respectively as follows (Fuller et al. 1980)

$$|M_{\rm F}|^2 = \frac{1}{2J_i+1}\sum_{m_i}\sum_{m-f}|\langle\psi_f m_f|\sum_N \tau_N^-|\psi_i m_i\rangle|^2 = T'(T'+1) - T_Z^{'i}(T_Z^{'i}-1)\,, \tag{5}$$

$$|M_{\rm GT}|^2 = \frac{1}{2J_i+1}\sum_{m_i}\sum_{m-f}|\langle\psi_f m_f|\sum_N \tau_N^-\sigma_N|\psi_i m_i\rangle|^2\,, \tag{6}$$

where T' is the nuclear isospin and $T'_Z = T_Z^{'i} = (Z-N)/2$ is its projection for the parent or the daughter nucleus. $|\psi_i m_i\rangle$ is the initial parent state, $\langle\psi_f m_f|$ is the final daughter state, and the Fermi matrix element is averaged over the initial and summed over the final nuclear spins. $\sum_N \tau_N^-$ is the negative component of the isovector, and the spatial scalar operator $T^{'-}$ commutes with the total isospin $T^{'2}$. σ is the Pauli spin operator and $\sum_N \tau_N^-\sigma_N$ is a spatial vector and an isovector.

The phase space factor in a UMF can be found in Liu & Luo (2007a,b, 2008b) and is defined as

$$f^B_{if} = \frac{b}{2}\sum_0^\infty \theta_n\,, \tag{7}$$

$$\theta_n = q_{n0}\int_{q_n}^\infty (Q_{if}+\varepsilon_n)^3 \frac{F(Z,\varepsilon_n)}{1+\exp(\frac{\varepsilon_n - U_{\rm F}-1}{kT})}dp\,, \tag{8}$$

where $Q_{if} = Q_{00} + E_i - E_f$ is the EC threshold energy; $Q_{00} = M_{\rm p}c^2 - M_{\rm d}c^2$, with $M_{\rm p}$ and $M_{\rm d}$ being the masses of the parent nucleus and the daughter nucleus respectively; E_i and E_f are the excitation energies of the ith state and fth state of the nucleus respectively; ε_n is the sum of rest mass and kinetic energy; $F(Z,\varepsilon_n)$ is the Coulomb wave correction which is the ratio of the square of the electron wave function distorted by the Coulomb scattering potential to the square of the wave function of the free electron. We assume that a UMF will have no effect on $F(Z,\varepsilon_n)$, which is only valid under the condition that the electron wave functions are locally approximated by plane wave functions (Dai et al. 1993). The condition requires that the Fermi wavelength $\lambda_{\rm F} \sim \frac{\hbar}{P_{\rm F}}$ ($P_{\rm F}$ is the Fermi momentum without a magnetic field) be smaller than the radius $\sqrt{2}\varrho$ (where $\varrho = \frac{\lambda_{\rm e}}{b}$) of the cylinder which corresponds to the lowest Landau level (Baym & Pethick 1975).

The q_n is defined as

$$q_n = \begin{cases} \sqrt{Q^2_{if} - \Theta^{1/2}} & (Q_{if} < -\Theta^{1/2}), \\ 0 & (\text{otherwise}). \end{cases} \tag{9}$$

where $\Theta = 1 + 2(n + \frac{1}{2} + \sigma)b$.

On the other hand, what really matters for stellar evolution is the electron abundance $Y_{\rm e}$, which is related to the RCEA in each nucleus. Therefore the RCEA due to EC on the kth nucleus is very important in a UMF. It is given by

$$\dot{Y_{\rm e}^{\rm ec}}(k) = \frac{dY_{\rm e}}{dt} = -\frac{X_k}{A_k}\lambda_k^{\rm ec}, \tag{10}$$

where $\lambda_k^{\rm ec}$ is the EC rates in a UMF, X_k is the mass fraction of the kth nucleus and A_k is the mass number of the kth nucleus.

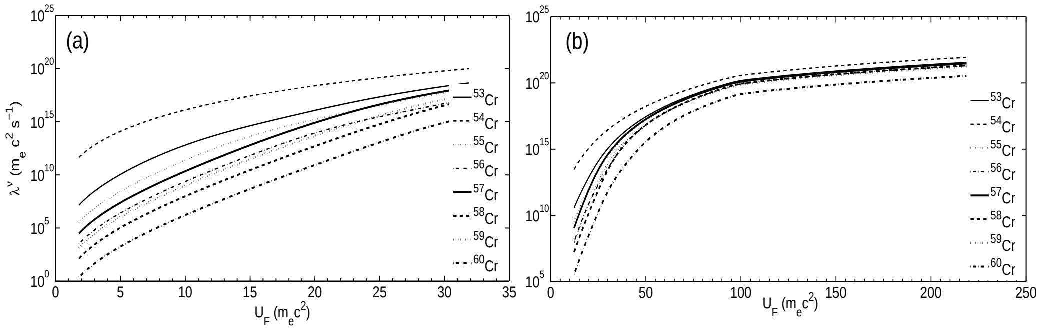

Fig. 1 The NEL rates as a function of the electron chemical potential $U_{\rm F}$ at the density, electron abundance and temperature of $\rho_7 = 5.86, Y_{\rm e} = 0.47$ and $T_9 = 7.33$ (a); $\rho_7 = 1.45 \times 10^4, Y_{\rm e} = 0.43$ and $T_9 = 9.43$ (b), respectively. The UMF strength is $10^{13}{\rm G} \leq B \leq 10^{18}$ G.

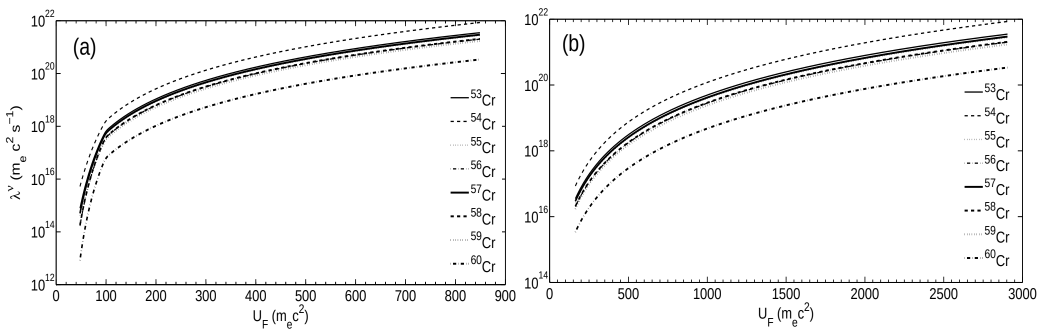

Fig. 2 The NEL rates as a function of the electron chemical potential $U_{\rm F}$ at the density, electron abundance and temperature of $\rho_7 = 3.46 \times 10^6, Y_{\rm e} = 0.40$ and $T_9 = 11.54$ (a); $\rho_7 = 4.86 \times 10^8, Y_{\rm e} = 0.39$ and $T_9 = 14.35$ (b), respectively. The UMF strength is $10^{13}{\rm G} \leq B \leq 10^{18}$ G.

3 THE EFFECT ON THE NEL RATES IN A UMF AND DISCUSSION

Figure 1 shows the NEL rates of $^{53-60}$Cr as a function of electron chemical potential at different stellar conditions of $\rho_7 = 5.86, Y_{\rm e} = 0.47$ and $T_9 = 7.33$ and $\rho_7 = 1.45 \times 10^4, Y_{\rm e} = 0.43$ and $T_9 = 9.43$ in magnetars (ρ_7 is the density in units of 10^7 g cm^{-3} and T_9 is the temperature in units of 10^9 K). We find that the NEL rates in a UMF have different effects at different densities and temperatures. The NEL rates greatly increase for these nuclides and even exceed nine orders of magnitude in the case of lower density and temperature (e.g. at $\rho_7 = 5.86, Y_{\rm e} = 0.47$ and $T_9 = 5$). The lower the density and temperature are, the larger the influence on NEL is by $U_{\rm F}$. The electron energy and electron chemical potential are so low at lower density and temperature that the UMF can strongly affect the NEL rates. On the other hand, one can see that with increasing density there are different effects on NEL for different nuclides. This is due to the fact that the nuclides have different Q–values and different transition orbits in the EC reactions.

Under the conditions $\rho_7 = 3.46 \times 10^6, Y_{\rm e} = 0.40$ and $T_9 = 11.54$ and $\rho_7 = 4.86 \times 10^8, Y_{\rm e} = 0.39$ and $T_9 = 14.35$, the NEL rates of these nuclides as a function of electron chemical potential $U_{\rm F}$ are found in panels (a) and (b) of Figure 2. Comparing the results from the two panels, one can see that a relativity lower density has a relatively larger effect on the NEL rates for the nuclides. The

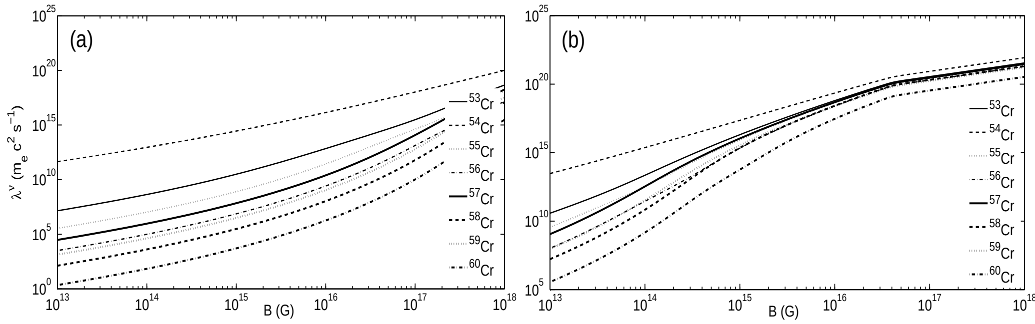

Fig. 3 The NEL rates as a function of the magnetic field strength B at the density, electron abundance and temperature of $\rho_7 = 5.86, Y_\mathrm{e} = 0.47$ and $T_9 = 7.33$ (a); $\rho_7 = 1.45 \times 10^4, Y_\mathrm{e} = 0.43$ and $T_9 = 9.43$ (b), respectively.

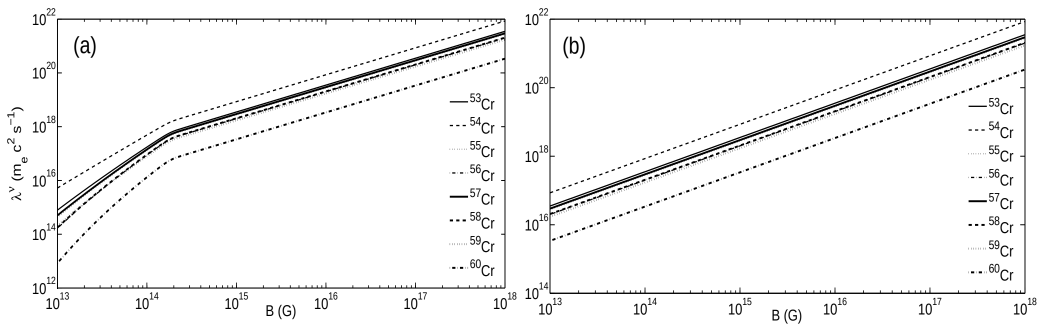

Fig. 4 The NEL rates as a function of the magnetic field strength B at the density, electron abundance and temperature of $\rho_7 = 3.46 \times 10^6, Y_\mathrm{e} = 0.40$ and $T_9 = 11.54$ (a); $\rho_7 = 4.86 \times 10^8, Y_\mathrm{e} = 0.39$ and $T_9 = 14.35$ (b), respectively.

NEL rates increase, but by no more than six orders of magnitude at $\rho_7 = 4.86 \times 10^8, Y_\mathrm{e} = 0.39$ and $T_9 = 14.35$. Furthermore, we can see that U_F has different effects on NEL rates at different densities and temperatures. The higher the density, the smaller the influence on NEL rate, because at relativity higher density, according to the theory of relativity in UMFs, the electron energy and the electron chemical potential are so high that the influence on NEL rate would be weakened in the EC reactions.

Figures 3 and 4 show the NEL rates as a function of the magnetic field strength B. One can find that the NEL rates are influenced greatly at relatively lower temperature and density. The NEL rates (e.g. $^{56-60}$Cr) may be increased more than nine orders of magnitude in the case of a magnetic field from 10^{13} G to 10^{18} G. However, the increase of the NEL rates in most nuclides is no more than six orders of magnitude for the case of relatively higher density and temperature (e.g. $\rho_7 = 4.86 \times 10^8$, $Y_\mathrm{e} = 0.39$ and $T_9 = 14.35$). This is because at higher temperature and density, the electron energy and chemical potential are also higher. This cripples the influence of the UMF on the EC reaction, so the increase in the NEL rates will evidently decrease.

Comparing the results in these figures, we find that the GT transition in the EC process may not be dominant at lower temperature. This process is dominated by the low-energy transition. Therefore,

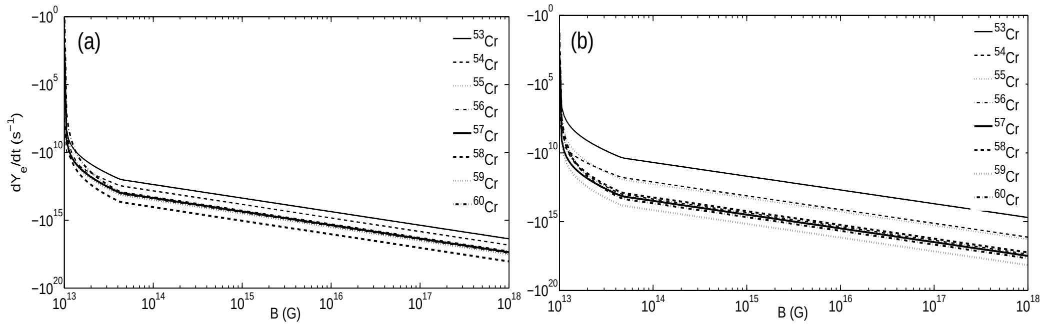

Fig. 5 The RCEA as a function of the magnetic field strength B at the density, electron abundance and temperature of $\rho_7 = 5.86, Y_\mathrm{e} = 0.47$ and $T_9 = 7.33$ (a); $\rho_7 = 4010, Y_\mathrm{e} = 0.41$ and $T_9 = 9.43$ (b), respectively.

the effect produced by this kind of density is very obvious by a UMF due to relativistic electron energy. We find that the distribution of the electron gas with high temperature and high density must satisfy the Fermi-Dirac distribution. The GT transition probability of a nuclide is distributed in the form of the centrosymmetric Gaussian function about the GT resonance point. The energies of the electrons taking part in the GT resonance transitions in the high-energy range are not symmetric in a UMF. The variance of the Gaussian distribution increases and includes more electrons that take part in the EC reactions. Therefore, increasing temperature obviously accelerates the progress of the electron capture process. However, weakening of the EC reactions by the UMF would inevitably lead to great decreases in the NEL rates.

As is well known, variation in electron abundance is one of the vital parameters in modeling supernovae. The RCEA in EC reactions is caused by each nucleus. The electronic abundance strongly influences the changes of electron degenerate pressure and entropy. The RCEA plays a key role at late stages of stellar evolution, especially in the process of supernova explosion.

Figure 5 shows the RCEA of these nuclides as a function of magnetic field. We find that the UMF has a great effect on the RCEA due to the EC for these nuclides. The RCEA greatly reduces, even by an amount exceeding ten orders of magnitude due to the UMF.

By analyzing the effect of a UMF on NEL rates for the different nuclides, we find that the UMF has different effects on NEL rates for different nuclides because of the difference in the nuclide's threshold energy and transition orbits in the EC reaction. The higher the magnetic fields are, the larger the NEL rates become for the case of relatively lower density and temperature.

4 CONCLUDING REMARKS

We have carried out an estimation on the NEL rates of nuclides $^{53-60}$Cr due to EC in a UMF. It is concluded that a UMF has a significant effect on the NEL rates of these iron group nuclei. The NEL rates greatly increase and even exceed nine orders of magnitude (e.g. $^{56-60}$Cr) by a UMF at relatively lower density and temperature (e.g. at $\rho_7 = 5.86$, $Y_\mathrm{e} = 0.47$ and $T_9 = 7.33$). However, the increase in the NEL rates for most nuclides is no more than six orders of magnitude in the case of relatively higher density and temperature (e.g. $\rho_7 = 4.86 \times 10^8$, $Y_\mathrm{e} = 0.39$ and $T_9 = 14.35$).

As is well known, due to the escape process of a great number of neutrinos from the EC reaction, the NEL gives one of the key contributions to the cooling in magnetar evolution. The NEL is also very helpful in facilitating the collapse and explosion of a supernova. The results we have obtained may have a significant influence on further research related to nuclear astrophysics and neutrino astrophysics, especially for work on the cooling systems of neutron stars and magnetars.

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