The bulk viscous string cosmology in an anisotropic universe with late time acceleration

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Abstract A model of a cloud formed by massive strings is used as a source of Bianchi type II cases. We assume that the expansion (θ) in the model is proportional to the shear (σ). To get an exact solution, we consider the equation of state of the fluid to be in the stiff form. It is found that the bulk viscosity played a very important role in the history of the universe. In the presence of bulk viscosity the particles dominate over strings whereas in the absence of it, strings dominate over the particles, which is not consistent with recent observations. Also we observe that the viscosity causes the expansion of the universe to be accelerating. Our models are evolving from an early decelerating phase to a late time accelerating phase. The physical and geometrical behaviors of these models are discussed.

Key words: cosmology: theory — viscous fluid — massive string

1 INTRODUCTION

In recent years, there has been considerable interest in string cosmology. One of the proposals of the Grand Unified Theories (GUTs) is that the universe underwent a phase transition as the temperature fell below $T_{\mathrm{GUT}} \sim 10^{28}$ K when the age of the universe was $t_{\mathrm{GUT}} \sim 10^{-36}$ s (Zeldovich et al. 1975; Kibble 1980; Everett 1981; Vilenkin 1981). There was a loss of symmetry when the universe underwent the GUT phase transition at t_{GUT}. At $T < T_{\mathrm{GUT}}$, the symmetry between the strong and electro-weak forces spontaneously broke. The phase transitions associated with loss of symmetry led to the formation of topological defects such as domain walls, cosmic strings, monopoles, etc. Cosmic strings are the important topologically stable defects which might have been found during a phase transition in the early universe (Kibble 1976). The existence of a large scale network of strings in the early universe does not contradict the present-day observations. The vacuum strings may generate density fluctuations sufficient to explain the formation of galaxies (Zeldovich 1980). The cosmic strings coupled stress-energy to the gravitational field. Therefore, the study of gravitational effects from such strings will be interesting. The general relativistic treatment of strings was initiated by Letelier (1979, 1983). Here we have considered gravitational effects that arose from strings by the coupling of stress-energy of strings to the gravitational field. Letelier (1979) defined the massive strings as the geometric strings (massless) with particles attached along their expansions.

The strings that form the cloud are the generalization of Takabayasi's relativistic model of strings (called p-strings). This is the simplest model wherein we have particles and strings together. In principle, we can eliminate the strings and end up with a cloud of particles. This is a desirable

property of a model of a string cloud that can be used in cosmology since strings are not observed at the present time due to evolution of the universe (Banerjee et al. 1990; Yadav et al. 2007; Saha & Visinescu 2008; Saha et al. 2010).

Most cosmological models assume that the matter in the universe can be described by 'dust' (a pressureless distribution) or at best a perfect fluid. To have realistic cosmological models we should consider the presence of a material distribution other than a perfect fluid. Cosmological models of a fluid with viscosity play a significant role in the study of evolution of the universe. The viscosity mechanism in cosmology can account for high entropy per baryon in the present universe (Weinberg 1972). It is well known that at an early stage of the universe when neutrino decoupling occurred, the matter behaved like a viscous fluid (Kolb & Turner 1990). Weinberg (1971, 1972) derived general formulae for bulk and shear viscosity and used these to evaluate the rate of cosmological entropy production. He deduced that the most general form of the energy-momentum tensor, allowed by rotational and space-inversion invariance, contains a bulk viscosity term proportional to the volume expansion of the model. Padmanabhan & Chitre (1987) also noted that viscosity may be relevant for the future evolution of the universe. If the coefficient of bulk viscosity decays sufficiently slowly, i.e. $\xi \sim \rho^n$, $n < \frac{1}{2}$, then the late epochs of the universe will be viscosity dominated, and the universe will enter a final inflationary era which has a steady-state character. Cosmological models with viscous fluid in the early universe have been widely discussed in the literature (for example see Pradhan et al. 2012; Pradhan & Lata 2011; Pradhan 2009; Pradhan & Kumhar 2009; Pradhan et al. 2008; Pradhan et al. 2007; Pradhan et al. 2005; Yadav 2011, 2010; Yadav et al. 2012).

Stiff fluid cosmological models create more interest in the study because, for these models, the speed of light is equal to the speed of sound and its governing equations have the same characteristics as those of a gravitational field (Zeldovich 1972). Barrow (1986) has discussed the relevance of a stiff equation of state $\rho = p$ to the matter content of the universe in its early stage of evolution. Wesson (1978) investigated an exact solution of Einstein's field equation with the stiff equation of state. Mohanty et al. (1982) investigated a cylindrically symmetric Zel'dovich fluid distribution in general relativity. Götz (1988) obtained a plane symmetric solution of Einstein's field equation for a stiff perfect fluid distribution. Pradhan & Kumhar (2009) investigated a locally rotationally symmetric (LRS) Bianchi type II (B-II) bulk viscous universe with decaying vacuum energy density in general relativity. Recently Yadav et al. (2011) have investigated a string LRS B-II universe in general relativity.

B-II space-time has a fundamental role in constructing cosmological models suitable for describing the early stages of evolution of the universe. Asseo & Sol (1987) emphasized the importance of a B-II universe. In the present paper we have considered an LRS model of spatially homogeneous B-II cosmology. To obtain exact solutions, the field equations have been solved for the case when the equation of state of the fluid is in the stiff form. The paper is organized as follows. The metric and the field equations are presented in Section 2. In Section 3, we deal with the solution of the field equations with a cloud of strings. In Subsection 3.1 we describe some physical and geometric properties of the model. In Subsection 3.2 we give the solution in the absence of bulk viscosity. A dark energy interpretation of the derived models is given in Section 4. Finally, in Section 5, concluding remarks are given.

2 THE METRIC AND FIELD EQUATIONS

We consider the B-II metric in the form

$$ds^2 = -dt^2 + B^2(dy + xdz)^2 + A^2(dx^2 + dz^2),\tag{1}$$

where A and B are functions of only t. The energy-momentum tensor for a cloud of strings in the presence of bulk viscosity is taken as

$$T_{ij} = (p + \rho)u_iu_j + pg_{ij} - \lambda x_ix_j + \xi\theta(g_{ij} + u_iu_j),\tag{2}$$

where u_i and x_i satisfy the condition

$$u_i u^i = -x^i x_i = -1, \qquad u^i x_i = 0,$$
(3)

p is the isotropic pressure, ρ is the proper energy density for a cloud of strings with particles attached to them, λ is the string tension density, u^i the four-velocity of the particles, and x^i is a unit space-like vector representing the direction of the string. In a co-moving coordinate system, we have

$$u^i = (0, 0, 0, 1), \qquad x^i = (\frac{1}{B}, 0, 0, 0).$$
(4)

The particle density of the configuration is given by

$$\rho_p = R_{ij} u^i u^j,$$
(5)

where ρ_p is the rest energy density of the particles attached to the strings. The string tension density, λ, can take positive or negative values. A negative value of λ represents a universe filled with no strings having only an anisotropic fluid, whereas its positive value represents strings loaded with particles forming the surface of a world sheet (Berman 1990a).

The Einstein's field equations (with $8\pi G = 1$ and $c = 1$)

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij}$$
(6)

for the metric (1) lead to the following system of equations:

$$G_{22} = 2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{3}{4}\frac{B^2}{A^4} = -p + \xi\theta + \lambda,$$
(7)

$$G_{11} = G_{33} = \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{1}{4}\frac{B^2}{A^4} = -p + \xi\theta,$$
(8)

$$G_{00} = 2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} - \frac{1}{4}\frac{B^2}{A^4} = \rho,$$
(9)

where an overdot stands for the first and a double overdot for the second derivative with respect to t.

The spatial volume for LRS B-II is given by

$$V = A^2 B.$$
(10)

We define $S = (A^2 B)^{\frac{1}{3}}$ as the average scale factor of the LRS B-II model (1) so that the Hubble's parameter is given by

$$H = \frac{\dot{S}}{S} = \frac{1}{3}\left(\frac{2\dot{A}}{A} + \frac{\dot{B}}{B}\right).$$
(11)

We define the generalized mean Hubble's parameter $\overline{H}$ as

$$\overline{H} = \frac{1}{3}(H_x + H_y + H_z),$$
(12)

where $H_x = \frac{\dot{A}}{A}$, $H_y = \frac{\dot{B}}{B}$ and $H_z = H_x$ are the directional Hubble's parameters in the directions of x, y and z respectively.

The deceleration parameter q is conventionally defined by

$$q = -\frac{S\ddot{S}}{\dot{S}^2}.$$
(13)

The scalar expansion θ, shear scalar σ^2 and the average anisotropy parameter A_m are defined by

$$\theta = \frac{2\dot{A}}{A} + \frac{\dot{B}}{B}, \tag{14}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right), \tag{15}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \tag{16}$$

where $\Delta H_i = H_i - H(i = 1, 2, 3)$.

The Raychaudhuri equation reads as

$$3\frac{\ddot{S}}{S} = -2\sigma^2 + \frac{3}{2}\xi\theta - \frac{1}{2}(\rho + 3p). \tag{17}$$

3 SOLUTION OF THE FIELD EQUATIONS

The field Equations (11)–(13) are a system of three equations with five unknown parameters A, B, p, ρ and λ. Two additional constraints relating to these parameters are required to obtain explicit solutions of the system. Firstly, we assume that the expansion θ in the model is proportional to the shear σ. This condition leads to

$$\frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = b\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right), \tag{18}$$

which yields

$$\frac{\dot{A}}{A} = m\frac{\dot{B}}{B}, \tag{19}$$

where $m = \frac{2b\sqrt{3}+1}{1-b\sqrt{3}}$ and b are constants. Equation (19), after integration, reduces to

$$A = B^m. \tag{20}$$

Secondly, we assume that the fluid obeys the stiff fluid equation of state, i.e.

$$p = \rho. \tag{21}$$

Using Equations (8) and (9) in Equation (21) we obtain

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + 3\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} - \xi\theta = 0. \tag{22}$$

In view of Equation (20), Equation (21) is taken as

$$2\ddot{B} + 4m\frac{\dot{B}^2}{B} = \frac{\chi}{(m+1)}B \tag{23}$$

where we have assumed that the coefficient of bulk viscosity is inversely proportional to expansion, i.e. $\xi\theta = \chi$ (say) = constant.

Let $\dot{B} = f(B)$ which implies that $\ddot{B} = ff'$, where $f' = \frac{df}{dB}$. Hence Equation (23) can be written as

$$\frac{d}{dB}(f^2) + 4m\frac{f^2}{B} = \frac{\chi}{(m+1)}B, \tag{24}$$

which on integration gives

$$dt = \frac{dB}{\sqrt{aB^2 + bB^{-4m}}}.$$

(25)

Here, $a = \frac{\chi}{(2m^2+3m+1)}$ and b is a positive constant of integration. Hence the model (1) is reduced to

$$ds^2 = -\frac{dB^2}{aB^2 + bB^{-4m}} + B^2(dx + zdy)^2 + B^{2m}(dy^2 + dz^2).$$

(26)

After using a suitable transformation of coordinates, the model (26) reduces to

$$ds^2 = -\frac{dT^2}{aT^2 + bT^{-4m}} + T^2(dx + zdy)^2 + T^{2m}(dy^2 + dz^2).$$

(27)

3.1 The Geometric and Physical Significance of the Model

Here we discuss some physical and kinematic properties of the string model (27).
The pressure (p), the energy density (ρ), the string tension (λ), and the particle density (ρ_p) for the model (27) are given by

$$p = \rho = \frac{m(m+2)}{2}\left[\frac{\chi}{(2m^2+3m+1)} + 2bT^{-2(1+2m)}\right] - \frac{1}{4}\frac{1}{T^{2(2m-1)}},$$

(28)

$$\lambda = \frac{1}{2}\left(\frac{2m^2-3m+1}{2m^2+3m+1}\right)\chi - \frac{1}{T^{2(2m-1)}},$$

(29)

$$\rho_p = -\frac{1}{2}\left(\frac{m^2-5m+1}{2m^2+3m+1}\right)\chi + m(m+2)bT^{-2(1+2m)} + \frac{3}{4}\frac{1}{T^{2(2m-1)}}.$$

(30)

From Equations (28) and (30), we observe that the energy density ρ and the particle density ρ_p are decreasing functions of time. This behavior of ρ and ρ_p is shown in Figure 1. Also the energy conditions, $\rho \geq 0$ and $\rho_p \geq 0$, are satisfied under conditions

$$T^{-4}\left[4b + \frac{2\chi}{(2m^2+3m+1)}T^{2(2m+1)}\right] \geq \frac{1}{m(m+2)},$$

(31)

and

$$T^{-4}\left[-4m(m+2)b + \left(\frac{m^2-5m+1}{2m^2+3m+1}\right)\chi T^{2(2m+1)}\right] \geq \frac{3}{2},$$

(32)

respectively. Also $\lambda > 0$ under

$$T > \left[2\left(\frac{2m^2+3m+1}{m^2-5m+1}\right)\frac{1}{\chi}\right]^{\frac{1}{2(2m-1)}}.$$

(33)

From Equation (29), it is observed that λ is an increasing function of time which is always negative and tends to zero at late time. It is pointed out by Letelier (1979) that λ may be positive or negative. When $\lambda < 0$, the string phase of the universe disappears, i.e. we have an anisotropic fluid of particles. This behavior of tension density λ is also depicted in Figure 1.

To study the behavior of strings and particles in the universe, here we define the following parameter

$$\frac{\rho_p}{|\lambda|} = \frac{-\frac{1}{2}\left(\frac{m^2-5m+1}{2m^2+3m+1}\right)\chi + m(m+2)bT^{-2(1+2m)} + \frac{3}{4}\frac{1}{T^{2(2m-1)}}}{|\frac{1}{2}\left(\frac{2m^2-3m+1}{2m^2+3m+1}\right)\chi - \frac{1}{T^{2(2m-1)}}|}.$$
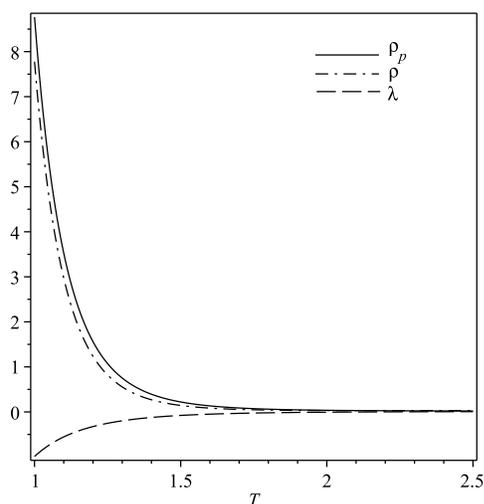
(34)

Fig. 1 The plot of energy density ρ, particle density ρ_p and tension density λ versus T for $m = 2$, $\chi = 0.1$ and $b = 1$.
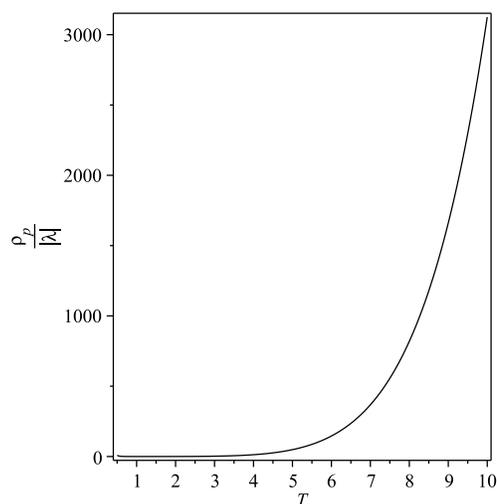
Fig. 2 The plot of $\frac{\rho_p}{|\lambda|}$ versus T for $m = 2$, $\chi = 0.1$ and $b = 1$.

As mentioned before, since strings are not observed at the present time due to evolution of the universe, in principle we can eliminate the strings and end up with a cloud of particles. In other words, we can say the particles dominate over the strings at the present time because of the evolution of the universe. Figure 2 clearly shows that in a universe which is described by the model (27), the strings dominate over the particles at the initial time whereas the particles dominate over the strings at late time. Also it is worth mentioning that from Figures 2 and 3, we observe that at the initial time, when the universe is in the decelerating phase, the strings dominate over the particles ($\rho_p < \lambda$) whereas when the universe is in the accelerating phase, particles dominate over the strings ($\rho_p > \lambda$). This is in agreement with the results obtained in Weinberg (1976) and Belinchón (2009) and also agrees with astronomical observations which predict that there is no direct evidence of strings in the present-day universe.

The expressions for the scalar of expansion θ, the average generalized Hubble's parameter H, magnitude of shear σ^2, proper volume V, deceleration parameter q and the average anisotropy parameter A_m for the model (27) are given by

$$\theta = 3H = (1 + 2m) \left[\frac{\chi}{2(2m^2 + 3m + 1)} + bT^{-2(1+2m)} \right]^{\frac{1}{2}}, \tag{35}$$

$$\sigma^2 = \frac{(m-1)^2}{3} \left[\frac{\chi}{2(2m^2 + 3m + 1)} + bT^{-2(1+2m)} \right], \tag{36}$$

$$V = T^{2m+1}, \tag{37}$$

$$q = \frac{3}{2m + 1} \left[\frac{2bT^{-2(1+2m)} - \frac{\chi}{2(2m^2+3m+1)}}{bT^{-2(1+2m)} + \frac{\chi}{2(2m^2+3m+1)}} \right], \tag{38}$$

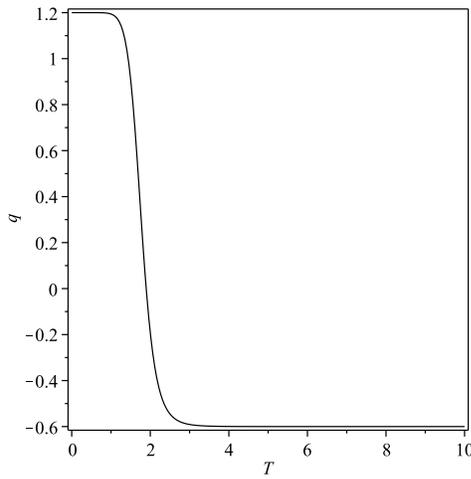$$A_m = 2 \left(\frac{1 - m}{1 + 2m} \right)^2. \tag{39}$$

Fig. 3 The plot of deceleration parameter q versus T for $m = 2$, $\chi = 0.1$ and $b = 1$.
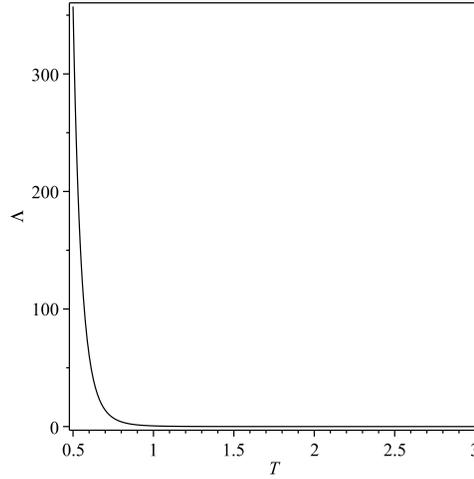
Fig. 4 The plot of cosmological constant Λ versus T for $m = 2$, $\chi = 0.1$ and $b = 1$.

From Equation (37) we observe that

$$q > 0 \text{ if } m > -\frac{1}{2} \text{ and } T < \left(\frac{2b}{a}\right)^{\frac{1}{2(1+2m)}} \quad \text{or } m < -\frac{1}{2} \text{ and } T > \left(\frac{2b}{a}\right)^{\frac{1}{2(1+2m)}}, \tag{40}$$

and

$$q < 0 \text{ if } m > -\frac{1}{2} \text{ and } T > \left(\frac{2b}{a}\right)^{\frac{1}{2(1+2m)}} \quad \text{or } m < -\frac{1}{2} \text{ and } T < \left(\frac{2b}{a}\right)^{\frac{1}{2(1+2m)}}. \tag{41}$$

A positive sign of q corresponds to the standard decelerating model whereas the negative sign $-1 \leq q < 0$ indicates inflation. Recent observations show that the deceleration parameter of the universe is in the range $-1 \leq q < 0$ and the present day universe is undergoing an accelerated expansion. From Figure 3 we observe that the model (27) successfully describes the expansion of our universe from the decelerating to accelerating phase.

Also we note that for

$$T = \left[\frac{(1-m)\chi}{b(2m+7)(2m^2+3m+1)}\right]^{-\frac{1}{2(1+2m)}},$$

$q = -1$ as in the case of a de Sitter universe.

In the absence of any curvature, matter/energy density (Ω_m) and dark energy (Ω_Λ) are related by the equation

$$\Omega_m + \Omega_\Lambda = 1, \tag{42}$$

where $\Omega_m = \frac{\rho}{3H^2}$ and $\Omega_\Lambda = \frac{\Lambda}{3H^2}$. Thus Equation (42) reduces to

$$\frac{\rho}{3H^2} + \frac{\Lambda}{3H^2} = 1. \tag{43}$$

Using Equations (28) and (35), in Equation (43), the cosmological constant is obtained as

$$\Lambda = \left[\frac{(m-1)^2}{2(2m^2+3m+1)}\chi \right] + \frac{(m-1)^2}{3}bT^{-2(1+2m)} + \frac{1}{4}\frac{1}{T^{2(2m-1)}} . \tag{44}$$

From Equation (44) we observe that Λ is a decreasing function of time and is always positive for $m > -0.5$ and $m < -1$. This behavior of cosmological constant Λ is clearly depicted in Figure 4. Recent cosmological observations suggest the existence of a positive cosmological constant Λ with the magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$. These observations on magnitude and redshift of type Ia supernovae suggest that our universe may be accelerating with induced cosmological density through the cosmological Λ-term. Thus, our model is consistent with the results of recent observations.

It is worth mentioning that for $m = 1$, from Equation (44) we find

$$\Lambda = \frac{1}{4}\frac{1}{T^2} . \tag{45}$$

This supports the views in favor of the dependence $\Lambda \propto T^{-2}$ first expressed by Bertolami (1986a,b) which was later observed by several authors (Abdel-Rahman 1990; Chen & Wu 1990; Berman 1990a,b; Berman & Som 1990; Pradhan & Kumar 2001). A relation like Equation (45) can also be found in Brans-Dicke theories when one supposes variable gravitational and cosmological "constant" (Peebles & Ratra 2003; Carmeli & Kuzmenko 2001; Gasperini 1987). We have derived the same variation of Λ with time in string viscous cosmology in this paper.

From the above results, it can be seen that the spatial volume is zero at $T = 0$ and it increases with the increase of T. This shows that the universe starts evolving with zero volume at $T = 0$ and expands with cosmic time T. In the derived model, the energy density ρ, particle density ρ_p, tension density λ and the cosmological constant Λ become zero as $T \to \infty$ and tend to infinity at $T = 0$. The model has a point-type singularity at $T = 0$ (MacCallum 1971). The expansion scalar and shear scalar both tend to zero as $T \to \infty$. The mean anisotropy parameter is uniform throughout the whole expansion of the universe when $m \neq -\frac{1}{2}$, but for $m = -\frac{1}{2}$ it tends to infinity. This shows that the universe is expanding with the increase of cosmic time but the rate of expansion and shear scalar decrease to zero and tend to be isotropic. Since $\frac{\sigma}{\theta}$ =constant provided $m \neq -\frac{1}{2}$, the model does not approach isotropy at any time. But for $m = 1$ our solution provides a totally isotropic universe.

3.2 Solutions in the Absence of Bulk Viscosity

In the absence of bulk viscosity, i.e. $\chi \to 0$ or $a \to 0$, the metric (26) reduces to

$$ds^2 = -\frac{dT^2}{bT^{-4m}} + T^2(dx + zdy)^2 + T^{2m}(dy^2 + dz^2) . \tag{46}$$

The pressure (p), the energy density (ρ), the string tension (λ), and the particle density (ρ_p) for the model (46) are given by

$$p = \rho = m(m+2)bT^{-2(1+2m)} - \frac{1}{4}\frac{1}{T^{2(2m-1)}} , \tag{47}$$

$$\lambda = -\frac{1}{T^{2(2m-1)}} , \tag{48}$$

$$\rho_p = m(m+2)bT^{-2(1+2m)} + \frac{3}{4}\frac{1}{T^{2(2m-1)}} . \tag{49}$$

From Equations (47)–(49) we observe that ρ, λ and ρ_p are decreasing functions of time and λ is always negative. The energy conditions, $\rho \geq 0$ and $\rho_p \geq 0$, are satisfied under
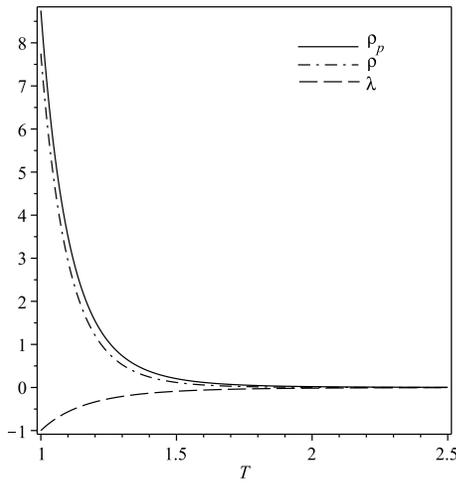
$$T \geq (4bm(m+2))^{\frac{1}{4}} , \tag{50}$$

Fig. 5 The plot of energy density ρ, particle density ρ_p and tension density λ versus T for $m = 2$ and $b = 1$.
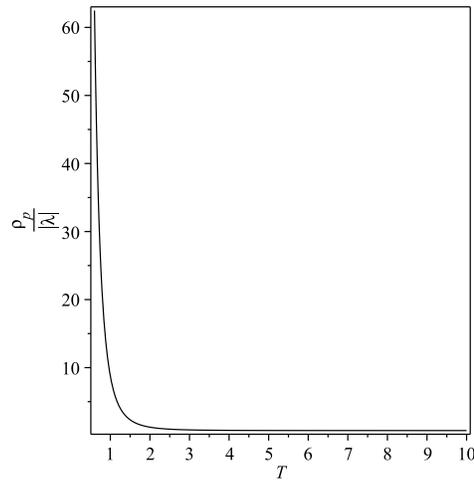
Fig. 6 The plot of $\frac{\rho_p}{|\lambda|}$ versus T for $m = 2$ and $b = 1$.

and

$$m > 0 \text{ and } m > -2 \text{ or } m < 0 \text{ and } m < -2 . \tag{51}$$

respectively. The behaviors of ρ, λ and ρ_p are clearly depicted in Figure 5 as a representative case with appropriate choice of constants of integration and other physical parameters using reasonably well-known situations.

From Equations (48) and (49) we obtain

$$\frac{\rho_p}{|\lambda|} = bm(m + 2)T^{-4} + 3/4 . \tag{52}$$

From Figure 6 and Equation (52) we observe that $\frac{\rho_p}{|\lambda|}$ is a decreasing function of time, i.e. as time goes on, the strings dominate over the particles, which contradicts the result obtained in the first case in the presence of bulk viscosity. This result is of course not consistent with astronomical observations, which predict that there is no direct evidence of strings in the present-day universe. Therefore, we conclude that the bulk viscosity may play an important role in the creation of particles from strings.

The expressions for the scalar of expansion θ, the average generalized Hubble's parameter H, magnitude of shear σ^2, proper volume V, deceleration parameter q and the average anisotropy parameter A_m for the model (46) are given by

$$\theta = 3H = (1 + 2m)\sqrt{b}T^{-(1+2m)} , \tag{53}$$

$$\sigma^2 = \frac{(m-1)^2}{3}bT^{-2(1+2m)} , \tag{54}$$

$$V = T^{2m+1} , \tag{55}$$

$$q = \frac{6}{2m+1} , \tag{56}$$

$$A_m = 2\left(\frac{1-m}{1+2m}\right)^2 . \tag{57}$$

From Equation (56) we observe that $q > 0$ if $m > -\frac{1}{2}$ and $q < 0$ if $m < -\frac{1}{2}$. But from Equation (55) we observe that $m < -\frac{1}{2}$ represents an accelerating collapsing universe with high blueshift. Since the recent observations (Riess et al. 2001) indicate that we live in an accelerating expanding universe with redshift, we conclude that in the absence of bulk viscosity, a universe with a decreasing rate of expansion is the only possible scenario.

4 DARK ENERGY INTERPRETATION OF THE MODELS

Figure 3 clearly shows that the presence of bulk viscosity in the cosmic fluid causes a decelerating to accelerating expansion of the universe. Also from Raychaudhuri's Equation (17), we observe that that bulk viscosity can play the role of an agent that drives the present acceleration of the universe.

In Eckart's theory (Eckart 1940) a viscous pressure is specified by

$$p^{\text{eff}} = p + \Pi . \tag{58}$$

Here $\Pi = -\xi\theta$ is the viscous pressure. Therefore in our models the effective pressure (stiff fluid plus viscous fluid) can be written as

$$p^{\text{eff}} = p - \chi = \frac{m(m+2)}{2}\left[\frac{\chi}{(2m^2 + 3m + 1)} + 2bT^{-2(1+2m)}\right] - \frac{1}{4}\frac{1}{T^{2(2m-1)}} - \chi . \tag{59}$$

Using Equations (28) and (59), the effective equation of state of the net fluid is obtained as

$$\omega^{\text{eff}} = \frac{p^{\text{eff}}}{\rho} = 1 - \frac{\chi}{\frac{m(m+2)}{2}\left[\frac{\chi}{(2m^2+3m+1)} + 2bT^{-2(1+2m)}\right] - \frac{1}{4}\frac{1}{T^{2(2m-1)}}} . \tag{60}$$

The behavior of effective equation of state, ω^{eff}, in terms of cosmic time T is shown in Figure 7. It is observed that the ω^{eff} parameter is a decreasing function of T and the rapidity of its decrease depends on the value of χ. We see that in the absence of bulk viscosity the models do not exhibit accelerating expansion (solid line), whereas in the presence of viscosity our models exhibit a decelerating to an accelerating expansion. From both Equation (60) and Figure 7 we observe that at the later stage of evolution, the effective equation of state tends to the same constant value, i.e. $\omega^{\text{eff}} = 1 - \frac{2(2m^2+3m+1)}{m(m+2)}$, independent of the value of χ.
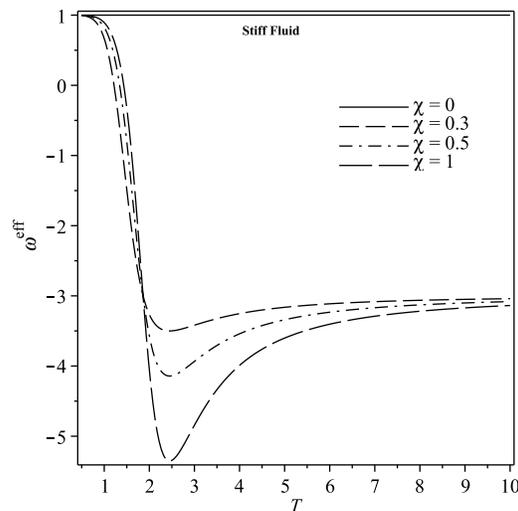

Fig. 7 The plot of effective equation of state ω^{eff} versus T for $m = 1$ and $b = 1$.

5 CONCLUDING REMARKS

In this paper we have presented a new exact solution of Einstein's field equations for LRS B-II space-time with a cloud of strings which is different from solutions presented by other authors. In general the models are expanding, shearing and non-rotating. It is found that in the presence of bulk viscosity, particles dominate over the strings at late time whereas in the absence of viscosity strings dominate over the particles, which is a contradictory result. On the other hand, from Raychaudhuri's Equation (17) we observe that bulk viscosity can play the role of an agent that drives the present acceleration of the universe. Hence we conclude that the bulk viscosity plays an important role in the evolution of the universe. For a universe which was decelerating in the past and is accelerating at the present time, the deceleration parameter must show signature flipping (see Padmanabhan & Choudhury 2003; Amendola 2003; Caldwell et al. 2006). Our models are evolving from an early decelerating phase to a late time accelerating phase (see Fig. 3) which is in good agreement with recent observations (Riess et al. 2001).

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