Accelerating dark energy models with anisotropic fluid in Bianchi type VI$_0$ space-time

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Abstract Motivated by the increasing evidence for the need of a geometry that resembles Bianchi morphology to explain the observed anisotropy in the *WMAP* data, we have discussed some features of Bianchi type VI$_0$ universes in the presence of a fluid that has an anisotropic equation of state (EoS) parameter in general relativity. We present two accelerating dark energy (DE) models with an anisotropic fluid in Bianchi type VI$_0$ space-time. To ensure a deterministic solution, we choose the scale factor $a(t) = \sqrt{t^n e^t}$, which yields a time-dependent deceleration parameter, representing a class of models which generate a transition of the universe from the early decelerating phase to the recent accelerating phase. Under suitable conditions, the anisotropic models approach an isotropic scenario. The EoS for DE ω is found to be time-dependent and its existing range for derived models is in good agreement with data from recent observations of type Ia supernovae (SNe Ia) (Knop et al. 2003), SNe Ia data combined with cosmic microwave background (CMB) anisotropy and galaxy clustering statistics (Tegmark et al. 2004a), as well as the latest combination of cosmological datasets coming from CMB anisotropies, luminosity distances of high redshift SNe Ia and galaxy clustering. For different values of n, we can generate a class of physically viable DE models. The cosmological constant Λ is found to be a positive decreasing function of time and it approaches a small positive value at late time (i.e. the present epoch), which is corroborated by results from recent SN Ia observations. We also observe that our solutions are stable. The physical and geometric aspects of both models are also discussed in detail.

Key words: cosmological models — dark energy — variable EoS parameter

1 INTRODUCTION

Recent cosmological observations obtained by type Ia supernovae (SNe Ia) (Riess et al. 1998; Perlmutter et al. 1999) suggested that the expansion of the universe is accelerating. Recent observations of SNe Ia with a high level of confidence (Tonry et al. 2003; Riess et al. 2004; Clocchiatti et al. 2006) have further confirmed this. In addition, measurements of the cosmic microwave background (CMB) anisotropies (Bennett et al. 2003; de Bernardis et al. 2000; Hanany et al. 2000), large scale structure (Tegmark et al. 2004a,b; Spergel et al. 2003), the Sloan Digital Sky Survey (SDSS) (Seljak et al. 2005; Adelman-McCarthy et al. 2006), the Wilkinson Microwave Anisotropy Probe (*WMAP*)

(Knop et al. 2003) and the Chandra X-ray Observatory (Allen et al. 2004) strongly indicate that our universe is dominated by a component with negative pressure, dubbed dark energy (DE), which constitutes $\simeq 3/4$ of the critical density. The cosmic acceleration arises from negative pressure and positive energy density that violate the strong energy condition. This violation gives a reverse gravitational effect. Due to this effect, the universe experiences a "cosmic jerk" and the transition from the earlier deceleration phase to the more recent acceleration phase takes place (Caldwell et al. 2006). A recent survey of more than $200\,000$ galaxies appears to confirm the existence of DE, although the exact physics behind it remains unknown (Rincon 2011).

During the last two decades, cosmology has quickly become an experimental part of physics. The theoretical models can be tested, and new and more accurate data in the near future will constrain our models of the universe to within an accuracy of a few percent. The simplest candidate for DE is the cosmological constant (Overduin & Cooperstock 1998; Sahni & Starobinsky 2000; Komatsu et al. 2009; Kachru et al. 2003), which suffers from conceptual problems such as fine-tuning and coincidence issues (Weinberg 1989). Other scenarios include quintessence (Wetterich 1988; Ratra & Peebles 1988), chameleon (Khoury & Weltman 2004), the k-essence (Chiba et al. 2000; Armendariz-Picon et al. 2000), which is based on earlier work of K-inflation (Armendáriz-Picón et al. 1999), modified gravity (Capozziello & Fang 2002; Carroll et al. 2004; Nojiri & Odintsov 2003, 2004; Abdalla et al. 2005; Rami El-Nabulsi 2011a), tachyons (Padmanabhan 2002) arising in string theory (Sen 2002), quintessential inflation (Peebles & Vilenkin 1999), Chaplygin gas as well as generalized Chaplygin gas (Srivastava 2005; Bertolami et al. 2004; Bento et al. 2002; Bilić et al. 2002; Avelino et al. 2003), and cosmological nuclear energy (Gupta & Pradhan 2010). Recently, Rami El-Nabulsi (2011b), Feng & Yang (2011), Biesiada et al. (2011), Singh & Chaubey (2012), Amirhashchi et al. (2011a) and Pradhan et al. (2011a) have studied DE models in different contexts. In spite of these attempts, cosmic acceleration is still a challenge in modern cosmology and astrophysics.

In general relativity, the evolution of the expansion rate is parameterized by the cosmological equation of state (EoS, the relationship between temperature, pressure, combined matter, energy, and vacuum energy density for any region of space). Measuring the EoS for DE is one of the biggest efforts in observational cosmology today. The DE model has been characterized in a conventional manner by the EoS parameter $\omega(t) = \frac{p}{\rho}$, which is not necessarily constant, where ρ is the energy density and p is the fluid pressure (Carroll et al. 2003). The present data seem to slightly favor an evolving DE with EoS $\omega < -1$ around the present epoch and $\omega > -1$ in the recent past. Obviously, ω cannot cross -1 for quintessence or phantoms alone. Some efforts have been made to build a DE model whose EoS can cross the phantom divide. The simplest DE candidate is vacuum energy ($\omega = -1$), which is mathematically equivalent to the cosmological constant (Λ). The other conventional alternatives, which can be described by minimally coupled scalar fields, are quintessence ($\omega > -1$) (Steinhardt & Wesley 2009), phantom energy ($\omega < -1$) (Caldwell 2002), and quintoms (that can cross from a phantom region to a quintessence region as they evolve) and have a time-dependent EoS parameter. Some other limits, obtained from observational results that come from SNe Ia data (Knop et al. 2003) and a combination of SNe Ia data with CMB anisotropy and galaxy clustering statistics (Tegmark et al. 2004a,b), are $-1.67 < \omega < -0.62$ and $-1.33 < \omega < -0.79$, respectively. The latest results in 2009, obtained after the combination of cosmological datasets coming from CMB anisotropies, luminosity distances of high redshift SNe Ia, and galaxy clustering, constrain the DE EoS to $-1.44 < \omega < -0.92$ at the 68% confidence level (Hinshaw et al. 2009; Komatsu et al. 2009). However, it is not at all obligatory to use a constant value of ω. Due to a lack of observational evidence in making a distinction between constant and variable ω, usually the EoS parameter is considered as a constant (Kujat et al. 2002; Bartelmann et al. 2005; Yadav 2011) with phase wise values $-1, 0, -\frac{1}{3}$ and $+1$ for a universe dominated by vacuum fluid, dust fluid, radiation and a stiff universe, respectively. However, in general, ω is a function of time, redshift z or scale factor a as well (Ratra & Peebles 1988; Jimenez 2003; Das et al. 2005). In earlier studies, various forms of time dependent ω have been used for variable Λ models by Mukhopadhyay et al. (2008).

Recently, DE models with a variable EoS parameter have been studied by Ray et al. (2011), Akarsu & Kılınç (2010a,b), Yadav et al. (2011), Yadav & Yadav (2011), Pradhan et al. (2011a), Pradhan et al. (2011b), Amirhashchi et al. (2011a,b) and Saha & Yadav (2012). In well-known reviews on modified gravity (Nojiri & Odintsov 2007, 2011), it is clearly indicated that any modified gravity may be represented as effective fluid with time dependent ω. The DE universe's EoS with an inhomogeneous, Hubble parameter dependent term is considered by Nojiri & Odintsov (2005). Later, Nojiri & Odintsov (2006) also presented the late-time cosmological consequences of DE with a time-dependent periodic EoS in an oscillating universe.

Today there is considerable evidence suggesting that the universe may be isotropic and homogeneous. After the discovery of CMB radiation, cosmology became a precise science. CMB radiation is also considered to be the major experimental evidence on which the most commonly accepted theory about the origin of the universe, i.e. "Big-Bang" cosmology, is based. Statistical Isotropy (SI) is usually assumed in almost all CMB studies. However, now, there exist many indications which suggest that CMB may violate this assumption. Apart from CMB there are some other indications of violation of SI which suggest the existence of a preferred direction in the universe. These indications include distributions of polarizations from radio galaxies (Birch 1982; Jain & Ralston 1999; Jain et al. 2004) and statistics of optical polarizations from quasars (Hutsemekers 1998; Hutsemékers & Lamy 2001; Jain et al. 2004; Ralston & Jain 2004). The polarization of electromagnetic waves coming from distant radio galaxies and quasars measured at radio and optical frequencies, respectively, are not consistent with the assumptions of SI; rather radio polarizations are organized coherently over the dome of the sky and optical polarizations are aligned in a preferential direction on very large scales, violating the assumed isotropy of the universe. These studies confirmed the strong significance of anisotropy and also claimed that the statistics are not consistent with isotropy at the 99.9% confidence level. It has also been observed that the quadrupole and the octopole have almost all their power perpendicular to a common axis in space pointing towards the Virgo cluster (Tegmark et al. 2003; de Oliveira-Costa & Tegmark 2006). The dipole, which is commonly attributed to our motion relative to the CMB rest frame, also aligns in the same direction as the quadrupole and the octopole, which is not expected under the condition of statistical isotropy. Another indication of anisotropy in CMB data is the presence of a cold spot with an improbably low temperature. It was found by Cruz et al. (2005) by using spherical Mexican hat wavelet analysis on *WMAP* data. Several authors have also searched for anisotropy using the SNe Ia data set. Jain et al. (2007) found violation of isotropy in this data. Subsequently, there have been a large number of studies (Bielewicz et al. 2004; Eriksen et al. 2004; Katz & Weeks 2004; Bielewicz et al. 2005; Prunet et al. 2005; Bernui et al. 2006; de Oliveira-Costa & Tegmark 2006; Freeman et al. 2006; Bernui et al. 2007; Land & Magueijo 2007) which claim the CMB is not consistent with isotropy. The possible violation of SI in the CMB has led to many theoretical studies. Several physical explanations for the observed anisotropy have been put forward (Cline et al. 2003; Contaldi et al. 2003; Kesden et al. 2003; Armendáriz-Picón 2004; Berera et al. 2004; Gordon et al. 2005; Abramo et al. 2006; Campanelli et al. 2007; Rodrigues 2008). Land & Magueijo (2005) found evidence that the detected anisotropy has positive mirror parity. The generation and evolution of primordial perturbations in an anisotropic universe have also been studied (Armendariz-Picon 2006; Battye & Moss 2006; Pereira et al. 2007) along with the possibility of anisotropic inflation (Hunt & Sarkar 2004; Buniy et al. 2006; Donoghue et al. 2009).

The possible violation of global isotropy in the CMB has been a subject of intense research after the publication of *WMAP* data. The possible alignment of axes corresponding to several diverse data sets in the direction of the Virgo cluster makes this extremely interesting. In recent years, there have been a large number of studies, which claim that the CMB temperature fluctuations are not consistent with statistical isotropy and thus question the cosmological principle. The CMB is considered to be major experimental evidence supporting the current/present models of the observed universe and, from these CMB observations, several people have found significant anisotropic scenarios. Based on these studies one may not preclude the possibility that our universe is anisotropic.

There is general agreement among cosmologists that CMB anisotropy on the small angle scale holds the key to the formation of discrete structure. The theoretical argument (Misner 1968) and modern experimental data support the existence of an anisotropic phase, which turns into an isotropic one. The anisotropy of DE within the framework of Bianchi type space-times is found to be useful in generating arbitrary ellipsoidality of the universe, and to fine tune the observed CMB anisotropies. Koivisto & Mota (2008a,b) have investigated cosmological models with the anisotropic EoS and have also shown that the present SNe Ia data allow large anisotropy. Recently, Akarsu & Kılınç (2010c) have described some features of the Bianchi type I universes in the presence of fluid that has an anisotropic EoS. Hence, for a realistic cosmological model one should consider spatially homogeneous and anisotropic space-times and then show whether they can evolve to the observed amount of homogeneity and isotropy. The only spatially homogeneous but anisotropic models other than Bianchi-type models are the Kantowski-Sachs locally symmetric family. See Ellis & van Elst (1999) for generalized, particularly anisotropic, cosmological models and Ellis (2006) for a concise review on Bianchi type models. The motivation for this investigation comes from the hints of statistical anisotropy in our universe that several observations seem to suggest.

Bianchi type VI_0 (B-VI_0) space-time, in connection with massive strings, is studied by Pradhan & Bali (2008) and Bali et al. (2008). Belinchón (2009) studied several cosmological models with B-VI_0 & III symmetries under the self similar approach. Given the growing interest of cosmologists, here, we propose to study the evolution of the universe within the framework of a B-VI_0 space-time. Recently, Amirhashchi et al. (2011c) and Pradhan et al. (2012) presented DE models in an anisotropic B-VI_0 space-time by considering constant and variable deceleration parameters (DPs) respectively. In this paper, we have investigated two new B-VI_0 DE models with variable ω by assuming different scale factors in such a way that they provide a time dependent DP in the presence of anisotropic fluid. The outline of the paper is as follows: in Section 2, the metric and the field equations are described. Section 3 deals with the solutions of the field equations. Section 4 covers physical and geometric behavior of the model. Section 5 addresses the stability of the corresponding solutions. In Section 6, we describe another DE model and its physical aspects. In Section 7, we again examine the stability of corresponding solutions for the second DE model. Finally, conclusions are summarized in Section 8.

2 THE METRIC AND FIELD EQUATIONS

We consider a totally anisotropic Bianchi type VI_0 line element, given by

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2x} dy^2 + C^2 e^{-2x} dz^2, \tag{1}$$

where the metric potentials A, B and C are functions of t alone. This ensures that the model is spatially homogeneous.

The simplest generalization of the EoS parameter of perfect fluid may be to determine the EoS parameter separately along each spatial axis by preserving the diagonal form of the energy momentum tensor in a consistent way with the considered metric. Therefore, the energy momentum tensor of fluid can be written, most generally, in an anisotropic diagonal form as follows

$$T_i^j = \text{diag}[T_0^0, T_1^1, T_2^2, T_3^3]. \tag{2}$$

Allowing for anisotropy in the pressure of the fluid, and thus in its EoS parameter, gives rise to new possibilities for the evolution of the energy source. To see this, we first parametrize the energy momentum tensor given in (2) as follows:

$$\begin{aligned}
T_i^j &= \text{diag}[\rho, -p_x, -p_y, -p_z] \\
&= \text{diag}[1, -\omega_x, -\omega_y, -\omega_z]\rho \\
&= \text{diag}[1, -\omega, -(\omega + \delta), -(\omega + \gamma)]\rho.
\end{aligned} \tag{3}$$

Here ρ is the proper energy density, p_x, p_y and p_z are the pressures, and ω_x, ω_y and ω_z are the directional EoS parameters along the x, y and z axes, respectively; ω is the deviation-free EoS parameter of the fluid. The deviation from isotropy is parametrized by setting $\omega_x = \omega$ and then introducing skewness parameters δ and γ which are the deviations from ω, respectively along the y and z axes. ω, δ and γ are not necessarily constants and might be functions of the cosmic time, t.

Einstein's field equations (with gravitational units, $8\pi G = 1$ and $c = 1$) read as

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j,\tag{4}$$

where the symbols have their usual meaning. In a comoving co-ordinate system, Einstein's field equation (4), with (3) for the B-VI$_0$ metric (1) subsequently leads to the following system of equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} = -\omega\rho,\tag{5}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} = -(\omega + \delta)\rho,\tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -(\omega + \gamma)\rho,\tag{7}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} = \rho,\tag{8}$$

$$\frac{\dot{C}}{C} - \frac{\dot{B}}{B} = 0.\tag{9}$$

Here and in what follows, an overhead dot denotes ordinary differentiation with respect to t.

The spatial volume for the model (1) is given by

$$V^3 = ABC.\tag{10}$$

We define $a = (ABC)^{\frac{1}{3}}$ as the average scale factor so that Hubble's parameter is anisotropic and may be defined as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right).\tag{11}$$

The DP q, the scalar expansion θ, the shear scalar σ^2, and the average anisotropy parameter A_m are defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2},\tag{12}$$

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C},\tag{13}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right),\tag{14}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2,\tag{15}$$

where $\Delta H_i = H_i - H (i = x, y, z)$.

3 SOLUTIONS OF THE FIELD EQUATIONS

Integrating Equation (9), we obtain

$$C = \ell B, \tag{16}$$

where ℓ is a constant of integration. Now if we put the value of Equation (16) in (7) and subtract the result from Equation (6), we obtain that the skewness parameters along the y and z axes are equal, i.e. $\delta = \gamma$.

Therefore, Equations (5)–(9) are reduced to

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{A^2} = -\omega\rho, \tag{17}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -(\omega + \gamma)\rho, \tag{18}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} = \rho. \tag{19}$$

The field Equations (17)–(19) are a system of three linearly independent equations with five unknown parameters A, B, ω, ρ and γ. Two additional constraints relating these parameters are required to obtain explicit solutions of the system.

In literature, it is common to use a constant DP (Akarsu & Kılınç 2010a,b; Amirhashchi et al. 2011c; Pradhan et al. 2011b; Kumar & Yadav 2011; Yadav 2011), as it duly gives a power law for the metric function or corresponding quantity. The motivation to choose such a time dependent DP is behind the fact that the universe shows accelerated expansion at present as observed in recent observations of SNe Ia (Riess et al. 1998; Perlmutter et al. 1999; Tonry et al. 2003; Riess et al. 2004; Clocchiatti et al. 2006) and CMB anisotropies (Bennett et al. 2003; de Bernardis et al. 2000; Hanany et al. 2000), but there was decelerated expansion in the past. Also, the transition redshift from decelerated expansion to accelerated expansion is about 0.5. Now for a universe which was decelerating in the past and is accelerating at the present time, the DP must show signature flipping (see the Refs. Padmanabhan & Choudhury 2003; Amendola 2003; Riess et al. 2001). So, in general, the DP is not a constant but rather is variable in time. This motivates us to choose a scale factor which yields a time-dependent DP. At this juncture, it should be stated that some authors first choose the scale factors as a power law, exponential or in another form, and then calculate other variables with some conditions under these solutions.

In this paper, following Saha et al. (2012) and Pradhan & Amirhashchi (2011), we take the following *ansatz* for the scale factor, where the increase in the term of time evolution is

$$a(t) = \sqrt{t^n e^t}, \tag{20}$$

where n is a positive constant. Saha et al. (2012) and Pradhan & Amirhashchi (2011) examined the relation (20) when studying a two-fluid scenario for DE models in an FRW universe and accelerating DE models in Bianchi type V space-times, respectively. This *ansatz* generalized the one proposed by Amirhashchi et al. (2011b). If we put $n = 0$ in Equation (20), it is reduced to $a(t) = \sqrt{e^t}$, i.e. an exponential law of variation for the scale factor. This choice of scale factor yields a time-dependent DP (see Eq. (30)) such that before the DE era, the corresponding solution gives the inflation and radiation/matter dominated era, with subsequent transition from deceleration to acceleration. Thus, our choice of scale factor is physically acceptable.

It is worth mentioning here that one can also select many other *ansatzes* than Equation (20) which mimic an accelerating universe. However, one should also be careful to check the physical acceptability and stability of their corresponding solutions, otherwise they do not prove any relation of such solutions with the observable universe. Equation (20) yields physically plausible solutions.

Secondly, we assume that the expansion (θ) is proportional to shear (σ). This condition and Equation (16) lead to

$$\frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = \alpha_0 \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right), \tag{21}$$

which yields

$$\frac{\dot{A}}{A} = m\frac{\dot{B}}{B}, \tag{22}$$

where $m = \frac{2\alpha_0\sqrt{3}+1}{1-\alpha_0\sqrt{3}}$ and α_0 are arbitrary constants. The above equation, after integration, reduces to

$$A = \beta(B)^m, \tag{23}$$

where β is a constant of integration. Here, for simplicity and without any loss of generality, we assume $\beta = 1$. Hence we have

$$A = (B)^m. \tag{24}$$

Collins et al. (1980) have pointed out that for a spatially homogeneous metric, the normal congruence with the homogeneous expansion satisfies the condition that $\frac{\sigma}{\theta}$ is constant.

Using Equations (16), (20) and (24) in (11), we obtain the expressions for metric functions as follows

$$B(t) = \ell_1(t^n e^t)^{\frac{3}{2(m+2)}}, \tag{25}$$

$$C(t) = \ell_2(t^n e^t)^{\frac{3}{2(m+2)}}, \tag{26}$$

$$A(t) = \ell_3(t^n e^t)^{\frac{3m}{2(m+2)}}, \tag{27}$$

where $\ell_1 = k^{-\frac{1}{(m+2)}}$, $\ell_2 = \ell\ell_1$, $\ell_3 = \ell_1^m$ and k is a constant of integration.

Hence the model (1) reduces to

$$ds^2 = -dt^2 + \ell_3^2(t^n e^t)^{\frac{6m}{(m+2)}} dx^2 + \ell_1^2(t^n e^t)^{\frac{6}{(m+2)}} dy^2 + \ell_2^2(t^n e^t)^{\frac{6}{(m+2)}} dz^2. \tag{28}$$

4 PHYSICAL ASPECTS OF THE DARK ENERGY MODEL

The expressions for the Hubble parameter (H), scalar of expansion (θ), shear scalar (σ), the spatial volume (V), and the average anisotropy parameter (A_m) for the model (28) are given by

$$\theta = 3H = \frac{3}{2}\left(1 + \frac{n}{t}\right), \tag{29}$$

$$q = \frac{2n}{(n+t)^2} - 1, \tag{30}$$

$$\sigma^2 = \frac{3}{4}\left(\frac{m-1}{m+2}\right)^2\left(1 + \frac{n}{t}\right)^2, \tag{31}$$

$$V = (t^n e^t)^{\frac{3}{2}}, \tag{32}$$

$$A_m = 2\left(\frac{m-1}{m+2}\right)^2. \tag{33}$$

From Equations (29)−(33), it is observed that at $t = 0$, the spatial volume vanishes and the other parameters, θ, σ and H, diverge. Hence the model starts with a big bang singularity at $t = 0$. This is a point type singularity (MacCallum 1971) since directional scale factor $A(t)$, $B(t)$ and $C(t)$ vanish at the initial time. Since $\frac{\sigma^2}{\theta^2} \neq 0$ except for $m = 1$, the model is anisotropic for all values of m

except for $m \neq 1$. The dynamics of the mean anisotropy parameter depend on the value of m. We observe that when $m = 1$, $A_m = 0$ (i.e. the case of isotropy). Thus, the observed isotropy of the model can be achieved in the region of a cosmological constant (see Fig. 2).

The energy density of the fluid can be found by using Equations (25) and (27) in (19)

$$\rho = \frac{9}{4}\left(\frac{2m+1}{m+2}\right)\left(1+\frac{n}{t}\right)^2 - \ell_0(t^n e^t)^{-\frac{3m}{(m+2)}}, \tag{34}$$

where $\ell_0 = \frac{1}{\ell_3^2}$. Using Equations (25), (27) and (34) in (17), the EoS parameter ω is obtained as

$$\omega = \frac{\frac{27}{4(m+2)^2}\left(1+\frac{n}{t}\right)^2 - \frac{3n}{(m+2)t^2} + \ell_0(t^n e^t)^{-\frac{3m}{(m+2)}}}{\ell_0(t^n e^t)^{-\frac{3m}{(m+2)}} + \frac{9}{4}\left(\frac{2m+1}{m+2}\right)\left(1+\frac{n}{t}\right)^2}. \tag{35}$$

Using Equations (25), (27), (34) and (35) in (18), the skewness parameters δ (or γ) (i.e. deviations from ω along the y and z axes) are computed as

$$\delta = \gamma = \frac{\frac{3}{4}\left(\frac{m-1}{m+2}\right)\left\{\left(1+\frac{n}{t}\right)^2 - \frac{2n}{t^2}\right\} - 2\ell_0(t^n e^t)^{-\frac{3m}{(m+2)}}}{\ell_0(t^n e^t)^{-\frac{3m}{(m+2)}} - \frac{9}{4}\left(\frac{2m+1}{m+2}\right)\left(1+\frac{n}{t}\right)^2}. \tag{36}$$

From Equation (35), it is observed that the EoS parameter ω is time dependent, and it can also be a function of redshift z or scale factor a (as already discussed in Sect. 1).

So, if the present work is compared with experimental results (Knop et al. 2003; Tegmark et al. 2004b; Hinshaw et al. 2009; Komatsu et al. 2009), then one can conclude that the limit of ω provided by Equation (35) may accommodate the acceptable range of the EoS parameter. Also it is observed that at $t = t_c$, ω vanishes, where t_c is a critical time given by the following relation

$$\frac{27}{4(m+2)^2}\left(1+\frac{n}{t_c}\right)^2 - \frac{3n}{(m+2)t_c^2} + \ell_0(t_c^n e^{t_c})^{-\frac{3m}{(m+2)}}. \tag{37}$$

Thus, for this particular time, our model represents a dusty universe. We also note that at the earlier time, when $t \leq t_c$ and $\omega \geq 0$, the universe was dominated by real matter, but later at $t > t_c$, and $\omega < 0$, the phase dominated by DE begins.

From Equation (34), we note that energy density of the fluid $\rho(t)$ is a decreasing function of time and $\rho \geq 0$ when

$$\left(1+\frac{n}{t}\right)^2 (t^n e^t)^{\frac{3m}{(m+2)}} \geq \frac{4\ell_0}{9}\left(\frac{m+2}{2m+1}\right). \tag{38}$$

Figure 1 is the plot of energy density for the fluid (ρ) versus time in the accelerating mode of the universe. Here we observe that ρ is a positive decreasing function of time and it approaches zero as $t \to \infty$.

Figure 2 depicts the variation of EoS parameter (ω) versus cosmic time (t) in the evolution of the universe, as a representative case with an appropriate choice of constants of integration and other physical parameters using reasonably well known situations (parameters are given in the Figure caption). For $m = 1$, we obtain the isotropic model that is studied here as a representative case. From Figure 2, we observed that at the initial time there is a quintessence ($\omega > -1$) region and at a late time it approaches the cosmological constant ($\omega = -1$) scenario. This is a situation in the early universe where a quintessence dominated universe (Caldwell 2002) may be playing an important role for the EoS parameter. Since ω approaches -1 for sufficiently large time, its value is consistent with the range of all the three observations (Knop et al. 2003; Tegmark et al. 2004b; Hinshaw et al. 2009; Komatsu et al. 2009).
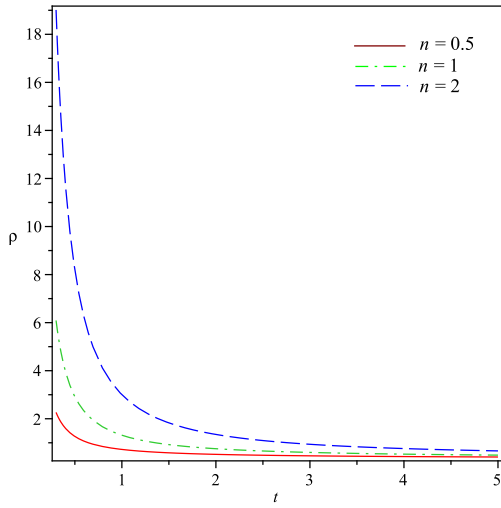
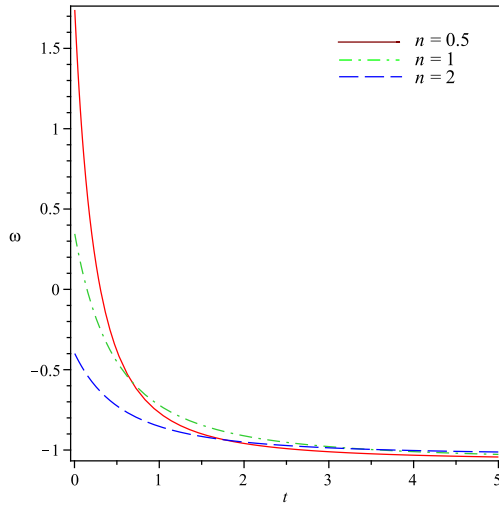**Fig. 1** The plot of energy density ρ versus t. Here $\ell_0 = 0.1$ and $m = 1$.

Fig. 2 The plot of EoS parameter ω versus t. Here $\ell_0 = 0.1$ and $m = 1$.

In the absence of any curvature, matter energy density Ω_m and DE Ω_Λ are related by the equation

$$\Omega_m + \Omega_\Lambda = 1, \tag{39}$$

where $\Omega_m = \frac{\rho}{3H^2}$ and $\Omega_\Lambda = \frac{\Lambda}{3H^2}$. Thus, Equation (39) reduces to

$$\frac{\rho}{3H^2} + \frac{\Lambda}{3H^2} = 1. \tag{40}$$

Using Equations (29) and (34) in (40), the cosmological constant is obtained as

$$\Lambda = -\frac{3}{4}\left(\frac{5m+1}{m+2}\right)\left(1+\frac{n}{t}\right)^2 + \ell_0(t^n e^t)^{-\frac{3m}{(m+2)}}. \tag{41}$$

From Equation (41), we observe that Λ is a decreasing function of time and is always positive when

$$\left(1+\frac{n}{t}\right)^2 (t^n e^t)^{\frac{3m}{(m+2)}} < \frac{4\ell_0}{3}\left(\frac{m+2}{5m+1}\right). \tag{42}$$

In general relativity, the Bianchi identities for Einstein's tensor G_{ij} and the vanishing covariant divergence of the energy momentum tensor T_{ij} together imply that the cosmological term Λ is constant. In theories with a variable Λ-term, one either introduces new terms (involving scalar fields, for instance) in the left hand side of the Einstein field equations to cancel the non-zero divergence of Λg_{ij} (Bergmann 1968; Wagoner 1970) or interpret Λ as a matter source and move it to the right hand side of the field equations (Zel'dovich 1968), in which case energy momentum conservation is understood to mean $T_{;j}^{*ij} = 0$, where $T_{ij}^* = T_{ij} - (\Lambda/8\pi G)g_{ij}$. It is here that the first assumption that leads to the cosmological constant problem is made. It is that the vacuum has a non-zero energy density. If such a vacuum energy density exists, Lorentz invariance requires that it have the form $\langle T_{\mu\nu}\rangle = -\langle\rho\rangle g_{\mu\nu}$. This allows the definition of an effective cosmological constant and a total effective vacuum energy density $\Lambda_{\text{eff}} = \Lambda + 8\pi G\langle\rho\rangle$ or $\rho_{\text{vac}} = \langle\rho\rangle + \Lambda/8\pi G$. Note at this point that only the effective cosmological constant, Λ_{eff}, is observable, not Λ, so the latter quantity may be referred to as 'bare.' The two approaches are of course equivalent for a given theory (Vishwakarma 2000).
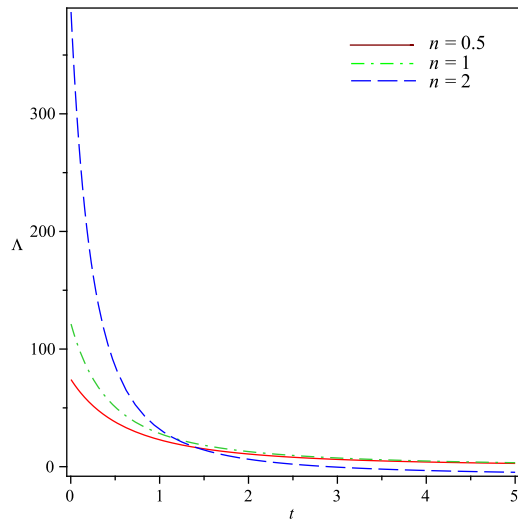
Fig. 3 The plot of cosmological constant Λ versus t. Here $\ell_0 = 0.1$ and $m = 1$.
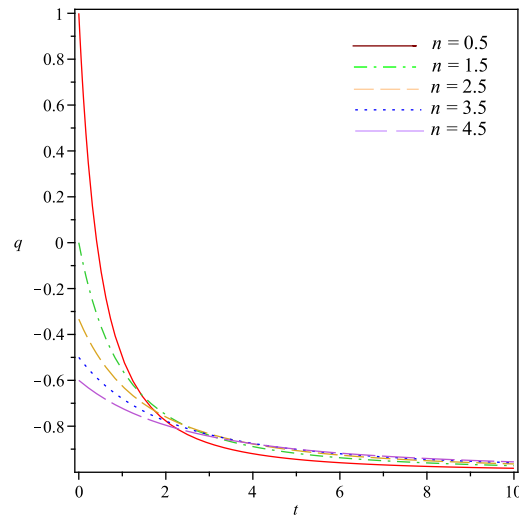
Fig. 4 The plot of DP q versus t.

For detailed discussions, readers are advised to see the references (Carroll et al. 1992; Abdussattar & Vishwakarma 1996; Peebles & Ratra 2003; Sahni & Starobinsky 2000; Padmanabhan 2003, 2008).

Figure 3 is the plot of cosmological constant Λ versus time t. We observe that the cosmological parameter is a decreasing function of time and it approaches a small positive value at late time (i.e. at the present epoch). Recent cosmological observations (Perlmutter et al. 1998, 1999; Riess et al. 1998, 2004; Tonry et al. 2003) suggest the existence of a positive cosmological constant Λ with the magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$. These observations of magnitude and redshift in SNe Ia suggest that our universe may be accelerating with induced cosmological density through the cosmological Λ-term. Thus, the nature of Λ in our derived DE model is supported by recent observations.

Figure 4 is the plot of DP q versus time t. From Figure 4, it is observed that q decreases very rapidly and reaches values -1, then afterwards it remains constant at -1 (like a de Sitter universe). From this figure, we observe that the DE model, for $0 < n < 1.5$, evolves from the matter dominated era to a quintessence era and ultimately approaches the cosmological constant era, whereas for $n \geq 1.5$, the universe evolves from the quintessence to the cosmological constant era. It is worth mentioning here that for $n < 1.5$, transition of the universe takes place from the early decelerating phase to the recent accelerating phase, whereas for $n \geq 1.5$, the expansion of the universe is always accelerating.

From these analyses, we conclude that it is the choice of scale factor that makes the model inflationary at the early stages of the universe and a radiation/matter dominated phase before the DE era. From Equation (29), we observe that when $t \to 0$, the expansion scalar θ becomes infinity, which indicates the inflationary scenario. Also from Figure 4, we observe that before $t \approx 1$, $q > 0$ and this indicates the radiation/matter dominated era of the universe. However, after $t \approx 1$, $q < 0$ which indicates the DE dominated era. The solution in our model is stable at any given epoch for the choice of the *ansatz* (20). Hence our derived model is physically acceptable.

The CMB is also considered to be major experimental evidence supporting the present models of the observed universe and from CMB observations several scientists found the signature of anisotropy. Based on these studies and observations, one may not preclude the possibility that our universe is anisotropic. We have already discussed this scenario in the Introduction.

5 STABILITY OF THE CORRESPONDING SOLUTIONS

A rigorous analysis on the stability of the corresponding solutions can be done by invoking a perturbative approach. Perturbations of the fields in a gravitational system against the background evolutionary solution should be checked to ensure the stability of the exact or approximate background solution (Chen & Kao 2001). Now we will study the stability of the background solution with respect to perturbations of the metric. Perturbations will be considered for all three expansion factors a_i via

$$a_i \to a_{Bi} + \delta a_i = a_{Bi}(1 + \delta b_i). \tag{43}$$

We will focus on the variables δb_i instead of δa_i from now on for convenience. Therefore, the perturbations of the volume scale factor $V_B = \Pi_{i=1}^3 a_i$, directional Hubble factors $\theta_i = \frac{\dot{a}_i}{a_i}$ and the mean Hubble factor $\theta = \sum_{i=3}^3 \frac{\theta_i}{3} = \frac{\dot{V}}{3V}$ can be shown to be

$$V \to V_B + V_B \sum_i \delta b_i, \qquad \theta_i \to \theta_{Bi} + \sum_i \delta b_i, \qquad \theta \to \theta_B + \frac{1}{3}\sum_i \delta b_i. \tag{44}$$

One can show that the metric perturbations δb_i, to the linear order in δb_i, obey the following equations

$$\sum_i \delta \ddot{b}_i + 2\sum \theta_{Bi}\delta \dot{b}_i = 0, \tag{45}$$

$$\delta \ddot{b}_i + \frac{\dot{V}_B}{V_B}\delta \dot{b}_i + \sum_j \delta \dot{b}_j \theta_{Bi} = 0, \tag{46}$$

$$\sum \delta \dot{b}_i = 0. \tag{47}$$

From the above three equations, we can easily find

$$\delta \ddot{b}_i + \frac{\dot{V}_B}{V_B}\delta \dot{b}_i = 0, \tag{48}$$

where V_B is the background volume scale factor. In our case, V_B is given by

$$V_B = t^{\frac{3}{2}n}e^{\frac{3}{2}t} \tag{49}$$

using the above equation in Equation (6) and after integration we get

$$\delta b_i = c_i t^{-\frac{3}{4}n}e^{-\frac{3}{4}t}\,\text{WittakerM}\left(-\frac{3}{4}n, -\frac{3}{4}n + \frac{1}{2}, \frac{3}{2}t\right), \tag{50}$$

where c_i is a constant of integration. Therefore, the "actual" fluctuations for each expansion factor $\delta a_i = a_{Bi}\delta b_i$ are given by

$$\delta a_i \to c_i t^{-\frac{n}{4}}e^{-\frac{t}{4}}\,\text{WittakerM}\left(-\frac{3}{4}n, -\frac{3}{4}n + \frac{1}{2}, \frac{3}{2}t\right). \tag{51}$$

From the above equation we see that for $n \gg 1$, δa_i approaches zero. Consequently, the background solution is stable against the perturbation of the graviton field.

6 OTHER DARK ENERGY MODELS

Now we take the following *ansatz* for the scale factor, where the increase in terms of time evolution is

$$a(t) = -\frac{1}{t} + t^2. \tag{52}$$

The above choice of scale factor yields a time dependent DP and the corresponding solutions are stable. The motivations for selecting such a type of scale factors for finding solutions were already described in Section 3. We define the DP q as usual

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -\frac{\ddot{a}}{aH^2}. \tag{53}$$

Using (52) in (53), we find

$$q = -2\left(\frac{t^3 - 1}{2t^3 + 1}\right)^2. \tag{54}$$

Using Equations (16), (24) and (52) in (11), we obtain the expressions for metric functions as follows

$$B(t) = \ell_4 \left(-\frac{1}{t} + t^2\right)^{\frac{3}{(m+2)}}, \tag{55}$$

$$C(t) = \ell_5 \left(-\frac{1}{t} + t^2\right)^{\frac{3}{(m+2)}}, \tag{56}$$

$$A(t) = \ell_6 \left(-\frac{1}{t} + t^2\right)^{\frac{3m}{(m+2)}}, \tag{57}$$

where $\ell_4 = l^{-\frac{1}{(m+2)}}$, $\ell_5 = \ell\ell_4$, $\ell_6 = \ell_4^m$ and l is a constant of integration.

Hence the model (1) reduces to

$$ds^2 = -dt^2 + \ell_6^2\left(-\frac{1}{t} + t^2\right)^{\frac{6m}{(m+2)}} dx^2 + \ell_4^2\left(-\frac{1}{t} + t^2\right)^{\frac{6}{(m+2)}} dy^2$$

$$+ \ell_5^2\left(-\frac{1}{t} + t^2\right)^{\frac{6}{(m+2)}} dz^2. \tag{58}$$

The expressions for the Hubble parameter (H), scalar of expansion (θ), shear scalar (σ), spatial volume (V) and the average anisotropy parameter (A_m) for the model (58) are given by

$$\theta = 3H = \frac{3}{t}\left(\frac{2t^3 + 1}{t^3 - 1}\right), \tag{59}$$

$$\sigma^2 = 3\left[\left(\frac{m-1}{m+2}\right)\frac{(2t^3 + 1)}{(t^3 - 1)t}\right]^2, \tag{60}$$

$$V = \left(-\frac{1}{t} + t^2\right)^3, \tag{61}$$

$$A_m = 2\left(\frac{m-1}{m+2}\right)^2. \tag{62}$$

From Equation (59), we observe that when $t \to 0$, $\theta \to \infty$ and this indicates the inflationary scenario at early stages of the universe. Since $\frac{\sigma^2}{\theta^2} \neq 0$ for all values of m except for $m = 1$, the model is

anisotropic except for $m = 1$. The dynamics of the mean anisotropy parameter depend on the value of m. The mean anisotropic parameter is constant. We observed that when $m = -2$, $A_m \to \infty$ and for $m = 1$, $A_m = 0$. Thus, the observed isotropy of the universe can be achieved in a phantom model (see Fig. 6).

The energy density of the fluid can be found by using Equations (55) and (57) in (19)

$$\rho = \frac{9(2m+1)}{(m+2)^2} \frac{(2t^3+1)^2}{(t^3-1)^2 t^2} - \ell_0 \left(-\frac{1}{t} + t^2\right)^{-\frac{6m}{(m+2)}}, \tag{63}$$

where $l_0 = \frac{1}{\ell_6^2}$. Using Equations (55), (57) and (63) in (17), the EoS parameter ω is obtained as

$$\omega = \frac{\frac{27}{(m+2)^2} \frac{(2t^3+1)^2}{(t^3-1)^2 t^2} - \frac{6}{(m+2)} \frac{(2t^6+8t^3-1)}{(t^3-1)^2 t^2} + \ell_0 \left(-\frac{1}{t} + t^2\right)^{-\frac{6m}{(m+2)}}}{\ell_0 \left(-\frac{1}{t} + t^2\right)^{-\frac{6m}{(m+2)}} + \frac{9(2m+1)}{(m+2)^2} \frac{(2t^3+1)^2}{(t^3-1)^2 t^2}}. \tag{64}$$

Using Equations (55), (57), (63) and (64) in (18), the skewness parameters δ (or γ) (i.e. deviations from ω along the y and z axes) are computed as

$$\delta = \gamma = \frac{6\left(\frac{m-1}{m+2}\right) \frac{(5t^6+2t^3+2)}{(t^3-)^2 t^2} - 2\ell_0 \left(-\frac{1}{t} + t^2\right)^{-\frac{6m}{(m+2)}}}{\ell_0 \left(-\frac{1}{t} + t^2\right)^{-\frac{6m}{(m+2)}} - \frac{9(2m+1)}{(m+2)^2} \frac{(2t^3+1)^2}{(t^3-1)^2 t^2}}. \tag{65}$$

So, if the present work is compared with experimental results (Knop et al. 2003; Tegmark et al. 2004b; Hinshaw et al. 2009; Komatsu et al. 2009), then one can conclude that the limit of ω provided by Equation (64) may be accommodated with an acceptable range for the EoS parameter. Also, it is observed that at $t = t_c$, ω vanishes, where t_c is a critical time given by the following relation

$$\frac{27}{(m+2)^2} \frac{(2t_c^3+1)^2}{(t_c^3-1)^2 t_c^2} - \frac{6}{(m+2)} \frac{(2t_c^6+8t_c^3-1)}{(t_c^3-1)^2 t_c^2} + \ell_0 \left(-\frac{1}{t_c} + t_c^2\right)^{-\frac{6m}{(m+2)}} = 0. \tag{66}$$

Thus, for this particular time, our model represents a dusty universe. We also note that the earlier baryonic matter dominated phase at $t \leq t_c$, where $\omega \geq 0$, is converted to the DE dominated phase of universe, at time $t > t_c$, where $\omega < 0$.

From Equation (63), we note that energy density of the fluid $\rho(t)$ is a decreasing function of time and $\rho \geq 0$ when

$$\frac{(2t^3+1)^2}{(t^3-1)^2 t^2} \left(-\frac{1}{t} + t^2\right)^{\frac{6m}{(m+2)}} \geq \frac{\ell_0(m+2)^2}{9(2m+1)}. \tag{67}$$

Figure 5 shows the variation of energy density (ρ) versus time t. Here we observe that ρ is a positive decreasing function of time and it approaches zero as $t \to \infty$.

Figure 6 displays parameter (ω) versus cosmic time (t) in the evolution of the universe, as a representative case with an appropriate choice of constants of integration and other physical parameters using reasonably well known values (parameters are given in the Figure caption). From Figure 6, we observe the following:

(i) for $m \leq 0.5$, the evolution of the universe starts from the quintessence era ($\omega > -1$) and approaches a phantom region ($\omega < -1$).

(ii) for $1 \leq m < 2$, the universe evolves from a phantom region ($\omega < -1$), then crosses PDL and ultimately approaches a quintessence region ($\omega > -1$).

(iii) for $2 \leq m \leq 3$, the evolution of the universe commences from the phantom region ($\omega < -1$), crosses PDL and then skips over to a non-dark region.

(iv) for $3 \leq m$, the evolution of the universe begins from the quintessence era ($\omega > -1$) and ultimately passes over to a non-dark region.

(v) for $m = 1$, we get $\omega \approx -0.65$, which is consistent with SN Ia data $-1.67 < \omega < -0.62$ (Knop et al. 2003).

(vi) for $m = 0.5$, we get $\omega \approx -1.1$ which is reproducible with the current observational realm (Knop et al. 2003; Tegmark et al. 2004b; Hinshaw et al. 2009; Komatsu et al. 2009).

Using Equations (59) and (63) in (40), the cosmological constant is obtained as

$$\Lambda = \frac{3(4m^2 + 10m + 13)}{(m+2)^2} \frac{(2t^3 + 1)^2}{(t^3 - 1)^2 t^2} + \ell_0 \left(-\frac{1}{t} + t^2\right)^{\frac{-6m}{(m+2)}}. \tag{68}$$

From Equation (68), we observe that Λ is a decreasing function of time and it is always positive when

$$\frac{(2t^3 + 1)^2}{(t^3 - 1)^2 t^2} \left(-\frac{1}{t} + t^2\right)^{\frac{6m}{(m+2)}} > -\frac{\ell_0(m+2)^2}{3(4m^2 + 10m + 13)}. \tag{69}$$

Figure 7 is the plot of cosmological constant Λ versus time t. It is observed that in all cases the cosmological parameter is a decreasing function of time and it approaches a small positive value at late time (i.e. at the present epoch). Thus, the nature of Λ in this derived DE model is also in good agreement with recent observations (Perlmutter et al. 1998, 1999; Riess et al. 1998, 2004; Tonry et al. 2003).

Figure 8 is the plot of DP q versus time t. From the figure we observe that the expansion of the universe starts from the accelerating phase and the rate of expansion decreases with time but then stops, and again starts accelerating to approach -0.5, which is very close to the value (≈ -0.7) predicted by the observations (Riess et al. 2004; Virey et al. 2005).

A convenient method to describe models close to Λ CDM is based on the "cosmic jerk" parameter j, a dimensionless third derivative of the scale factor with respect to the cosmic time (Chiba & Nakamura 1998; Sahni 2002; Blandford et al. 2005; Visser 2004, 2005). A deceleration-to-acceleration transition occurs for models with a positive value of j_0 and negative q_0. Flat Λ CDM
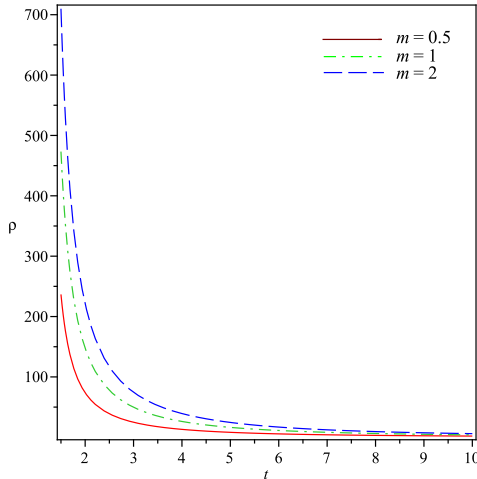

Fig. 5 The plot of energy density ρ versus t. Here $\ell_0 = 0.1$.
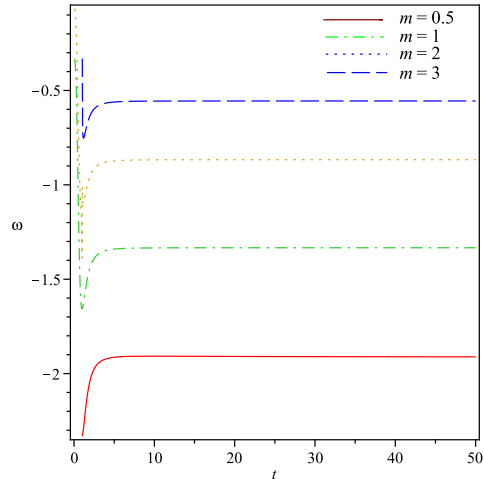
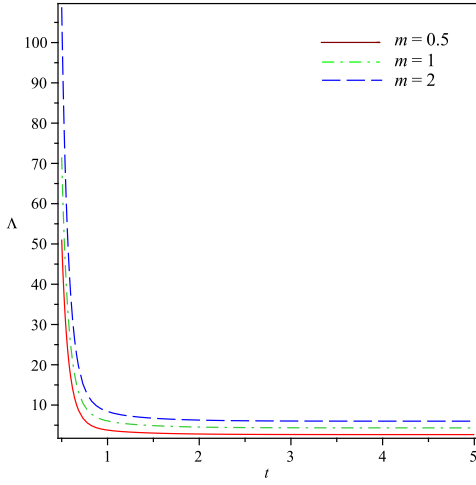**Fig. 6** The plot of EoS parameter ω versus t. Here $\ell_0 = 0.1$.

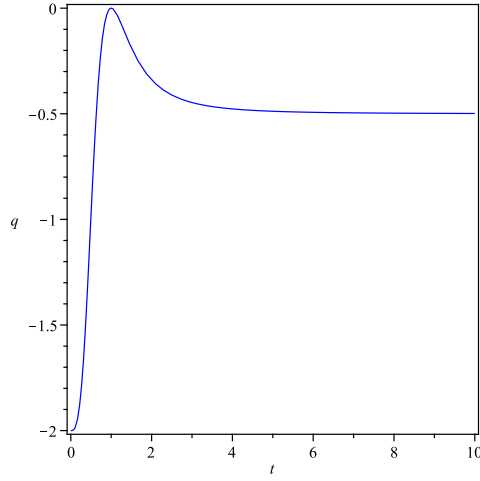**Fig. 7** The plot of cosmological constant Λ versus t. Here $\ell_0 = 0.1$.

Fig. 8 The plot of DP q versus t.

models have a constant jerk $j = 1$. We obtain the jerk parameter as

$$j(t) = \frac{2t^5 + 2t^4 - 2t^2 - t - 2}{(t+1)(1+t^2)}.$$ (70)

This value is consistent with observational value $j \simeq 2.16$ obtained from the combination of three kinematical data sets: the gold sample of SNe Ia (Riess et al. 2004), the SNe Ia data from the SN Legacy Survey (SNLS) project (Astier et al. 2006), and the X-ray galaxy cluster distance measurements (Rapetti et al. 2007) for $t = 1.50$.

7 STABILITY OF THE CORRESPONDING SOLUTIONS

The method to study the stability of the background solution with respect to perturbations of the metric was already given in Section 5. From Equations (45)$-$(47), we can easily derive

$$\ddot{\delta b_i} + \frac{\dot{V}_B}{V_B}\dot{\delta b_i} = 0,$$ (71)

where V_B is the background volume scale factor. In our case, V_B is given by

$$V_B = t^6.$$ (72)

Using the above expression in Equation (71), after integration we get

$$\delta b_i = c_i t^{-5},$$ (73)

where c_i is a constant of integration. Therefore, the "actual" fluctuations for each expansion factor $\delta a_i = a_{Bi}\delta b_i$ are given by

$$\delta a_i \rightarrow c_i t^{-3},$$ (74)

where $a_{Bi} \rightarrow t^2$. From the above equation it is obvious that δa_i approaches zero as $t \rightarrow \infty$. Consequently, the background solution is stable against the perturbation of the graviton field.

8 CONCLUDING REMARKS

A new class of anisotropic B-VI$_0$ DE models with a variable EoS parameter ω has been investigated by using a time-dependent DP. In the literature, it is common to use a constant DP. Now, for a universe which was decelerating in the past and is accelerating during the present epoch, the DP must show signature flipping, as discussed in Section 2. Therefore our consideration of defining DP as variable is physically justified.

The main features of the models are as follows:

– DE models present the dynamics of the EoS parameter ω provided by Equations (35) and (55) whose range is in good agreement with the acceptable range by the recent observations (Knop et al. 2003; Tegmark et al. 2004b; Hinshaw et al. 2009; Komatsu et al. 2009).
– It can be easily seen that in both DE models, the mean anisotropic parameter vanishes at $m = 1$. Thus, both our anisotropic models approach isotropy at $m = 1$.
– We obtain a cosmological constant dominated universe, a quintessence and phantom fluid dominated universe (Chevallier & Polarski 2001), representing the different phases of the universe throughout the evolution process for different cosmic times. These fits suggest that $\omega > -1$ for a long (quintessence-like) period in the past, and at the same time they suggest that the universe has just entered a phantom phase $\omega < -1$ near our present era.
– Unlike the Robertson-Walker (RW) metric, Bianchi type metrics can admit a DE that has an anisotropic EoS parameter according to the characteristics. Therefore, one cannot rule out the possibility of the anisotropic nature of DE in the framework of B-VI$_0$ space-time.
– In the first case, the observed isotropy of the universe can be achieved in a model that incorporates a cosmological constant (see, Fig. 2) whereas in the second case, the observed isotropy of the universe can be achieved in a phantom model (see, Fig. 6). Thus, Bianchi type VIpreferable in terms of academical interest.
– Our DE models are of great importance in the sense that the nature of decaying vacuum energy density $\Lambda(t)$ is supported by recent cosmological observations (Perlmutter et al. 1998, 1999; Riess et al. 1998, 2004; Tonry et al. 2003).
– Though there are many candidates, such as the cosmological constant, vacuum energy, the scalar field, the brane world model, cosmological nuclear-energy, etc., as reported in the vast literature for DE, the proposed models in this paper favor the EoS parameter as a possible suspect for DE.
– The cosmic jerk parameter in our derived models is also found to be in good agreement with the recent data from astrophysical observations, namely the gold sample of SNe Ia (Riess et al. 2004), the SN Ia data from the SNLS project (Astier et al. 2006), and distance measurements of the X-ray gas in galaxy clusters (Rapetti et al. 2007).
– For a different choice of n, we can generate a class of DE models in Bianchi type VI$_0$ space-time. It is observed that such DE models are also in good harmony with current observations. Our study is continuing and we shall generate some other interesting physically viable models for other values of n.
– Our corresponding solutions have an inflationary scenario at the early stages of the universe and also a radiation/matter era before the DE era.
– Our corresponding solutions are physically acceptable and the solutions are stable.

Thus, the solutions demonstrated in this paper may be useful for better understanding the characteristics of anisotropic DE in the evolution of the universe within the framework of Bianchi type VI$_0$ space-time.

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