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# Observational constraints on the new generalized Chaplygin gas model \*

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Abstract We use the latest data to investigate observational constraints on the new generalized Chaplygin gas (NGCG) model. Using the Markov Chain Monte Carlo method, we constrain the NGCG model with type Ia supernovae from the Union2 set (557 data), the usual baryonic acoustic oscillation (BAO) observation from the spectroscopic Sloan Digital Sky Survey data release 7 galaxy sample, the cosmic microwave background observation from the 7-year Wilkinson Microwave Anisotropy Probe results, newly revised data on H(z), as well as a value of  $\theta_{\text{BAO}}(z = 0.55) = (3.90^{\circ} \pm 0.38^{\circ})$  for the angular BAO scale. The constraint results for the NGCG model are  $\omega_X = -1.0510^{+0.1663}_{-0.1685}(1\sigma)^{+0.2226}_{-0.2398}(2\sigma)$ ,  $\eta = 1.0117^{+0.0469}_{-0.0502}(1\sigma)^{+0.0693}_{-0.0716}(2\sigma)$  and  $\Omega_X = 0.7297^{+0.0229}_{-0.0276}(1\sigma)^{+0.0329}_{-0.0402}(2\sigma)$ , which give a rather stringent constraint. From the results, we can see that a phantom model is slightly favored and the probability that energy transfers from dark matter to dark energy is a little larger than the inverse.

**Key words:** new generalized Chaplygin gas — angular BAO scale — cosmological observations

# **1 INTRODUCTION**

Cosmic observations suggest that the present universe is undergoing an accelerated state (Riess et al. 1998; Perlmutter et al. 1999; Pope et al. 2004). In order to explain this, a component with negative pressure known as dark energy was proposed. The most simple and popular model for dark energy is the cosmological constant ( $\Lambda$ ). This model has successfully explained many phenomena, but it also encounters some theoretical problems; for example, the coincidence problem and the "fine-tuning" problem. Therefore, many other models have been proposed including quintessence (Peebles & Ratra 1988; Ratra & Peebles 1988), holographic dark energy (Cohen et al. 1999; Li 2004), quintom (Guo et al. 2005; Feng et al. 2005), phantom (Caldwell 2002), brane world (Dvali et al. 2000; Zhu & Alcaniz 2005) and so on. Among these, the generalized Chaplygin gas (GCG) model, acting as a unification of dark energy and dark matter, is a good candidate (Bento et al. 2002, 2004). It has been widely studied and seems to be in agreement with different observational data (Zhu 2004; Lu et al. 2009; Xu & Lu 2010; Park et al. 2010). Moreover, GCG and the original Chaplygin gas (CG)

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model (Kamenshchik et al. 2001) can be connected to string and brane theory (Bilić et al. 2002). The equation of state (EoS) of GCG is expressed as

$$p_{\rm GCG} = -\frac{A}{\rho_{\rm GCG}^{\alpha}}.$$
 (1)

Under the Friedmann-Robertson-Walker (FRW) metric with the energy momentum conservation equation, we can obtain (Amendola et al. 2003)

$$\rho_{\rm GCG} = \rho_{\rm GCG,0} \left[ A_* + (1 - A_*) a^{-3(1+\alpha)} \right]^{\frac{1}{(1+\alpha)}}, \tag{2}$$

where  $A_* \equiv A/\rho_{GCG,0}^{1+\alpha}$  and  $\rho_{GCG,0}$  is the energy density today. In addition to the parameter A,  $\alpha$  is also a new parameter that makes the GCG model different from the original CG model. When  $\alpha = 1$ , it becomes CG. The core aspect of CG and GCG is when the scale factor was very small in the early universe, it acted as a dust, but recently it has behaved like a cosmological constant. The interacting CG model was discussed in Zhang & Zhu (2006) and the interacting generalized Chaplygin gas model was addressed in Setare (2007a,b). Other models relevant to Chaplygin gas were proposed including the modified Chaplygin gas (MCG) model (Chimento 2004; Chimento & Lazkoz 2006), variable Chaplygin gas (VCG) model (Guo & Zhang 2007), extended Chaplygin gas model (Meng et al. 2005) and new modified Chaplygin gas (NMCG) model (Chattopadhyay & Debnath 2008).

Since the GCG model can be equal to the interacting  $\Lambda$ CDM model (Fabris et al. 2004; Zhang et al. 2006), a new generalized Chaplygin gas (NGCG) model which is equivalent to a kind of interacting XCDM model, was proposed as a unification of X-type dark energy with the EoS parameter  $\omega_X$  and cold dark matter (Zhang et al. 2006). The interacting new generalized Chaplygin gas model has been discussed in Jamil (2010).

There are different kinds of observational data which can be used to constrain cosmological models: type Ia supernovae (SNe Ia), observational Hubble parameter data, baryon acoustic oscillation (BAO), cosmic microwave background (CMB) data, lensing (Liao & Zhu 2012) and so on. For BAO, using spectroscopic determinations of the redshifts of galaxies, several detections at different redshifts have been studied (Percival et al. 2007, 2010). This method examines the three-dimensional averaged distance parameter  $D_V$ . The radial BAO data were discussed by Gaztañaga et al. (2009b). Recently, a new determination of the BAO scale using the photometric sample of luminous red galaxies in the Sloan Digital Sky Survey (SDSS) data release 7 (DR7) was performed (Crocce et al. 2011; Carnero et al. 2012). They derived a value of  $\theta_{BAO}(z = 0.55) = (3.90^{\circ} \pm 0.38^{\circ})$ , including systematic errors, for the angular BAO scale. It is the first direct measurement of the pure angular BAO signal. Combined with previous BAO signals, it can break the degeneracies in the determination of model parameters.

In this paper, we use the Markov Chain Monte Carlo (MCMC) method to constrain the NGCG model from the latest data including the BAO data at z = 0.55. Throughout the paper, the unit with light velocity c = 1 is used.

In Section 2, we give a brief introduction of the NGCG model. In Section 3, we introduce the observational data. The full parameter space using different combinations of data is provided in Section 4. Finally, we summarize our results in Section 5.

### 2 THE NEW GENERALIZED CHAPLYGIN GAS MODEL

We give a very brief introduction to the new generalized Chaplygin gas model in this section. For details of this model, please refer to Zhang et al. (2006). Assuming the universe is flat with the FRW metric, the EoS of NGCG fluid (Zhang et al. 2006) is

$$p_{\rm NGCG} = -\frac{\hat{A}(a)}{\rho_{\rm NGCG}^{\alpha}},\tag{3}$$

where a is the scale factor and  $\alpha$  is the parameter similar to the one in the GCG model. NGCG fluid consists of dark energy  $\rho_X \sim a^{-3(1+w_X)}$ , where  $w_X$  is the EoS parameter, and dark matter  $\rho_{\rm DM} \sim a^{-3}$ . Naturally, the energy density of the NGCG can be considered as

$$\rho_{\text{NGCG}} = \left[ A a^{-3(1+w_X)(1+\alpha)} + B a^{-3(1+\alpha)} \right]^{\frac{1}{1+\alpha}} , \qquad (4)$$

with

$$A + B = \rho_{\text{NGCG},0}^{\eta} , \qquad (5)$$

where A and B are positive constants. Considering the energy conservation equation,  $\tilde{A}(a)$  can be derived

$$\tilde{A}(a) = -w_X A a^{-3(1+w_X)(1+\alpha)} .$$
(6)

We can easily see that it becomes GCG when the EoS parameter of dark energy component  $w_X$  is -1. On the other hand it becomes XCDM if  $\alpha = 0$ . It is remarkable that  $\alpha$  describes the interaction between dark energy and dark matter. When  $\alpha > 0$ , the energy transfers from dark matter to dark energy. By contrast, the energy transfers from dark energy to dark matter in the case  $\alpha < 0$ . Based on Zhang et al. (2006), we take the radiation component into consideration, which dominated the early universe. The whole density consists of the NGCG component, the baryon matter component and the radiation component  $\rho_{tot} = \rho_{NGCG} + \rho_b + \rho_r$ . The Friedmann equation can be expressed as

$$H(a) = H_0 E(a),\tag{7}$$

where

$$E(a)^{2} = (1 - \Omega_{\rm b} - \Omega_{\rm r})a^{-3} \left[ 1 - \frac{\Omega_{X}}{1 - \Omega_{\rm b} - \Omega_{\rm r}} (1 - a^{-3w_{X}\eta}) \right]^{1/\eta} + \Omega_{\rm b}a^{-3} + \Omega_{\rm r}a^{-4} .$$
(8)

Here  $H_0$ ,  $\Omega_X$ ,  $\Omega_b$  and  $\Omega_r$  represent the Hubble constant, dimensionless dark energy, baryonic matter and radiation today respectively. Consistent with the original paper, the parameter  $\eta = 1 + \alpha$ .

# **3 CURRENT OBSERVATIONAL DATA**

Now, we introduce the methods of constraints on the NGCG model by using the latest data.

# **3.1 Baryon Acoustic Oscillation Including Data at** z = 0.55

For BAO, the distance scale is defined as (Eisenstein et al. 2005)

$$D_V(z) = \frac{1}{H_0} \left[ \frac{z}{E(z)} \left( \int_0^z \frac{dz}{E(z)} \right)^2 \right]^{1/3} , \qquad (9)$$

and baryons were released from photons at the so-called drag epoch. The corresponding redshift  $z_d$  is given by

$$z_{\rm d} = \frac{1291(\Omega_{\rm m0}h^2)^{0.251}}{\left[1 + 0.659(\Omega_{\rm m0}h^2)^{0.828}\right]} \left[1 + b_1(\Omega_{\rm b}h^2)^{b_2}\right],\tag{10}$$

where

$$b_1 = 0.313 (\Omega_{\rm m0} h^2)^{-0.419} \left[ 1 + 0.607 (\Omega_{\rm m0} h^2)^{0.674} \right]^{-1}$$

and

$$b_2 = 0.238 (\Omega_{\rm m0} h^2)^{0.223}$$

(Eisenstein & Hu 1998). For observations of BAO, we choose the measurement of the distance ratio  $(d_z)$  at z = 0.2 and z = 0.35 (Percival et al. 2010). The definition is given by

$$d_z = \frac{r_{\rm s}(z_{\rm d})}{D_V(z)},\tag{11}$$

where  $r_s(z_d)$  is the comoving sound horizon. The SDSS DR7 galaxy sample gives the best-fit values of the data set  $(d_{0.2}, d_{0.35})$  (Percival et al. 2010)

$$\bar{\mathbf{P}}_{\text{matrix}} = \begin{pmatrix} \bar{d}_{0.2} \\ \bar{d}_{0.35} \end{pmatrix} = \begin{pmatrix} 0.1905 \pm 0.0061 \\ 0.1097 \pm 0.0036 \end{pmatrix}.$$
 (12)

The  $\chi^2$  value of this BAO observation from SDSS DR7 can be calculated as (Percival et al. 2010)

$$\chi^2_{\text{matrix}} = \Delta \mathbf{P}_{\text{matrix}}^{\text{T}} \mathbf{C}_{\text{matrix}}^{-1} \Delta \mathbf{P}_{\text{matrix}}, \tag{13}$$

where the corresponding inverse covariance matrix is

$$\mathbf{C_{matrix}}^{-1} = \begin{pmatrix} 30124 & -17227\\ -17227 & 86977 \end{pmatrix}.$$
 (14)

Moreover, a new determination of the BAO scale using a photometric sample of luminous red galaxies in the DR7 is presented (Crocce et al. 2011; Carnero et al. 2012). It makes use of  $\sim 1.5 \times 10^6$  luminous red galaxies with photometric redshifts. They get a value of  $\theta_{BAO}(z = 0.55) = (3.90^\circ \pm 0.38^\circ)$  for the angular BAO scale including systematic errors. It is the first direct measurement of the pure angular BAO signal. The definition of  $\theta_{BAO}$  is expressed as (Sánchez et al. 2011)

$$\theta_{\rm BAO} = \frac{r_{\rm s}(z_{\rm d})}{\chi(z)},\tag{15}$$

where the comoving radial distance  $\chi(z)$  depending on the NGCG model is defined as

$$\chi(z) = \frac{1}{H_0} \int_0^z \frac{dz}{E(a)} \,. \tag{16}$$

The corresponding  $\chi^2$  of these BAO data is expressed as

$$\chi_{\text{angular}}^2 = \frac{[\theta_{\text{BAO}} - 0.0681]^2}{0.00663^2},\tag{17}$$

where the unit has been transformed into radians. Since these data are independent of previous signals, the total  $\chi^2$  can be expressed as

$$\chi_{\rm BAO}^2 = \chi_{\rm matrix}^2 + \chi_{\rm angular}^2.$$
(18)

# 3.2 Observational Hubble Parameter Data

It is known that we use the distance scale of SNe Ia, CMB and BAO to constrain cosmological parameters. However, the distance scale is determined by integrating the Hubble parameter, which cannot reflect the fine structure of H(z). Thus, investigating the H(z) data directly reveals a more realistic evolution of the Universe. Many works have been done using the newly revised H(z) (Xu & Wang 2011; Ma & Zhang 2011). The measurement of Hubble parameter data as a function of redshift z depends on the differential ages of red-envelope galaxies

$$H(z) = -\frac{1}{1+z}\frac{dz}{dt}.$$
 (19)

We choose 12 data from this method (Riess et al. 2009; Stern et al. 2010). In addition, the data can be obtained from the BAO scale as a standard ruler in the radial direction. So, we choose another three data at different redshifts (Gaztañaga et al. 2009a).

The  $\chi^2$  value of the H(z) data can be expressed as

$$\chi_H^2 = \sum_{i=1}^{15} \frac{[H(z_i) - H_{\text{obs}}(z_i)]^2}{\sigma_i^2},$$
(20)

where  $\sigma_i$  is the  $1\sigma$  uncertainty of the observational H(z) data.

# 3.3 Type Ia Supernovae

SNe Ia have been playing an important role in studying cosmology since they first revealed the accelerated expansion of the universe. The recent data (Union2) are given by the Supernova Cosmology Project collaboration including 557 values (Amanullah et al. 2010). The new data have been used to constrain cosmological models (Xu & Wang 2010a,b; Liao et al. 2012). In practice, people use the distance moduli of supernovae to reflect the cosmological model and constrain the cosmological parameters. The distance moduli are determined by the luminosity distance

$$\mu = 5 \log(d_{\rm L}/{\rm Mpc}) + 25, \tag{21}$$

where  $d_{\rm L}$  is the luminosity distance. In the flat universe, it is connected with redshift which is an observable quantity

$$d_{\rm L} = (1+z) \int_0^z dz' / H(z').$$
(22)

We choose the marginalized nuisance parameter (Nesseris & Perivolaropoulos 2005) for  $\chi^2$ 

$$\chi_{\rm SNe}^2 = A - \frac{B^2}{C},\tag{23}$$

where

$$A = \sum_{i}^{557} (\mu^{\text{data}} - \mu^{\text{theory}})^2 / \sigma_i^2,$$
  

$$B = \sum_{i}^{557} (\mu^{\text{data}} - \mu^{\text{theory}}) / \sigma_i^2,$$
  

$$C = \sum_{i}^{557} 1 / \sigma_i^2$$

and  $\sigma_i$  is the  $1\sigma$  uncertainty of the observed data.

#### 3.4 Cosmic Microwave Background

For CMB, the acoustic scale is related to the distance ratio and is expressed as

$$l_{\rm a} = \pi \frac{\Omega_{\rm k}^{-1/2} sinn \left[ \Omega_{\rm k}^{1/2} \int_{0}^{z_{*}} \frac{dz}{E(z)} \right] / H_{0}}{r_{\rm s}(z_{*})},$$
(24)

where  $sinn(\sqrt{|\Omega_k|}x)$  denotes  $sin(\sqrt{|\Omega_k|}x)$ ,  $\sqrt{|\Omega_k|}x$  and  $sinh(\sqrt{|\Omega_k|}x)$ , for  $\Omega_k < 0$ ,  $\Omega_k = 0$  and  $\Omega_k > 0$ , respectively.  $r_s(z_*) = H_0^{-1} \int_{z_*}^{\infty} c_s(z)/E(z)dz$  is the comoving sound horizon at the epoch where photons decoupled. The CMB shift parameter R is expressed as (Bond et al. 1997)

$$R = \Omega_{\rm m0}^{1/2} \Omega_{\rm k}^{-1/2} sinn \left[ \Omega_{\rm k}^{1/2} \int_0^{z_*} \frac{dz}{E(z)} \right], \tag{25}$$

where the redshift  $z_*$  corresponding to the decoupling epoch of photons is given by (Hu & Sugiyama 1996)

$$z_* = 1048 \Big[ 1 + 0.00124 \Big( \Omega_{\rm b} h^2 \Big)^{-0.738} \Big( 1 + g_1 \Big( \Omega_{\rm m0} h^2 \Big)^{g_2} \Big) \Big], \tag{26}$$

where

$$g_1 = 0.0783(\Omega_{\rm b}h^2)^{-0.238}(1+39.5(\Omega_{\rm b}h^2)^{-0.763})^{-1},$$
  

$$g_2 = 0.560(1+21.1(\Omega_{\rm b}h^2)^{1.81})^{-1}.$$

For the CMB observation, we choose the data set including the acoustic scale  $(l_a)$ , the shift parameter (R) and the redshift of recombination  $(z_*)$ . The 7-year Wilkinson Microwave Anisotropy Probe (WMAP7) measurement gives the best-fit values of the data set (Komatsu et al. 2011)

$$\bar{\mathbf{P}}_{\rm CMB} = \begin{pmatrix} \bar{l}_{\rm a} \\ \bar{R} \\ \bar{z}_{*} \end{pmatrix} = \begin{pmatrix} 302.09 \pm 0.76 \\ 1.725 \pm 0.018 \\ 1091.3 \pm 0.91 \end{pmatrix}.$$
(27)

The  $\chi^2$  value of the CMB observation can be calculated as (Komatsu et al. 2011)

$$\chi^2_{\rm CMB} = \Delta \mathbf{P}_{\rm CMB}^{\rm T} \mathbf{C}_{\rm CMB}^{-1} \Delta \mathbf{P}_{\rm CMB}, \qquad (28)$$

where  $\Delta P_{\rm CMB} = P_{\rm CMB} - \bar{P}_{\rm CMB}$  and the corresponding inverse covariance matrix is

$$\mathbf{C}_{\rm CMB}^{-1} = \begin{pmatrix} 2.305 & 29.698 & -1.333\\ 29.698 & 6825.270 & -113.180\\ -1.333 & -113.180 & 3.414 \end{pmatrix}.$$
 (29)

### **4 CONSTRAINTS ON THE NGCG MODEL VIA THE MCMC METHOD**

In order to constrain the parameters of the NGCG model, we use the usual maximum likelihood method of the  $\chi^2$  fitting with the MCMC method. We implement the Metropolis-Hastings algorithm with a uniform prior probability distribution. By using the Monte Carlo method, we generate a chain of sample points distributed over the parameter space according to the posterior probability, then repeat the process until the established convergence accuracy is achieved. In our testing, the convergence of the chains R - 1 is set to be less than 0.003 which is small enough to satisfy the required accuracy. The code we use is based on CosmoMCMC (Lewis & Bridle 2002). We combine the SNe, BAO, CMB and H(z) data by multiplying the likelihood functions to constrain the NGCG model. The total  $\chi^2$  can be expressed as

$$\chi^2 = \chi^2_{\rm SNe} + \chi^2_{\rm BAO} + \chi^2_{\rm CMB} + \chi^2_H.$$
 (30)



Fig. 1 The 2-D regions and the 1-D marginalized distribution with the  $1\sigma$  and  $2\sigma$  contours of parameters  $\Omega_{\rm b}h^2$ ,  $\Omega_{\rm DM}h^2$ ,  $\omega_X$ ,  $\eta$ ,  $\Omega_X$ , Age/Gyr,  $\Omega_m$  and  $H_0$  in the NGCG model, for the data sets SNe+CMB+BAO+H(z).

We show the 1-D probability of each parameter  $(\Omega_b h^2, \Omega_{DM} h^2, \omega_X, \eta, \Omega_X, Age/Gyr, \Omega_m, H_0)$  $(\Omega_b, \Omega_{DM}, \Omega_X \text{ and } \Omega_m \text{ correspond to baryon matter, dark matter, dark energy and the total matter respectively) and 2-D plots for parameters in the NGCG model with SNe + CMB + BAO + <math>H(z)$  in Figure 1.

The best-fit values of NGCG model parameters with the four kinds of data above are

$$\begin{split} \omega_X &= -1.0510^{+0.1563}_{-0.1685}(1\sigma)^{+0.2226}_{-0.2398}(2\sigma) \,, \\ \eta &= 1.0117^{+0.0469}_{-0.0502}(1\sigma)^{+0.0693}_{-0.0716}(2\sigma) \,, \\ \Omega_b h^2 &= 0.0224^{+0.0012}_{-0.0010}(1\sigma)^{+0.0017}_{-0.0014}(2\sigma) \,, \\ \Omega_X &= 0.7297^{+0.0229}_{-0.0276}(1\sigma)^{+0.0329}_{-0.0402}(2\sigma) \,, \end{split}$$

with the  $\chi^2_{\text{min}} = 543.668$ . We also constrain the NGCG model with other combinations of data for comparison: SNe + CMB + BAO and CMB + BAO + H(z). They are shown in Figures 2 and 3 respectively.

The best-fit values of each parameter with the  $1\sigma$  and  $2\sigma$  uncertainties and  $\chi^2_{\min}$  are presented in Table 1. We can see the combination of the latest data gives a rather tight constraint on NGCG parameters, especially for  $\eta$ , which describes the interaction between dark energy and dark matter. The best-fit of  $\omega_X$  is slightly smaller than -1, which is a little different from Zhang et al. (2006),



**Fig. 2** The 2-D regions and 1-D marginalized distribution with the  $1\sigma$  and  $2\sigma$  contours of parameters  $\Omega_{\rm b}h^2$ ,  $\Omega_{\rm DM}h^2$ ,  $\omega_X$ ,  $\eta$ ,  $\Omega_X$ , Age/Gyr,  $\Omega_m$  and  $H_0$  in the NGCG model, for the data sets SNe+CMB+BAO.



**Fig.3** The 2-D regions and 1-D marginalized distribution with the  $1\sigma$  and  $2\sigma$  contours of parameters  $\Omega_{\rm b}h^2$ ,  $\Omega_{\rm DM}h^2$ ,  $\omega_X$ ,  $\eta$ ,  $\Omega_X$ , Age/Gyr,  $\Omega_m$  and  $H_0$  in the NGCG model, for the data sets CMB+BAO+H(z).

**Table 1** The best-fit values of parameters  $\Omega_{\rm b}h^2$ ,  $\Omega_{\rm DM}h^2$ ,  $\omega_X$ ,  $\eta$ ,  $\Omega_X$ , Age/Gyr,  $\Omega_m$  and  $H_0$  for the NGCG model with the  $1\sigma$  and  $2\sigma$  uncertainties, as well as  $\chi^2_{\rm min}$ , for the data sets SNe+CMB+BAO+H(z), SNe+BAO+CMB and CMB+BAO+H(z).

	The NGCG Model		
	SNe+CMB+BAO+ $H(z)$	SNe+BAO+CMB	CMB+BAO+H(z)
$\Omega_{\rm b}h^2$	$0.0224^{+0.0012}_{-0.0010}(1\sigma)^{+0.0017}_{-0.0014}(2\sigma)$	$0.0224^{+0.0012}_{-0.0011}(1\sigma)^{+0.0017}_{-0.0015}(2\sigma)$	$0.0225^{+0.0011}_{-0.0011}(1\sigma)^{+0.0016}_{-0.0015}(2\sigma)$
$\Omega_{\rm DM} h^2$	$0.1139^{+0.0079}_{-0.0075}(1\sigma)^{+0.0109}_{-0.0102}(2\sigma)$	$0.1144^{+0.0072}_{-0.0087}(1\sigma)^{+0.0106}_{-0.0118}(2\sigma)$	$0.1153^{+0.0084}_{-0.0097}(1\sigma)^{+0.0113}_{-0.0138}(2\sigma)$
$\omega_X$	$-1.0510^{+0.1563}_{-0.1685}(1\sigma)^{+0.2226}_{-0.2398}(2\sigma)$	$-1.0297^{+0.1765}_{-0.1829}(1\sigma)^{+0.2474}_{-0.2640}(2\sigma)$	$-1.1156^{+0.2786}_{-0.3150}(1\sigma)^{+0.3588}_{-0.4495}(2\sigma)$
$\eta$	$1.0117^{+0.0469}_{-0.0502}(1\sigma)^{+0.0693}_{-0.0716}(2\sigma)$	$1.0076^{+0.0502}_{-0.0504}(1\sigma)^{+0.0731}_{-0.0759}(2\sigma)$	$1.0170^{+0.0477}_{-0.0598}(1\sigma)^{+0.0673}_{-0.0907}(2\sigma)$
$\Omega_X$	$0.7297^{+0.0229}_{-0.0276}(1\sigma)^{+0.0329}_{-0.0402}(2\sigma)$	$0.7232^{+0.0305}_{-0.0298}(1\sigma)^{+0.0408}_{-0.0446}(2\sigma)$	$0.7367^{+0.0351}_{-0.0382}(1\sigma)^{+0.0458}_{-0.0543}(2\sigma)$
Age/Gyr	$13.73^{+0.14}_{-0.12}(1\sigma)^{+0.20}_{-0.18}(2\sigma)$	$13.75^{+0.16}_{-0.14}(1\sigma)^{+0.24}_{-0.20}(2\sigma)$	$13.70^{+0.21}_{-0.14}(1\sigma)^{+0.33}_{-0.19}(2\sigma)$
$\Omega_{\mathrm{m}}$	$0.2703^{+0.0276}_{-0.0229}(1\sigma)^{+0.0402}_{-0.0329}(2\sigma)$	$0.2768^{+0.0298}_{-0.0305}(1\sigma)^{+0.0446}_{-0.0408}(2\sigma)$	$0.2633^{+0.0382}_{-0.0351}(1\sigma)^{+0.0543}_{-0.0458}(2\sigma)$
$H_0$	$71.03^{+3.20}_{-3.37}(1\sigma)^{+4.55}_{-4.96}(2\sigma)$	$70.29^{+3.86}_{-3.76}(1\sigma)^{+5.61}_{-5.34}(2\sigma)$	$72.36^{+6.16}_{-5.89}(1\sigma)^{+8.45}_{-7.67}(2\sigma)$
$\chi^2_{\rm min}$	543.668	533.586	12.701

and  $\eta$  is a little larger than 1. Considering the uncertainty, these indicate that the evolution of the universe that the NGCG model represents is near the  $\Lambda$ CDM model. On the other hand, the results still accommodate an interacting XCDM model. In the 1- $\sigma$  range for SNe + CMB + BAO + H(z),  $\omega_X = (-1.2195, -0.8947)$ ,  $\alpha = \eta - 1 = (-0.0385, 0.0585)$ . The result of  $\omega_X$  indicates that the dark energy acting as a phantom is slightly favored. The constraint on  $\alpha$  indicates the probability that energy transfers from dark matter to dark energy is a little larger than the inverse. Moreover, since  $\alpha$  is constrained to a very small range near 0, the interaction between dark sectors seems very weak. The results also totally rule out the original CG model and the VCG model which require  $\alpha = 1$ . This is in agreement with Zhang et al. (2006); Wu & Yu (2007).

# **5** CONCLUSIONS

With the MCMC method, we give constraints on the NGCG model from the latest data including the SNe Ia data from Union2, BAO data from the SDSS DR7 galaxy sample and CMB observations from WMAP7, as well as the newly obtained angular BAO signal  $\theta_{BAO}(z = 0.55) = (3.90^{\circ} \pm 0.38^{\circ})$ , which is the first direct measurement of the pure angular BAO signal. The best-fit values of the parameters for NGCG are  $\omega_X = -1.0510^{+0.1563}_{-0.1685}(1\sigma)^{+0.2226}_{-0.2292}(2\sigma)$ ,  $\eta = 1.0117^{+0.0469}_{-0.0502}(1\sigma)^{+0.0693}_{-0.0716}(2\sigma)$ ,  $\Omega_b h^2 = 0.0224^{+0.0012}_{-0.0010}(1\sigma)^{+0.0017}_{-0.0014}(2\sigma)$  and  $\Omega_X = 0.7297^{+0.0229}_{-0.0276}(1\sigma)^{+0.0329}_{-0.0402}(2\sigma)$ , with the  $\chi^2_{min} = 543.668$ , which give a rather tight constraint. From the results we can see that the  $\Lambda$ CDM model is near the best-fit point and it remains a good choice for explaining the observation. However, the NGCG model permits an interacting XCDM model. The constraint results seem to slightly favor a phantom and the energy transfers from dark matter to dark energy. Moreover, since  $\alpha$  is constrained to a very small range near 0, the interaction between dark sectors seems very weak. The results are consistent with the situations in references (Guo et al. 2007; Chen et al. 2010; Feng et al. 2008; Cao et al. 2011a; Cao et al. 2011b). These papers as well as ours demonstrate that current observations cannot distinguish the directions of energy transformation well, thus the coincidence problem has not been solved so far in this way. For future study on the coincidence problem, we hope more data and other independent observations can improve the distinguishability. Besides, since the constraint results for these interacting XCDM models slightly favor the energy transfers from dark matter to dark energy transfers from dark matter to dark energy we may suspect this method might not be valid for explaining the coincidence problem.

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