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The effects of strong magnetic fields on neutron star structure: lowest order constrained variational calculations

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Abstract We investigate the effects of strong magnetic fields upon the large-scale properties of neutron and protoneutron stars. In our calculations, the neutron star matter was approximated by pure neutron matter. Using the lowest order constrained variational approach at zero and finite temperatures, and employing AV_{18} potential, we present the effects of strong magnetic fields on the gravitational mass, radius, and gravitational redshift of neutron and protoneutron stars. It is found that the equation of state for a neutron star becomes stiffer with an increase of magnetic field and temperature. This leads to larger values of the maximum mass and radius for the neutron stars.

Key words: dense matter — equation of state — stars: magnetic fields — stars: fundamental parameters (masses, radii, temperatures)

1 INTRODUCTION

Compression of magnetic flux inherited from the progenitor star could form a strong magnetic field in the interior of a neutron star (Reisenegger 2007). Using this point of view, Woltjer predicted a magnetic field strength of order 10^{15} G for neutron stars (Woltjer 1964). In the core of high density inhomogeneous gravitationally bound neutron stars, the magnetic field strength can be as large as 10^{20} G (Ferrer 2010). In addition, considering the formation of a quark core in the high density interior of a neutron star, the maximum field reaches up to about 10^{20} G (Ferrer 2010; Tatsumi 2000). According to the scalar virial theorem, which is based on Newtonian gravity, the magnetic field strength is allowed to have values up to 10^{18} G in the interior of a magnetar (Lai & Shapiro 1991). On the other hand, general relativity predicts the allowed maximum value of the neutron star's magnetic field to be about $10^{18}\ - 10^{20}$ G (Shapiro & Teukolsky 1983). By comparing with the observational data, Yuan et al. obtained a magnetic field strength of order 10^{19} G for neutron stars (Yuan & Zhang 1998).

A strong magnetic field could have an important influence on the structure of a neutron star. Some authors have studied the effects of strong magnetic fields on the properties of neutron stars. Bocquet et al. extended the numerical code for computing rotating neutron stars containing perfect fluid in the context of general relativity to include the electromagnetic fields and studied rapidly rotating neutron stars endowed with magnetic fields (Bocquet et al. 1995). The results show that for a magnetic field $B \sim 10^{18}$ G, the maximum mass increases by 13% to 29% (depending upon the equation of state) with respect to the maximum mass of non-magnetized stars. Within a relativistic Hartree approach in a simple linear $\sigma-\omega-\rho$ model, Chakrabarty et al. studied the large-scale properties of cold symmetric nuclear matter and nuclear matter in beta equilibrium under the influence of strong magnetic fields (Chakrabarty et al. 1997). They showed that for magnetic fields $B_m = 0$, 4.4×10^{17} and 10^{20} G, the maximum masses are $M_{\rm max} = 3.10$, 2.99 and $2.91\ M_\odot$, with radii $R_{M_{\rm max}} = 15.02, 14.95$ and 12.25 km, respectively. Based on two nonlinear $\sigma-\omega$ models of nuclear matter, Yuan et al. considered the properties of neutron stars under the influence of strong magnetic fields (Yuan & Zhang 1999). They found that the equation of state became softer with an increase of the magnetic field. The results show that for the ZM model, the maximum masses are $M_{\rm max} = 1.70$ and $1.62\ M_\odot$ for $B = 0$ and 10^{20} G respectively, with corresponding radii $R_{M_{\rm max}} = 9.82$ and 8.70 km. Furthermore, for the BB model, the maximum masses are $M_{\rm max} = 2.26$ and $2.07\ M_\odot$ for $B = 0$ and 10^{20} G, with radii $R_{M_{\rm max}} = 12.07$ and 10.09 km respectively. Cardall et al. studied static neutron stars with magnetic fields and a simple class of electric current distributions consistent with the stationarity requirement (Cardall et al. 2001). It has been demonstrated that the maximum mass of static neutron stars with magnetic fields determined by a constant current function is noticeably larger than that attainable with uniform rotation and no magnetic field. Within a relativistic field theory, Mao et al. considered neutron-star matter consisting of neutrons, protons and electrons interacting through the exchange of σ, ω and ρ mesons in the presence of a magnetic field which decreases from the center to the surface of a neutron star (Mao et al. 2003). It has been found that the equation of state becomes stiffer by increasing the magnetic field, which leads to an increase of 40% in the maximum mass of the neutron star.

In our previous studies, we have investigated the properties of neutron stars and protoneutron stars in the absence of a magnetic field (Bordbar et al. 2006; Bordbar & Hayati 2006; Bordbar et al. 2009; Yazdizadeh & Bordbar 2011). We have recently calculated the properties of spin polarized neutron matter in the presence of strong magnetic fields at zero (Bordbar et al. 2011) and finite temperatures (Bordbar & Rezaei 2012) by using the LOCV technique employing AV_{18} potential. In the present work, the neutron star matter is approximated by the pure neutron matter to investigate the effects of strong magnetic fields on the large-scale properties of neutron stars and protoneutron stars using the equations of state of neutron matter in the presence of strong magnetic fields (Bordbar et al. 2011; Bordbar & Rezaei 2012).

2 NEUTRON STAR STRUCTURE IN THE PRESENCE OF STRONG MAGNETIC FIELDS

In the present study, we calculate the neutron star and protoneutron star structure by using the equations of state of cold and hot neutron matter in the presence of strong magnetic fields (Bordbar et al. 2011; Bordbar & Rezaei 2012). In our study, we employ AV_{18} nuclear potential (Wiringa et al. 1995) and use the lowest order constrained variational method to calculate the equation of state. For more details, we refer the reader to Bordbar et al. (2011); Bordbar & Rezaei (2012). Our results for the equation of state of neutron matter in the presence of strong magnetic fields are given in Figures 1–3.

Figures 1(b) and 2(b) indicate that for the cold and hot neutron matter, at each density, the pressure increases with an increase of the magnetic field. This stiffening of the equation of state is due to the inclusion of neutron anomalous magnetic moments. In other words, in the presence of high magnetic fields, the fraction of polarized neutrons at the equilibrium state increases and therefore the degeneracy pressure grows. This is in agreement with the results obtained in references (Broderick et al. 2000; Yue & Shen 2006). We have found that at low densities, the influence of the magnetic field on the pressure is negligible.

Figure 3(b) shows that at each density, the pressure grows by increasing the temperature. Consequently, for hot neutron matter, the equation of state is stiffer compared to the cold one.

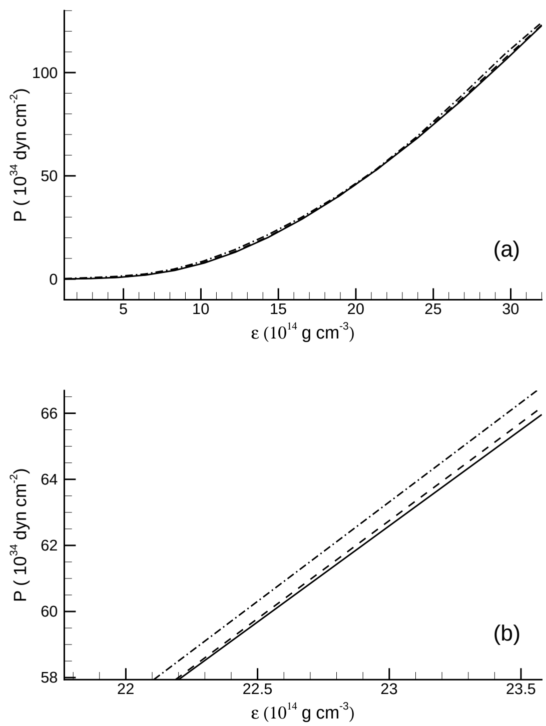

Fig. 1 (a) Pressure, P, versus energy density, ε, for the cases $B = 0$ G (*solid curve*), $B = 5\times10^{18}$ G (*dashed curve*) and $B = 10^{19}$ G (*dash-dotted curve*) at a fixed value of the temperature, $T = 0$ MeV. (b) Same as in the top panel but for a different range of energy density.

Figure 3(a) also shows that the effect of finite temperature on the equation of state is more significant at high densities.

The equilibrium configurations could be obtained by solving the general relativistic equations of hydrostatic equilibrium, called the Tolman-Oppenheimer-Volkoff (TOV) equations (Shapiro & Teukolsky 1983),

$$\begin{aligned} \frac{dm}{dr} &= 4\pi r^2 \varepsilon(r), \\ \frac{dP}{dr} &= -\frac{Gm(r)\varepsilon(r)}{r^2}\left(1+\frac{P(r)}{\varepsilon(r)c^2}\right)\left(1+\frac{4\pi r^3 P(r)}{m(r)c^2}\right)\left(1-\frac{2Gm(r)}{c^2 r}\right)^{-1}, \end{aligned} \tag{1}$$

where $\varepsilon(r)$ is the energy density, G is the gravitational constant and

$$m(r) = \int_0^r 4\pi r'^2 \varepsilon(r')dr' \tag{2}$$

gives the gravitational mass inside a radius r. By selecting a central energy density $\varepsilon_{\rm c}$, under the boundary conditions $P(0) = P_{\rm c}$, $m(0) = 0$, we integrate the TOV equations outwards to a radius $r = R$, at which P vanishes. This yields the radius R and mass $M = m(R)$ (Shapiro &

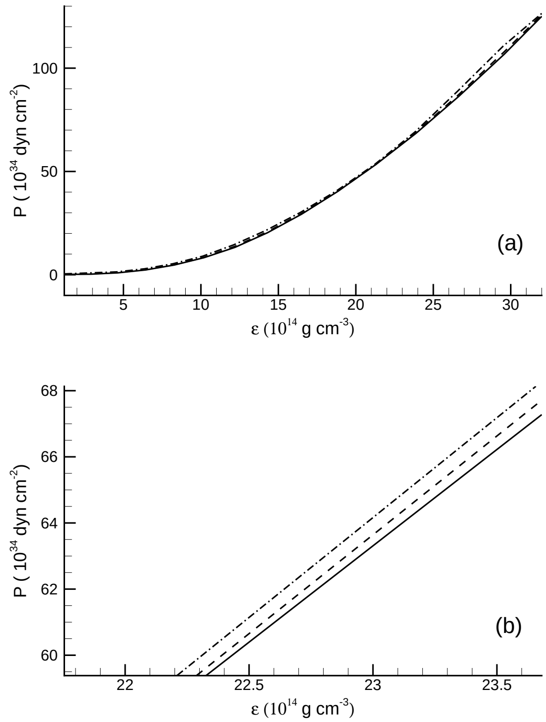

Fig. 2 (a) Pressure, P, versus energy density, ε, for the cases $B = 0$ G (*solid curve*), $B = 5\times10^{18}$ G (*dashed curve*) and $B = 10^{19}$ G (*dash-dotted curve*) at a fixed value of the temperature, $T = 15$ MeV. (b) Same as in the top panel but for a different range of energy density.

Teukolsky 1983). Gravitational redshift, the criterion for the star compactness, is given by

$$Z = \left[1 - 2\left(\frac{GM}{c^2R}\right)\right]^{-1/2} - 1, \tag{3}$$

where R is the radius of the neutron star. In our calculations of neutron star structure, for densities greater than 0.05 fm^{-3}, we use the equations of state presented in Figures 1–3. However, at lower densities, because the magnetic field and finite temperature have insignificant effects on the equation of state, we employ the equation of state calculated by Baym et al. (1971) for all magnetic fields and temperatures.

The effects of magnetic fields on the gravitational masses of neutron stars and protoneutron stars at a temperature of about 15 MeV with different central densities are presented in Figure 4. Obviously, at very low central densities, the gravitational masses are independent of the equation of state; but at higher densities, the gravitational mass increases by increasing both magnetic field and temperature. The limit of neutron star mass (maximum mass) also reaches a larger value when the magnetic field and temperature rise.

For a cold neutron star at $B = 10^{19}$ G, the maximum mass is about 1.17% larger than the cold non magnetized one. Considering two stars (a protoneutron star at $T = 15$ MeV and a cold neutron star) in the presence of a magnetic field $B = 10^{19}$ G, the protoneutron star's maximum mass is about 1.16% greater than the cold neutron star. Besides, for a protoneutron star at $T = 15$ MeV and

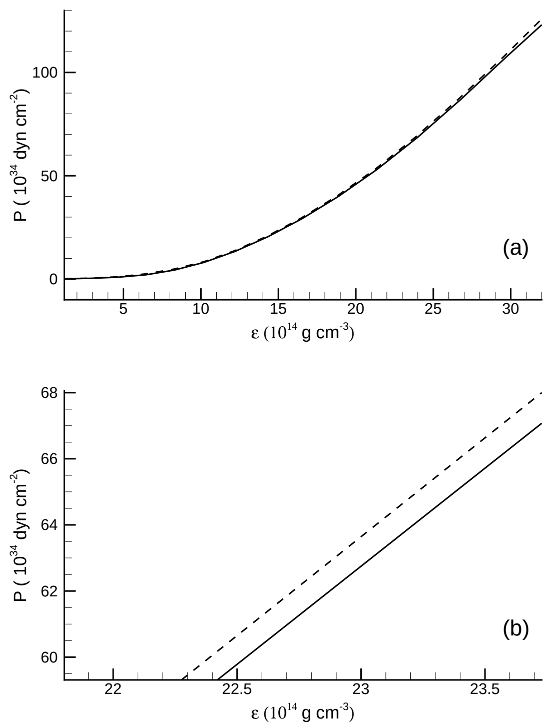

Fig. 3 (a) Pressure, P, versus energy density, ε, for the cases $T = 0$ MeV (*solid curve*) and $T = 15$ MeV (*dashed curve*) at a fixed value of the magnetic field, $B = 5 \times 10^{18}$ G. (b) Same as in the top panel but for a different range of energy density.

$B = 10^{19}$ G, the maximum mass increases about 2.36% compared to a cold non magnetized one. These results are due to the stiffening of the equation of state (Figs. 1–3).

Figure 5 presents the gravitational mass versus radius (M-R relation) for different magnetic fields at zero and finite temperatures. For all magnetic fields and temperatures, the neutron star mass decreases by increasing the radius.

It is clear from Figure 5 that for a given radius, the gravitational mass increases whenever the equation of state becomes stiffer. We have found that the effect of the equation of state on the M-R relation is more significant for neutron stars with a smaller radius.

Figure 6 shows the gravitational redshift versus the gravitational mass of the neutron star for different magnetic fields at zero and finite temperatures. Clearly, the stiffness of the equation of state reduces the gravitational redshift.

Figure 6 also indicates that the maximum redshift (redshift corresponding to the maximum mass) decreases with the increase of maximum mass.

For a cold and a hot neutron star at $T = 15$ MeV with $B = 10^{19}$ G, the values of maximum redshift are $z_s^{\max} = 0.49$ and $z_s^{\max} = 0.47$, respectively. In addition, we have found that in the case of a cold neutron star, for magnetic fields $B = 0$, 5×10^{18} and 10^{19} G, the values of $z_s^{\max}$ are 0.56, 0.53 and 0.49, respectively. Therefore, the maximum surface redshift of our calculations, i.e. 34.18% (for a cold neutron star at $B = 10^{19}$ G), is lower than the upper bound on the surface redshift for a subluminal equation of state, i.e. $z_s^{\mathrm{CL}} = 0.8509$ (Haensel et al. 1999).

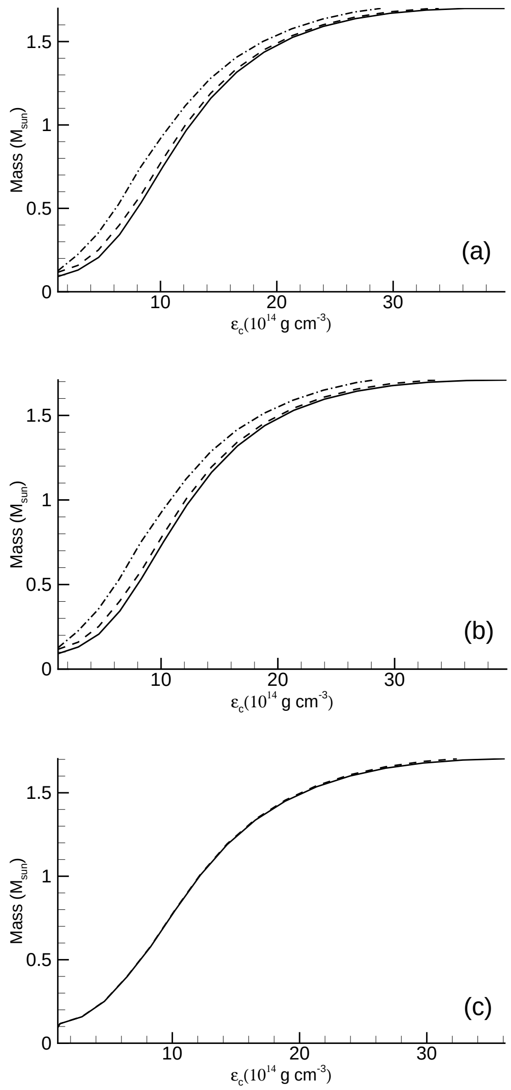

Fig. 4 (a) Gravitational mass of a neutron star (in units of the solar mass, $M_\odot$) versus central energy density, ε_c, at $T = 0$ MeV. All curves correspond to those in Fig. 1. (b) Same as (a) but at $T = 15$ MeV. All curves correspond to those in Fig. 2. (c) Gravitational mass of a neutron star (in units of the solar mass, $M_\odot$) versus central energy density, ε_c, at $B = 5 \times 10^{18}$ G. All curves correspond to those in Fig. 3.

Tables 1 and 2 show a summary of our results for the maximum mass and the corresponding radius predicted for different neutron stars. We have found that the effects of magnetic fields with magnitude $B \leq 10^{18}$ G are almost negligible. Obviously, for cold neutron stars as well as protoneutron stars, the maximum mass and the corresponding radius increase by increasing the magnetic field. Tables 1 and 2 show that at any magnetic field, the maximum mass and the corresponding radius of the protoneuton star are larger than those of the cold neutron star. Therefore, we conclude that the

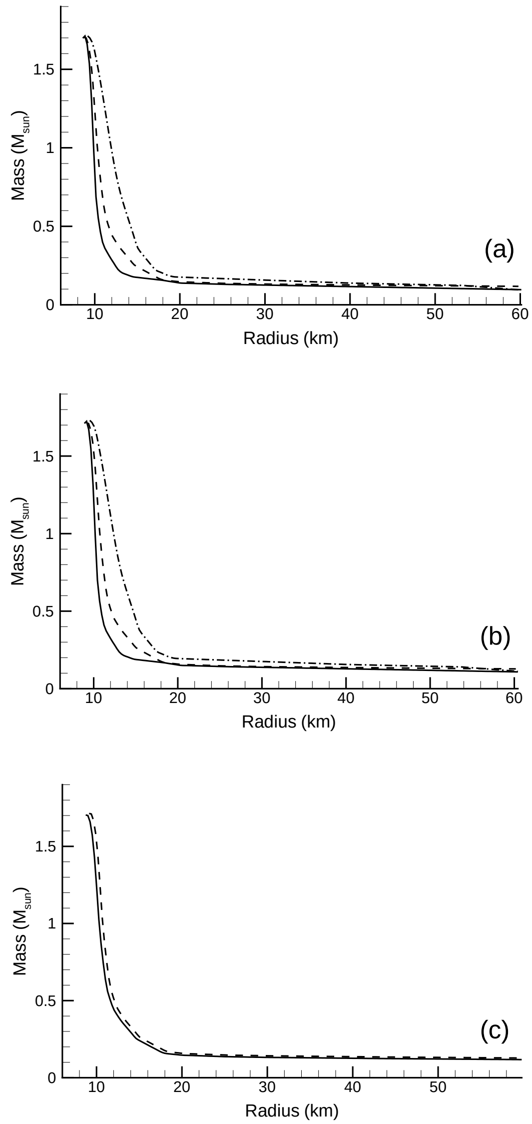

Fig. 5 (a) Mass-radius relation at $T = 0$ MeV. All curves correspond to those in Fig. 1. (b) Same as (a) but at $T = 15$ MeV. All curves correspond to those in Fig. 2. (c) Mass-radius relation at $B = 5 \times 10^{18}$ G. All curves correspond to those in Fig. 3.

stiffer equation of state leads to a neutron star with a larger maximum mass and radius. According to our results, for a cold neutron star, the maximum mass can vary between 1.69 and $1.71\ M_\odot$, depending on the interior magnetic field, but for a protoneutron star with $T = 15$ MeV, this variation is between 1.70 and $1.73\ M_\odot$. Therefore, the effect of magnetic field on the protoneutron star's maximum mass is more important than for the cold neutron star's maximum mass. Our results for the neutron star's maximum mass are higher than the observational results from X-ray binaries presented in Table 3. Moreover, the study of the statistics of 61 measured masses of neutron stars in binary pulsar systems gives an average mass of $M = 1.46 \pm 0.3\ M_\odot$ (Zhang et al. 2011). Their results indicate

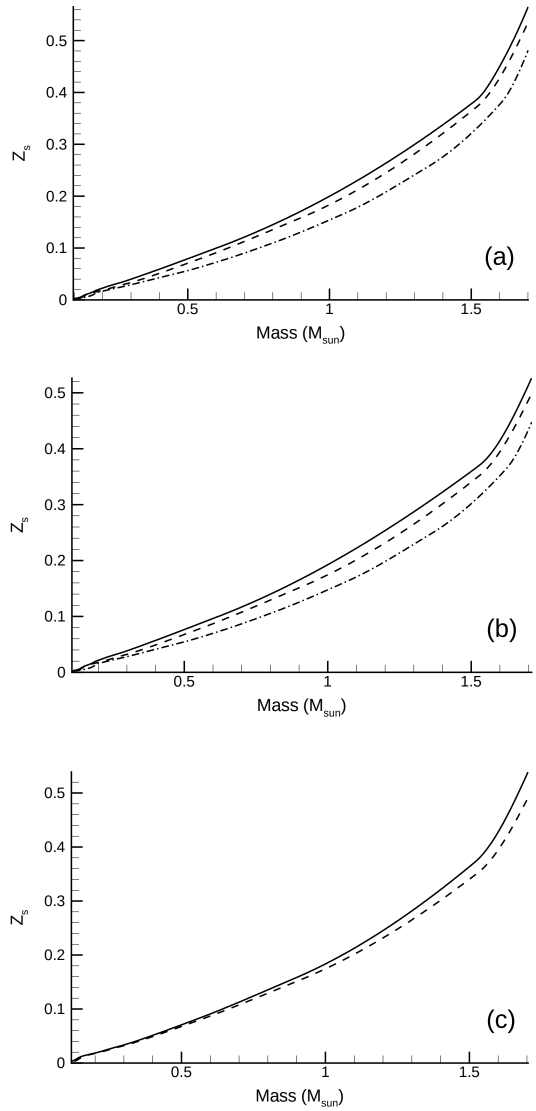

Fig. 6 Gravitational redshift, Z_s, vs. total mass for neutron stars at $T = 0$ MeV. All curves correspond to those in Fig. 1. (b) Same as (a) but at $T = 15$ MeV. All curves correspond to those in Fig. 2. (c) Gravitational redshift, Z_s, vs. total mass for neutron stars at $B = 5 \times 10^{18}$ G. All curves correspond to those in Fig. 3.

that the average mass of the more rapidly rotating millisecond pulsars is $M = 1.57 \pm 0.35\ M_\odot$. In the present work, the values of the radius for a protoneutron star at higher magnetic fields are near the values obtained using M-R relationships (Zhang et al. 2007), which show that the neutron star's radius varies in the range of $10 - 20$ km. We have also found that the effect of magnetic field on the radius of the protoneutron star is less important in the case of the cold neutron star.

Table 1 Maximum gravitational mass, $M_{\max}$, and the corresponding radius, $R_{M_{\max}}$, obtained for different values of magnetic field, B, at $T = 0$ MeV.

B (G)	$M_{\max}$ $(M_{\odot})$	$R_{M_{\max}}$ (km)
0	1.69	8.59
5×10^{18}	1.70	8.73
10^{19}	1.71	9.16

Table 2 Same as Table 1 but at $T = 15$ MeV

B (G)	$M_{\max}$ $(M_{\odot})$	$R_{M_{\max}}$ (km)
0	1.70	8.70
5×10^{18}	1.71	8.83
10^{19}	1.73	9.22

Table 3 Measured Masses of Neutron Stars in X-ray Binaries

System	M $(M_{\odot})$	Reference
SMC X-1	1.05 ± 0.09	van der Meer et al. 2005
Cen X-3	1.24 ± 0.24	van der Meer et al. 2005
LMC X-4	1.31 ± 0.14	van der Meer et al. 2005
V395 CAR/2S 0921C630	1.44 ± 0.10	Steeghs et al. 2007

3 SUMMARY AND CONCLUSIONS

Different properties in terms of the structures of a neutron star and protoneutron star have been investigated using the equation of state of neutron matter in the presence of strong magnetic fields. In our calculations, we have employed the lowest order constrained variational method and applied AV_{18} potential to find the equation of state at zero and finite temperature in the presence of strong magnetic fields. Our results show that the stiffer equation of state at higher magnetic fields and larger values of temperatures lead to higher values for the maximum mass and radius. For the maximum value of the magnetic field considered in this study, i.e. 10^{19} G, the maximum masses of a cold neutron star and protoneutron star at $T = 15$ MeV are 1.71 and 1.73 $M_{\odot}$, respectively. The corresponding radii are also 9.16 and 9.22 km. Our results indicate that the effects of magnetic field on the maximum mass of the protoneutron stars are more important than in the case of cold neutron stars, but the effects of magnetic fields are more visible on the radius of cold neutron stars. It has been shown that the effects of the equation of state on the M-R relation are more important for neutron stars with smaller radii. Our calculations also demonstrate that the maximum value of the gravitational redshift at the surface decreases by increasing the neutron star's maximum mass.

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