# Interacting two-fluid viscous dark energy models in a non-flat universe

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Abstract We study the evolution of the dark energy parameter within the scope of a spatially non-flat and isotropic Friedmann-Robertson-Walker model filled with barotropic fluid and bulk viscous stresses. We have obtained cosmological solutions that do not have a Big Rip singularity, and concluded that in both non-interacting and interacting cases the non-flat open Universe crosses the phantom region. We find that during the evolution of the Universe, the equation of state for dark energy  $\omega_D$  changes from  $\omega_D^{\text{eff}} > -1$  to  $\omega_D^{\text{eff}} < -1$ , which is consistent with recent observations.

Key words: cosmology: theory — dark energy — viscous fluid

## **1 INTRODUCTION**

Observations of distant Type Ia Supernovae (SNe Ia) (Perlmutter et al. 1997, 1998, 1999; Riess et al. 1998; Riess 2000; Garnavich et al. 1998a,b; Schmidt et al. 1998; Tonry et al. 2003; Clocchiatti et al. 2006), fluctuations of cosmic microwave background radiation (CMB) (de Bernardis et al. 2000; Hanany et al. 2000), large scale structure (LSS) (Spergel et al. 2003; Tegmark et al. 2004), the Sloan Digital Sky Survey (SDSS) (Seljak et al. 2005; Adelman-McCarthy et al. 2006), the Wilkinson Microwave Anisotropy Probe (WMAP) (Bennett et al. 2003) and the Chandra X-ray Observatory (Allen et al. 2004) by means of ground-based experiments and those at high altitude have established that our Universe is undergoing a late-time accelerated expansion, and we live in a spatially flat Universe composed of approximately 4% baryonic matter, 22% dark matter and 74% dark energy (DE). The simplest candidate for DE is the cosmological constant. Recently, a great number of themes have explored the current accelerating Universe, such as the scalar field model, an exotic equation of state (EoS), modified gravity, and the inhomogeneous cosmology model. There are several DE models which can be distinguished by, for instance, their EoS ( $\omega = \frac{p_{\rm de}}{\rho_{\rm de}}$ ) during the evolution of the Universe.

The introduction of viscosity into cosmology has been investigated from different viewpoints (Gron 1990; Padmanabhan & Chitre 1987; Barrow 1986; Zimdahl 1996; Adabi et al. 2012). Misner

(1968, 1967) noted that the "measurement of the isotropy of the cosmic background radiation represents the most accurate observational datum in cosmology." An explanation of this isotropy was provided by showing that in a large class of homogeneous but anisotropic Universes, the anisotropy rapidly decreases. It was found that the most important mechanism in reducing the anisotropy is neutrino viscosity at temperatures just above  $10^{10}$  K (when the Universe was about 1 s old, e.g. Zeldovich & Novikov 1971). The astrophysical observations also indicate some evidence that the cosmic medium is not a perfect fluid (Jaffe et al. 2005), and the viscosity effect could be involved in the evolution of the Universe (Brevik & Gorbunova 2005; Brevik et al. 2005; Cataldo et al. 2005). On the other hand, in the standard cosmological model, if the EoS parameter  $\omega$  is less than -1, a case called phantom cosmology, the Universe shows a future finite time singularity called the Big Rip (Caldwell et al. 2003; Nojiri et al. 2005) or Cosmic Doomsday. Several mechanisms are proposed to prevent a future Big Rip, like considering quantum effects terms in the action (Nojiri & Odintsov 2004; Elizalde et al. 2004), or including viscosity effects in the evolution of the Universe (Meng et al. 2007). A well known result of the Friedmann-Robertson-Walker (FRW) cosmological solutions, corresponding to Universes filled with perfect fluid and bulk viscous stresses, is the possibility of violating the dominant energy condition (Barrow 1986, 1988; Folomeev & Gurovich 2008; Ren & Meng 2006; Brevik & Gorbunova 2005; Nojiri & Odintsov 2005). Setare (2007a,b,c) and Setare & Saridakis (2009) have studied the interacting models of DE in a different context. New interacting agegraphic viscous DE with varying G has been studied by Sheykhi & Setare (2010).

Recently, Amirhashchi et al. (2011a,b), Pradhan et al. (2011), and Saha et al. (2012) have studied the two-fluid scenario for DE in an FRW Universe in a different context. Singh & Chaubey (2012) examined interacting DE in Bianchi type I space-time. Some experimental data implied that our Universe is not perfectly flat and recent papers (Spergel et al. 2003; Bennett et al. 2003; Ichikawa et al. 2006) have favored a Universe with spatial curvature. Setare et al. (2009) studied the tachyon cosmology in non-interacting and interacting cases in a non-flat FRW Universe. Due to these considerations and motivations, in this paper, we study the evolution of the DE parameter within the framework of an FRW open cosmological model filled with two fluids (i.e. barotropic fluid and bulk viscous stresses). In doing so we consider both interacting and non-interacting cases. The outline of the paper is as follows: in Section 2, the metric and the field equations are described. Section 3 deals with the non-interacting two-fluid model and its physical significance. Section 4 covers the interacting two-fluid model and its physical significance. Section 4 covers the interacting two-fluid model and its physical significance.

### **2** THE METRIC AND FIELD EQUATIONS

We consider the spherically symmetric FRW metric given by

$$ds^{2} = -dt^{2} + a^{2}(t) \left[ \frac{dr^{2}}{1 - kr^{2}} + r^{2}(d\theta^{2} + \sin^{2}\theta d\phi^{2}) \right],$$
(1)

where a(t) is the scale factor and the curvature constant k is -1, 0, +1 respectively for open, flat and closed models of the Universe.

Einstein's field equations (with  $8\pi G = 1$  and c = 1) read as

$$R_i^j - \frac{1}{2}R\delta_i^j = -T_i^j, \qquad (2)$$

where the symbols have their usual meaning and  $T_i^j$  is the two-fluid energy-momentum tensor due to bulk viscous dark and barotropic fluids written in the form

$$T_{i}^{j} = (\rho + \bar{p})u_{i}^{j} + \bar{p}g_{i}^{j}, \qquad (3)$$

where

$$\bar{p} = p - \xi u_{;i}^i \tag{4}$$

and

$$u^{i}u_{i} = -1, \qquad (5)$$

where  $\rho$  is the energy density; p, the pressure;  $\xi$ , the bulk viscosity coefficient; and  $u^i$ , the fourvelocity vector of the distribution. Hereafter a semi-colon denotes covariant differentiation.

The expansion factor  $\theta$  is defined by  $\theta = u_{i}^i = 3\frac{\dot{a}}{a}$ . Hence Equation (4) leads to

$$\bar{p} = p - 3\xi H \,, \tag{6}$$

where H is Hubble's constant defined by

$$H = \frac{\dot{a}}{a} \,. \tag{7}$$

Now with the aid of Equations (3)–(5) and metric (1), the surviving field equations (2) take the explicit forms

$$\rho = 3\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right),\tag{8}$$

and

$$\bar{p} = -\left(\frac{\dot{a}^2}{a^2} + 2\frac{\ddot{a}}{a} + \frac{k}{a^2}\right). \tag{9}$$

Also in space-time (1) the Bianchi identity for the bulk-viscous fluid distribution  $G_{ij}^{;j} = 0$  leads to  $T_{ij}^{;j} = 0$  which yields

$$\rho u^i + (\rho + \bar{p}) u^i_{;i} \tag{10}$$

which gives

$$\dot{\rho} + 3H(\rho + \bar{p}) = 0.$$
 (11)

Using Equation (7) in Equations (8) and (9) we get

$$\rho = \left(\frac{3k}{A^2}\mathrm{e}^{-2Ht} + 3H^2\right)\,,\tag{12}$$

and

$$\bar{p} = -\left(\frac{k}{A^2}\mathrm{e}^{-2Ht} + 3H^2\right)\,,\tag{13}$$

where  $\bar{p} = p_m + \bar{p}_D$  and  $\rho = \rho_m + \rho_D$ . Here  $p_m$  and  $\rho_m$  are pressure and energy density of barotropic fluid and  $p_D$  and  $\rho_D$  are pressure and energy density of dark fluid respectively.

The EoS for the barotropic fluid  $\omega_m$  and dark field  $\omega_D$  are given by

$$\omega_m = \frac{p_m}{\rho_m},\tag{14}$$

and

$$\omega_{\rm D} = \frac{\bar{p}_{\rm D}}{\rho_{\rm D}}\,,\tag{15}$$

respectively.

From Equations (11)–(13) we obtain

$$\frac{\dot{\rho}}{3H} = \frac{2k}{a^2} \mathrm{e}^{-2Ht} \,. \tag{16}$$

Now we assume

$$\rho = \alpha \theta^2 \text{ or } \rho = 9\alpha H^2 \,, \tag{17}$$

where  $\alpha$  is an arbitrary constant. Equation (17) ensures that our Universe approaches homogeneity (Collins 1977). This condition has also been used by Banerjee et al. (1986) for deriving a viscous-fluid cosmological model with Bianchi type II space-time.

Putting Equation (17) in Equation (16) and after integrating we get

$$e^{-2Ht} = -\frac{3\alpha A^2}{2kt^2},$$
 (18)

which yields

$$H = \frac{1}{2t} \ln \left( -\frac{2kt^2}{3\alpha A^2} \right) \,, \tag{19}$$

where A is an arbitrary constant. From Equation (19), we observe that the condition given by Equation (17) restricts our study to the case when k = -1 (i.e. only for an open Universe). In the following sections we deal with two cases, (i) a non-interacting two-fluid model and (ii) an interacting two-fluid model.

### **3 NON-INTERACTING TWO-FLUID MODEL**

In this section we assume that two fluids do not interact with each other. Therefore, the general form of conservation Equation (11) leads us to write the conservation equation for the dark and barotropic fluid separately as,

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}(\rho_m + p_m) = 0, \qquad (20)$$

and

$$\dot{\rho}_{\rm D} + 3\frac{\dot{a}}{a}\left(\rho_{\rm D} + \bar{p}_{\rm D}\right) = 0.$$
 (21)

Integrating Equation (20) and using (7) leads to

$$\rho_m = \rho_0 a^{-3(1+\omega_m)} \quad \text{or} \quad \rho_m = \rho_0 B e^{-3H(1+\omega_m)t},$$
(22)

where  $\rho_0$  is a constant of integration and  $B = A^{-3(1+\omega_m)}$ . By using Equation (22) in Equations (12) and (13), we first obtain the  $\rho_D$  and  $p_D$  in terms of Hubble's constant H as

$$\rho_{\rm D} = \left(\frac{3k}{A^2} \mathrm{e}^{-2Ht} + 3H^2\right) - \rho_0 B \mathrm{e}^{-3H(1+\omega_m)t} \,, \tag{23}$$

and

$$\bar{p}_{\rm D} = \left(\frac{k}{A^2} e^{-2Ht} + 3H^2\right) - \omega_m \rho_0 B e^{-3H(1+\omega_m)t} \,. \tag{24}$$

respectively. By using Equations (23) and (24) in Equation (15), we can find the EoS of DE in terms of time is

$$\omega_{\rm D} = -\frac{\left(\frac{k}{A^2} \mathrm{e}^{-2Ht} + 3H^2\right) + \omega_m \rho_0 B \mathrm{e}^{-3H(1+\omega_m)t}}{\left(\frac{3k}{A^2} \mathrm{e}^{-2Ht} + 3H^2\right) - \rho_0 B \mathrm{e}^{-3H(1+\omega_m)t}}.$$
(25)

Therefore the effective EoS parameter for viscous DE can be written as

$$\omega_{\rm D}^{\rm eff} = \omega_{\rm D} - \frac{3\xi H}{\rho_{\rm D}} = -\frac{\left(\frac{k}{A^2} \mathrm{e}^{-2Ht} + 3H^2\right) + 3\xi H + \omega_m \rho_0 B \mathrm{e}^{-3H(1+\omega_m)t}}{\left(\frac{3k}{A^2} \mathrm{e}^{-2Ht} + 3H^2\right) - \rho_0 B \mathrm{e}^{-3H(1+\omega_m)t}} \,. \tag{26}$$

The expressions for the matter-energy density  $\Omega_m$  and DE density  $\Omega_D$  are given by

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{4t^2 \rho_0 B e^{-\frac{3}{2} \ln(\frac{2t^2}{3\alpha A^2})(1+\omega_m)}}{3\ln^2(\frac{2t^2}{3\alpha A^2})},$$
(27)



Fig.1 The plot of  $\rho_D$  versus t for  $\alpha = 0.1$ , A = 100 and  $\omega_m = 0.5$  in both the non-interacting and interacting two-fluid models.

and

$$\Omega_{\rm D} = \frac{\rho_{\rm D}}{3H^2} = -\frac{6\alpha}{\ln^2(\frac{2t^2}{3\alpha A^2})} + 1 - \frac{4t^2\rho_0 B \mathrm{e}^{-\frac{3}{2}\ln(\frac{2t^2}{3\alpha A^2})(1+\omega_m)}}{3\ln^2(\frac{2t^2}{3\alpha A^2})}, \qquad (28)$$

respectively. Adding Equations (27) and (28), we obtain

$$\Omega = \Omega_m + \Omega_D = -\frac{6\alpha}{\ln^2(\frac{2t^2}{3\alpha A^2})} + 1.$$
<sup>(29)</sup>

From the right hand side of Equation (29), it is clear that for an open Universe,  $\Omega < 1$  but at late times we see that  $\Omega \rightarrow 1$ , i.e. the flat Universe scenario. This result is also compatible with the observational results. Since our model predicts a flat Universe for large times and the present-day Universe is very close to being flat, the derived model is thus compatible with observational results.

Figure 1 depicts the energy density of DE ( $\rho_D$ ) versus t. From this figure, in both non-interacting and interacting cases, we observe that  $\rho_D$  is a decreasing function of time which approaches a small positive value at late times and never goes to infinity. Thus, in both cases the Universe is free from the Big Rip.

The behavior of EoS for DE in terms of cosmic time t is shown in Figure 2. It is observed that for an open Universe, the  $\omega_{\rm D}^{\rm eff}$  is a decreasing function of time, and the rapidity of its decrease at the early stage depends on a large value of the bulk viscosity coefficient. The EoS parameter of the DE begins in the non-dark ( $\omega_{\rm D} > -\frac{1}{3}$ ) region at the early stage and crosses the phantom divide or the cosmological constant ( $\omega_{\rm D} = -1$ ) region and then passes over into the phantom ( $\omega_{\rm D} < -1$ ) region. The property of DE is a violation of the null energy condition (NEC) since the DE crosses the Phantom Divide Line (PDL), in particular depending on the direction (Rodrigues 2008; Kumar & Yadav 2011; Pradhan & Amirhashchi 2011). In theory, despite the observational constraints, extensions of general relativity are a prime candidate for theories consistent with PDL crossing (Nesseris & Perivolaropoulos 2007). On the other hand, while the current cosmological data from SNe Ia (Supernova Legacy Survey, gold sample of Hubble Space Telescope) (Riess et al. 2004; Astier et al. 2006). CMB (WMAP, BOOMERANG) (Komatsu et al. 2009; MacTavish et al.



Fig. 2 The plot of EoS parameter  $\omega_{\rm D}^{\rm eff}$  versus t for  $\rho_0 = 10, \omega_m = 0.5, \alpha = 0.01$  and B = 1 in the non-interacting two-fluid model.



Fig. 3 The plot of density parameter ( $\Omega$ ) versus t for A = 1 and  $\alpha = 0.01$  in the non-interacting two-fluid model.

2006) and LSS (SDSS) (Eisenstein et al. 2005) rule out  $\omega_D \ll -1$ , they mildly favor dynamically evolving DE crossing the PDL (see Rodrigues 2008; Kumar & Yadav 2011; Pradhan & Amirhashchi 2011; Nesseris & Perivolaropoulos 2007; Zhao et al. 2007; Copeland et al. 2006) for the theoretical and observational status of crossing the PDL. Thus our DE model is in good agreement with well established theoretical results as well as the recent observations. From Figure 2, it is observed that

in the absence of viscosity (i.e. for  $\xi = 0$ ), the Universe does not cross the PDL but approaches a cosmological constant ( $\omega_D = -1$ ) scenario. Thus, it is clear that viscosity impacts the evolution of the Universe.

The variation of density parameter  $(\Omega)$  with cosmic time t for an open Universe has been shown in Figure 3. From the figure, it can be seen that in an open Universe,  $\Omega$  is an increasing function of time and at late time, it approaches the scenario of a flat Universe.

#### **4 INTERACTING TWO-FLUID MODEL**

In this section we consider the interaction between dark viscous and barotropic fluids. For this purpose we can write the continuity equations for barotropic and dark viscous fluids as

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}(\rho_m + p_m) = Q,$$
(30)

and

$$\dot{\rho}_{\rm D} + 3\frac{\dot{a}}{a}(\rho_{\rm D} + \bar{p}_{\rm D}) = -Q,$$
(31)

where the quantity Q expresses the interaction between the dark components. Since we are interested in an energy transfer from DE to dark matter, we consider Q > 0 which ensures that the second law of thermodynamics is fulfilled (Pavón & Wang 2009). Here we emphasize that the continuity Equations (11) and (30) imply that the interaction term (Q) should be proportional to a quantity with units of inverse of time, i.e.  $Q \propto \frac{1}{t}$ . Therefore, a first and natural candidate can be the Hubble factor H multiplied by the energy density. Following Amendola et al. (2007) and Guo et al. (2007), we consider

$$Q = 3H\sigma\rho_m\,,\tag{32}$$

where  $\sigma$  is a coupling constant. Using Equation (32) in Equation (30) and after integrating, we obtain

$$\rho_m = \rho_0 a^{-3(1+\omega_m - \sigma)} \text{ or } \rho_m = \rho_0 B e^{-3H(1+\omega_m - \sigma)t}.$$
(33)

By using Equation (33) in Equations (12) and (13), we again obtain the  $\rho_D$  and  $p_D$  in terms of Hubble's constant H as

$$\rho_{\rm D} = \left(\frac{3k}{A^2} {\rm e}^{-2Ht} + 3H^2\right) - \rho_0 B {\rm e}^{-3H(1+\omega_m-\sigma)t} , \qquad (34)$$

and

$$\bar{p}_{\rm D} = \left(\frac{k}{A^2} {\rm e}^{-2Ht} + 3H^2\right) - (\omega_m - \sigma)\rho_0 B {\rm e}^{-3H(1+\omega_m - \sigma)t}, \qquad (35)$$

respectively. By using Equations (34) and (35) in Equation (15), we can find the EoS of DE in terms of time as

$$\omega_{\rm D} = -\frac{\left(\frac{k}{A^2} \mathrm{e}^{-2Ht} + 3H^2\right) + (\omega_m - \sigma)\rho_0 B \mathrm{e}^{-3H(1+\omega_m - \sigma)t}}{\left(\frac{3k}{A^2} \mathrm{e}^{-2Ht} + 3H^2\right) - \rho_0 B \mathrm{e}^{-3H(1+\omega_m - \sigma)t}} \,.$$
(36)

Again we can write the effective EoS parameter of viscous DE as

$$\omega_{\rm D}^{\rm eff} = -\frac{\left(\frac{k}{A^2} \mathrm{e}^{-2Ht} + 3H^2\right) - 3\xi H + (\omega_m - \sigma)\rho_0 B \mathrm{e}^{-3H(1+\omega_m - \sigma)t}}{\left(\frac{3k}{A^2} \mathrm{e}^{-2Ht} + 3H^2\right) - \rho_0 B \mathrm{e}^{-3H(1+\omega_m - \sigma)t}} \,. \tag{37}$$

The expressions for the matter-energy density  $\Omega_m$  and DE density  $\Omega_{\rm D}$  are given by

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{4t^2 \rho_0 B \mathrm{e}^{-\frac{3}{2} \ln(\frac{2t^2}{3\alpha A^2})(1+\omega_m - \sigma)}}{3\ln^2(\frac{2t^2}{3\alpha A^2})},$$
(38)



Fig.4 The plot of EoS parameter  $\omega_{\rm D}^{\rm eff}$  versus t for  $\rho_0 = 10, \omega_m = 0.5, \alpha = 0.01, B = 1$  and  $\sigma = 0.3$  in the interacting two-fluid model.

and

$$\Omega_{\rm D} = \frac{\rho_{\rm D}}{3H^2} = -\frac{6\alpha}{\ln^2(\frac{2t^2}{3\alpha A^2})} + 1 - \frac{4t^2\rho_0 B e^{-\frac{3}{2}\ln(\frac{2t^2}{3\alpha A^2})(1+\omega_m-\sigma)}}{3\ln^2(\frac{2t^2}{3\alpha A^2})},$$
(39)

respectively. Adding Equations (38) and (39), we obtain

$$\Omega = \Omega_m + \Omega_D = -\frac{6\alpha}{\ln^2(\frac{2t^2}{3\alpha A^2})} + 1, \qquad (40)$$

which is the same expression as in the previous case of two non-interacting fluids. Figure 4 shows a plot of EoS parameter ( $\omega_{\rm D}^{\rm eff}$ ) versus t. The characteristic of  $\omega_{\rm D}^{\rm eff}$  in this case is the same as in the previous case.

## **5** CONCLUSIONS

In this paper, we have studied the evolution of the DE parameter within the framework of an open FRW space-time filled with barotropic and bulk viscous dark fluid. In both non-interacting and interacting cases, we have observed that for all values of the bulk viscosity coefficient, the Universe has transitioned from the non-dark region ( $\omega_{\rm D}^{\rm eff} > -\frac{1}{3}$ ) to the phantom region ( $\omega_{\rm D}^{\rm eff} < -1$ ). In summary, we have investigated the possibility of constructing two-fluid DE models which have the EoS ( $\omega_D^{eff}$ ) crossing -1 by using the two fluids (barotropic and bulk viscous dark fluid). Therefore, the two-fluid scenario discussed in the present paper is a viable candidate for DE. It is also worth mentioning here that in both interacting and non-interacting cases, our models are free from the Big Rip.

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