

## Mysteries of the geometrization of gravitation <sup>\*</sup>

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**Abstract** As we now know, there are at least two major difficulties with general relativity (GR). The first one is related to its incompatibility with quantum mechanics, in the absence of a consistent, widely accepted theory that combines the two theories. The second problem is related to the requirement of the dark sectors—inflaton, dark matter and dark energy by the energy-stress tensor, which are needed to explain a variety of astronomical and cosmological observations. Research has indicated that the dark sectors themselves do not have any non-gravitational or laboratory evidence. Moreover, the dark energy poses, in addition, a serious confrontation between fundamental physics and cosmology. Guided by theoretical and observational evidences, we are led to an idea that the source of gravitation and its manifestation in GR should be modified. The result is in striking agreement with not only the theory, but also the observations, without requiring the dark sectors of the standard approach. Additionally, it provides natural explanations to some unexplained puzzles.

**Key words:** general relativity and gravitation — fundamental problems and general formalism — cosmology: observations

### 1 INTRODUCTION

Einstein's theory of general relativity (GR), which provides the current description of gravitation in modern physics, ranks as one of the crowning intellectual achievements of the twentieth century. It is a geometric theory of gravitation which describes gravity not as a 'force' in the usual sense but as a manifestation of the curvature of spacetime. In particular, the curvature of spacetime is directly related to the energy-stress tensor  $T^{\mu\nu}$  through the Einstein field equations defined by

$$R^{\mu\nu} - \frac{1}{2}R g^{\mu\nu} = -\frac{8\pi G}{c^4}T^{\mu\nu}, \quad (\mu, \nu = 0, 1, 2, 3). \quad (1)$$

It should be noted that Equation (1) attributes the source of curvature entirely to matter, as the tensor  $T^{\mu\nu}$  does not include the energy, momenta or stresses associated with the gravitational field itself (since a proper energy-stress tensor of the gravitational field does not exist), though it does incorporate all the candidates of material fields including dark energy.

Although GR is not the only relativistic theory of gravitation, it is the simplest theory that has survived the tests of nearly a century of observational confirmation ranging from the solar system to

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the largest scales, including the universe itself. However, this success is achieved provided we admit three completely independent new components in the energy-stress tensor – inflaton, dark matter and dark energy, which are believed to play significant roles in the dynamics of the universe during their turns. However, there has been, until now, no non-gravitational or laboratory evidence for any of these dark sectors. Additionally, the mysterious dark energy, which has been evoked primarily to fit the observations of type Ia supernovae (SNeIa), poses a serious confrontation between fundamental physics and cosmology.

Despite the remarkable success of GR, many researchers interpret the observations supporting the requirement of the dark sectors as a failure of the theory. This reminds us of Einstein's 'biggest blunder' when he forced his equations to predict the unstable static universe (by imposing the cosmological constant) though a more natural implication of GR – the expanding universe – was already known to him. It appears that we have encountered a similar situation when we are trying to explain the observations in terms of the dark sectors. Though there have been other simpler explanations, they have not been paid proper attention. For example, it has been known since the first generation of SNeIa observations that the data are consistent with the 'vacuum' Friedmann-Robertson-Walker model ( $\Omega_m = 0 = \Lambda$ ).

While Einstein's blunder was perhaps motivated by his religious conviction that the universe must be eternal and unchanging, in the present case it is one's deep-rooted conviction that space would remain empty unless it is filled with  $T^{\mu\nu}$ . But what are the reasons to doubt this obvious and well-established notion? We shall see in the following that expressions derived from Equation (1) do not necessarily represent an empty space in the absence of  $T^{\mu\nu}$  and the sources of gravitation do exist there in the form of the geometry itself. Although this may appear orthogonal to the usual understanding, it is strongly supported by observations, ranging from the solar system to the largest scales, which seem to favor Equation (1) without  $T^{\mu\nu}$ , implying that the tensor is not needed.

Then, let us first see how the 'vacuum' field equations are supported by observations. What are the observations/experiments which have directly tested the complete Einstein's Equation (1)? The classical tests of GR consider  $T^{\mu\nu} = 0$ . The same is true for the more precise tests of GR made through the observations of radio pulsars, which are rapidly rotating strongly magnetized neutron stars. The pulsar tests assume the neutron stars to be point-like objects and look for the relativistic corrections in the post-Keplerian parameters by measuring the pulsar timing. The tests do not even require knowledge of the exact nature of the matter that pulsars and other neutron stars are made of. As  $T^{\mu\nu} = 0$  implies  $T = 0 = R$  in which case Equation (1) reduces to

$$R^{\mu\nu} = 0, \quad (2)$$

all we can claim is that it is only Equation (2) which has been verified by the classical tests of GR. As these tests have only been limited to our galaxy, let us see how this equation fairs against the cosmological observations. For this purpose, let us first solve Equation (2) for a homogeneous and isotropic spacetime, as is expected on a large enough scale. Obviously, the considered symmetry of homogeneity and isotropy requires the metric to be the Robertson-Walker one given by

$$ds^2 = c^2 dt^2 - S^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right), \quad (3)$$

where  $S(t)$  is the scale factor of the universe. For the metric (3), expressions derived from Equation (2) yield

$$R^0_0 = \frac{3}{c^2} \frac{\ddot{S}}{S} = 0, \quad (4)$$

$$R^1_1 = R^2_2 = R^3_3 = \frac{1}{c^2} \left( \frac{\ddot{S}}{S} + 2 \frac{\dot{S}^2}{S^2} + 2kc^2 \frac{1}{S^2} \right) = 0, \quad (5)$$

which uniquely determine

$$S = ct \text{ with } k = -1, \quad (6)$$

so that the final solution reduces to

$$ds^2 = c^2 dt^2 - c^2 t^2 \left( \frac{dr^2}{1+r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right). \quad (7)$$

## 2 SUPPORT FROM THE COSMOLOGICAL OBSERVATIONS TO $R^{\mu\nu} = 0$

### 2.1 Observations of SNeIa

In order to study the compatibility of Equation (7) with the cosmological observations, let us first consider the observations of SNeIa. The solution given in Equation (6) is efficient enough to define uniquely, without requiring any inputs from the matter fields, the luminosity distance  $d_L$  of a source with redshift  $z$  by

$$d_L = cH_0^{-1}(1+z) \sinh[\ln(1+z)], \quad (8)$$

where  $H_0$  represents the present value of the Hubble parameter  $H = \dot{S}/S$ . It is already known that this model, albeit non-accelerating (and not decelerating), is consistent with the observations of SNeIa *without requiring any dark energy*. As early as 1998, the Supernova Cosmology Project team noticed from the analysis of their first-generation SNeIa data that the performance of the empty model ( $\Omega_m = 0 = \Omega_\Lambda$ ) is practically identical to that of the best-fit unconstrained cosmology with a positive  $\Lambda$  (Perlmutter et al. 1999). Let us consider a newer dataset<sup>1</sup>, for example, the ‘new gold sample’ of 182 SNeIa (Riess et al. 2007), which is a reliable set of SNeIa with reduced calibration errors arising from the systematics. The model (8) provides an excellent fit to the data with a value of  $\chi^2$  per degree of freedom (DoF) =  $174.29/181 = 0.96$  and a probability of the goodness of fit  $Q = 63\%$ . Obviously the standard  $\Lambda$ CDM model has an even better fit as it has more free parameters:  $\chi^2/\text{DoF} = 158.75/180 = 0.88$  and  $Q = 87\%$  obtained for the values  $\Omega_m = 1 - \Omega_\Lambda = 0.34 \pm 0.04$ . The best-fitting model given in Equation (8) and the  $\Lambda$ CDM model have been compared with this data sample in Figure 1.

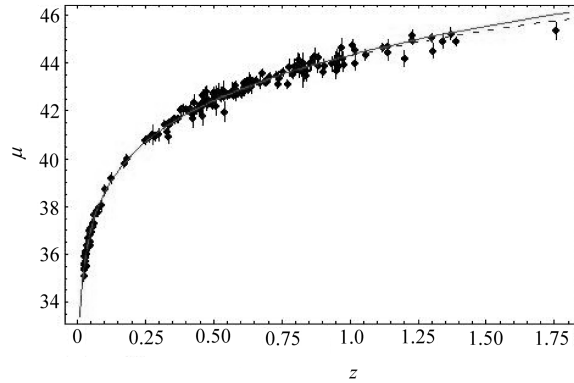
### 2.2 Observations of High-Redshift Radio Sources

Let us now consider the data on the angular size and redshift of radio sources compiled by Jackson & Dodgson (1997), which have 256 sources with their redshift in the range 0.5–3.8. These sources are ultra-compact radio objects with angular sizes of the order of a few milliarcseconds (mas), deeply embedded in galactic nuclei and have a very short lifetime compared with the age of the universe. Thus they are expected to be free from evolutionary effects and hence may be treated as standard rods, at least in the statistical sense. These sources are distributed into 16 redshift bins, with each bin containing 16 sources. This compilation has recently been used by many authors to test different cosmological models (Banerjee & Narlikar 1999; Vishwakarma 2000; Vishwakarma & Singh 2003; Vishwakarma 2007). In order to fit the data to the model, we derive the  $\Theta - z$  relation in the following. The angle  $\Theta$  subtended in a telescope, by a source with the proper diameter  $d$ , is given by

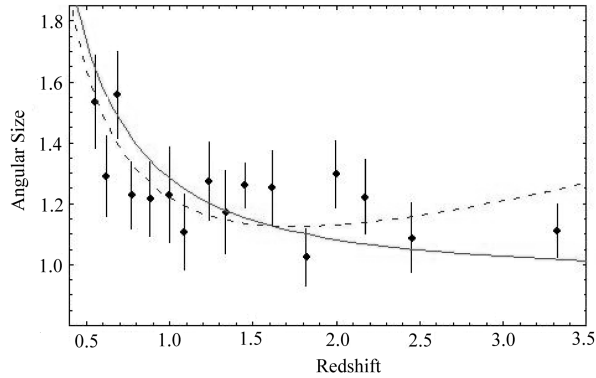
$$\Theta(z) = \frac{0.0688dh}{H_0 d_A} \text{ mas}, \quad (9)$$

where  $d$  is measured in pc,  $h$  is the present value of the Hubble parameter in units of  $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , and the angular diameter distance  $d_A = d_L/(1+z)^2$ .

<sup>1</sup> Although various newer SNeIa datasets are available, however, the way they are analyzed has left little scope for testing a theoretical model with them. This issue has been addressed in Vishwakarma & Narlikar (2010).



**Fig. 1** The ‘new gold sample’ of 182 SNeIa from Riess et al. (2007) is compared with some best-fitting models. The solid curve corresponds to the model (8) and the dashed curve corresponds to the spatially-flat  $\Lambda$ CDM model  $\Omega_m = 1 - \Omega_\Lambda = 0.34 \pm 0.04$ .



**Fig. 2** The data on the ultra-compact radio sources compiled by Jackson & Dodgson (1997) are compared with some best-fitting models. The solid curve corresponds to the model represented by Equation (7) and the dashed curve corresponds to the spatially-flat  $\Lambda$ CDM model  $\Omega_m = 1 - \Omega_\Lambda = 0.21 \pm 0.08$ .

We find that the present model has a satisfactory fit to the data with  $\chi^2/\text{DoF} = 20.78/15 = 1.39$  and  $Q = 14\%$ . In order to compare, we find that the best-fitting  $\Lambda$ CDM model has a slightly better fit:  $\chi^2/\text{DoF} = 16.03/14 = 1.15$  and  $Q = 31\%$  obtained for the values  $\Omega_m = 1 - \Omega_\Lambda = 0.21 \pm 0.08$ . These models are shown in Figure 2.

### 2.3 Observations on $H_0$ and the Age of the Oldest Objects

The age of the universe  $t_0$ , in big bang-like theories, is the time that has elapsed since the big bang. It depends on the expansion dynamics of the model and is given by

$$t_0 = \int_0^\infty \frac{H^{-1}(z)}{(1+z)} dz. \quad (10)$$

**Table 1** Magnitudes, with Uncertainties, of 13 HII Starburst Galaxies

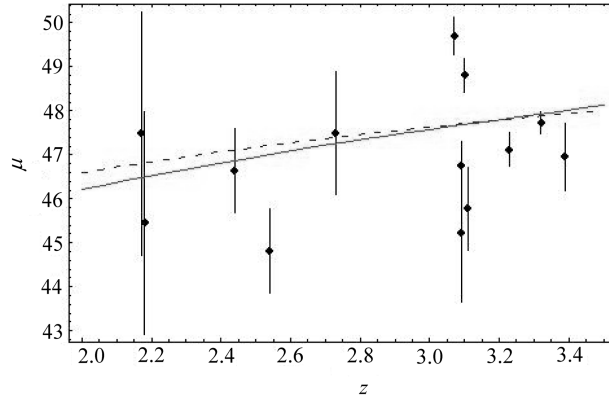
Galaxy	$z$	$\mu \pm \sigma$
Q0201-B13	2.17	$47.49^{+2.10}_{-3.43}$
Q1623-BX432	2.18	$45.45^{+1.97}_{-3.07}$
Q1623-MD107	2.54	$44.82^{+0.31}_{-1.58}$
Q1700-BX717	2.44	$46.64^{+0.31}_{-1.58}$
CDFa C1	3.11	$45.77^{+0.31}_{-1.58}$
Q0347-383 C5	3.23	$47.12^{+0.44}_{-0.32}$
B2 0902+343 C12	3.39	$46.96^{+0.71}_{-0.81}$
Q1422+231 D81	3.10	$48.81^{+0.38}_{-0.40}$
SSA22a-MD46	3.09	$46.76^{+0.56}_{-0.51}$
SSA22a-D3	3.07	$49.71^{+0.43}_{-0.41}$
DSF2237+116a C2	3.32	$47.73^{+0.25}_{-0.25}$
B2 0902+343 C6	3.09	$45.22^{+1.38}_{-1.76}$
MS1512-CB58	2.73	$47.49^{+1.22}_{-1.57}$

Hence, the Hubble parameter controls the age of the universe, which in turn depends on the free parameters of the model. For example, in the standard cosmology,  $H(z) = H_0\{\Omega_m(1+z)^3 + \Omega_\Lambda + (1 - \Omega_m - \Omega_\Lambda)(1+z)^2\}^{1/2}$ . Although  $t_0$  is a model-based parameter, a lower limit is put on it by requiring that the universe must be at least as old as the oldest objects in it. This is done through  $t_{GC}$ , the age of globular clusters in the Milky Way which are among the oldest objects we know so far. The parameter  $H_0$  can be estimated in a model-independent way, for example, from the observations of the low-redshift SNeIa, in which case the predicted magnitude does not depend on the model-parameters. One can use this value to calculate the age of the universe in a particular theory which can be compared with the age of the oldest objects. Thus the measurements of  $H_0$  and  $t_{GC}$  provide a powerful tool to test the underlying theory.

For example, by using the current measurements of  $H_0 = 71 \pm 6 \text{ km s}^{-1} \text{ Mpc}^{-1}$  from the Hubble Space Telescope Key Project (Mould et al. 2000), Equation (10) gives  $t_0$  for the Einstein-de Sitter model ( $\Omega_m = 1, \Lambda = 0$ ) as 9.18 Gyr. This cannot be reconciled with the age of the oldest globular cluster estimated to be  $t_{GC} = 12.5 \pm 1.2 \text{ Gyr}$  (Gnedin et al. 2001) and the age of the Milky Way as  $12.5 \pm 3 \text{ Gyr}$  coming from the latest uranium decay estimates (Cayrel et al. 2001). However, for the concordance  $\Lambda$ CDM model with  $\Omega_m = 1 - \Omega_\Lambda = 0.27$  (as estimated by the WMAP project (Larson et al. 2011)), Equation (10) gives a satisfactory age of the universe as  $t_0 = 13.67 \text{ Gyr}$  which is well above the age of the globular clusters. The age of the universe in the present model is given by  $t_0 = H_0^{-1}$ , as can be checked from Equation (6). For the above-mentioned value of  $H_0$ , this gives  $t_0 = 13.77 \text{ Gyr}$  which is even higher than the value from the concordance model.

## 2.4 Observations of Starburst Galaxies

Let us now consider the data on the apparent magnitude and redshift of starburst galaxies. Recent work has indicated that HII starburst galaxies might be considered as standard candles because of a correlation between their velocity dispersion, H luminosity and metallicity (see, for example, Mania & Ratra (2012) and references therein). Siegel et al. (2005) compiled a sample of 15 HII-like starburst galaxies with redshifts in the range 2.17–3.39 (by using the data available in the literature) in order to constrain  $\Omega_m$ . Mania & Ratra (2012) modified this sample by excluding two HII galaxies (Q1700-MD103 and SSA22a-MD41) that show signs of a considerable rotational velocity compo-



**Fig. 3** The HII-like starburst galaxy data (Mania & Ratra 2012) are compared with some best-fitting models. The solid curve corresponds to the model represented by Equation (8) and the dashed curve corresponds to the  $\Lambda$ CDM model with  $\Omega_m = 0.19$  and  $\Omega_\Lambda = 0.98$ .

nent and used the resulting sample (shown in Table 1) to constrain the cosmological models of dark energy.

For this sample, the present model provides the minimum value of  $\chi^2/\text{DoF} = 53.54/12 = 4.46$  with  $Q = 3.4 \times 10^{-7}$ , whereas the standard  $\Lambda$ CDM model gives the minimum  $\chi^2/\text{DoF} = 53.32/10 = 5.33$  with  $Q = 6.5 \times 10^{-8}$  for the values  $\Omega_m = 0.19$  and  $\Omega_\Lambda = 0.98$ . It would not be fair to claim that any of these models fit the data well. Perhaps the inherent scatter of the data is large, and a large sample size is required to perform the test and to get any meaningful constraint on the cosmological parameters. However, it is clear from the fitting results that compared to that of the standard model, the performance of the present model is better. The results are shown in Figure 3.

### 3 EVASION OF THE PROBLEMS OF STANDARD COSMOLOGY

The field equation  $R^{\mu\nu} = 0$  registers success, not only on the observational front, but also on the theoretical front. As we shall see in the following, the theory circumvents the long-standing problems of standard cosmology, for example, the horizon, flatness and the cosmological constant.

#### 3.1 Horizon Problem

The (particle) horizon distance, given by

$$d_H(t) = S(t) \int_0^t \frac{cdt'}{S(t')}, \quad (11)$$

sets a limit on the observable or causally connected part of the universe at time  $t$ . As a finite value exists for  $d_H$  in the standard cosmology, this means that the universe has a horizon in this theory. This is in conflict with the observed smoothness of the cosmic microwave background (CMB) at the largest scales in all directions, indicating that even the parts of the universe outside the horizon have been in causal contact. Since no physical process propagating at or below light speed could have brought them into thermal equilibrium, it appears that the universe required special initial conditions, which are supposed to be provided by inflation. This problem does not exist in the present theory as  $d_H = \infty$  at any time, as can be checked from Equations (6) and (11). Hence, the whole universe is always causally connected, which explains the observed uniformity of CMB without invoking the hypothetical inflaton field.

### 3.2 Flatness and Cosmological Constant Problems

The flatness problem of standard cosmology requires the initial density of matter (represented by the energy-stress tensor) to be extremely fine-tuned to its critical value (corresponding to a spatially flat universe). Even a tiny deviation from this value would have had drastic effects on the nature of the present universe. This problem is evaded in the present model owing to the fact that the matter tensor does not explicitly appear in the dynamical equations and all the fields are represented through the geometry.

The cosmological constant problem is circumvented for the same reason because its origin lies in a conflict between the energy-stress tensor and the vacuum expectation values derived from quantum field theory. Let us recall that the cosmological constant is represented by the energy-stress tensor of a perfect fluid (through a particular equation of state) and hence it is absent in the present theory. Hence, any other candidate of dark energy also does not exist in the present theory.

## 4 GRAVITY OF THE ‘VACUUM’ FIELD EQUATION $R^{\mu\nu} = 0$

It thus seems that equation  $R^{\mu\nu} = 0$  gets strong support not only from the observations but also from the theory. One may ignore this as chance happenings. However, we adopt the view that it is unlikely to have so many coincidences happen together and the theoretical as well as observational evidence supports equation  $R^{\mu\nu} = 0$  at all scales; these perhaps point towards some missing link of the theory, unnoticed so far. Let us then see if equation  $R^{\mu\nu} = 0$  can describe the real universe with matter, as the observations suggest. As the source of curvature is invariably matter, there have already been evidences available since the very inception of the theory, from a variety of solutions of equation  $R^{\mu\nu} = 0$ , which indicate that the space described by these equations does not necessarily need to be empty. For example, let us consider the following well-known solutions of equation  $R^{\mu\nu} = 0$  which have non-vanishing curvature.

### 4.1 Curved Solutions of $R^{\mu\nu} = 0$

#### Schwarzschild Solution:

Discovered by Karl Schwarzschild in 1915 immediately after GR was formulated, the solution forms the cornerstone of GR. It is believed to represent the spacetime structure outside an isotropic mass in an empty space (Hawking & Ellis 1973)

$$ds^2 = \left(1 + \frac{K}{r}\right) c^2 dt^2 - \frac{dr^2}{(1 + K/r)} - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2, \quad (12)$$

where  $K$  is a constant of integration. In fact, all the experiments which have so far been carried out to test GR are based on the predictions by this solution (except the Gravity Probe B experiments, which are based on the predictions of the Kerr solution).

#### Kerr Solution:

Discovered by Roy Kerr in 1963, the solution describes the spacetime surrounding a spherical mass  $m$  spinning with angular momentum per unit mass  $= \alpha$  (so that its total angular momentum  $= m\alpha$ ). In the Boyer-Lindquist coordinates (Hawking & Ellis 1973), the solution takes the form

$$ds^2 = \left(1 - \frac{r_S r}{\rho^2}\right) c^2 dt^2 - \frac{\rho^2}{\Delta} dr^2 - \rho^2 d\theta^2 - \left(r^2 + \alpha^2 + \frac{r_S r \alpha^2}{\rho^2} \sin^2 \theta\right) \sin^2 \theta d\phi^2 + \frac{2r_S r \alpha}{\rho^2} \sin^2 \theta d\phi c dt, \quad (13)$$

where  $\rho^2 = r^2 + \alpha^2 \cos^2 \theta$ ,  $\Delta = r^2 - r_S r + \alpha^2$  and  $r_S = 2Gm/c^2$  is the Schwarzschild radius. When  $\alpha = 0$ , the solution reduces to the Schwarzschild solution.

It may be mentioned that solutions (12) and (13) cannot be transformed to the flat Minkowski metric by any possible coordinate transformation, as the Riemann-Christoffel curvature tensor  $R_{\lambda\mu\nu\delta} \neq 0$  in these cases. Let us decipher the source of curvature in these solutions.

#### 4.2 Sources of Curvature in $R^{\mu\nu} = 0$

As Equation (2) is devoid of the source term  $T^{\mu\nu}$ , one may wonder from where the solution (12) derives its curvature. It is believed that the mystery of the presence of this curvature is related to the central singularity of the spherically symmetric space represented by (12) (which is associated, through a correspondence between the Newtonian and Einsteinian theories of gravitation in the case of a weak field, with an isotropic mass sitting at the center  $r = 0$ ). Thus, the source of curvature in Einstein's theory is regarded to be either  $T^{\mu\nu}$  or a singularity in  $g_{\mu\nu}$ . However, it should be noted that the metric (12) represents space exterior to the central mass at  $r = 0$  and *not* the point  $r = 0$  itself, where the metric breaks down. So, how can a mass situated at the point  $r = 0$  (which is not even represented by the metric) curve the space of (12) at points for which  $r > 0$ ? Obviously, one cannot expect the Newtonian theory of action-at-a-distance to work in the framework of GR which is a local theory.

A little reflection suggests that the agent responsible for the curvature in (12) at the points for  $r > 0$  must be the gravitational energy, which can definitely exist in an empty space. However, if this is true, one should be able to calculate the gravitational energy from the metric (12). We can certainly do this by the following two simple observations:

- (1) The metric (12) departs from flat spacetime in the term  $K/r$ , implying that this term must be the source of curvature.
- (2) We have shown that the source of curvature in (12), at the points  $r > 0$ , must be the gravitational energy.

Taken together, these two points imply that  $K/r$  must be the gravitational energy (in the units with  $c = 1$ ) in (12). This is in perfect agreement with the way the value of the constant  $K$  is determined. Let us recall that the constant  $K$  in Equation (12) is specified in terms of the Newtonian gravitational potential energy (by requiring that in the case of a weak gravitational field, Newton's law should hold) giving  $g_{00} = 1 + 2\psi/c^2$  where  $\psi = -Gm/r$  is the gravitational energy (per unit mass) at a distance  $r$  from the central mass  $m$  producing the field. It is thus established that the source of the curvature of spacetime in (12) is the energy of the gravitational field present at the points exterior to  $r = 0$ .

However, the fact remains that no formulation of the gravitational energy has been incorporated in equation  $R^{\mu\nu} = 0$  (or in Eq. (1)). This implies that the gravitational energy already exists there implicitly in the geometry, through the non-linearity of the field equations, and no additional incorporation thereof is needed. This fits very well in the story of the failure to discover an energy-stress tensor for the gravitational field<sup>2</sup>. As mentioned earlier, the energy-stress tensor of the gravitational field is not included in  $T^{\mu\nu}$  in Einstein's field Equation (1), as this tensor does not exist. Here we find the answer to this mystery - the energy-stress tensor of the gravitational field does not exist simply because the tensor is not needed in the geometric framework of GR; it already exists there inherently in the geometry of equation  $R^{\mu\nu} = 0$ . We should note that in the weak-field approximation, the GR equations do reduce to the usual Newtonian dynamical equations; gravitational energy, force, etc. do emerge respectively from the metric tensor, the Christoffel symbol, etc., *without adding any formulation of the gravitational field energy to the Einstein equations.*

<sup>2</sup> It may be mentioned that despite the century-long dedicated efforts of many luminaries, the attempts to discover a unanimous formulation of the gravitational field energy in GR have failed. This is primarily because of the non-tensorial character of the energy-stress pseudotensors of the gravitational field and the lack of a unique agreed-upon formula for it. Secondly, this is because of the inherent difficulty in the localization of the gravitational energy.



The insight gained above about the futility of the energy-stress tensor of the gravitational field is also corroborated by the Kerr metric given in Equation (13) wherein the angular momentum also contributes to its curvature. This is an entirely new occurrence for GR. It may be mentioned that there is no place for the angular momentum in  $T^{\mu\nu}$  in the framework of Einstein’s theory, which needs to be extended to non-Riemannian curved spacetime with torsion (as in the Einstein-Cartan theory) to support asymmetric Ricci and metric tensors, so that an asymmetric energy-stress tensor of spin can appear on the right hand side of the field equations. (However, when the right hand side is vanishing, the Ricci tensor need not be asymmetric and the Einstein-Cartan ‘vacuum’ equations reduce to  $R^{\mu\nu} = 0$ .)

One may not show much inhibition to agree that expressions given by Equation (2) do contain energy of the gravitational field (by leveraging the controversial nature of the subject, as has been done), despite the monumental works of Tolman, Papapetrou, Landau-Lifshitz, Møller and Weinberg on the (pseudo) energy-stress tensors of the gravitational field. However, one would maintain that although expressions defined by Equation (2) contain energy of the gravitational field, they describe spacetime structures in an *otherwise empty* space. Further, the source of curvature in the solutions of Equation (2) must be the singularity which fuels the gravitational energy. Let us now consider another curved solution of Equation (2), which may not fit this interpretation very well.

**Kasner Solution:**

This important cosmological solution of equation  $R^{\mu\nu} = 0$  was discovered by Edward Kasner in 1921. Later it was rediscovered by V. V. Narlikar and K. R. Karmarkar in 1946 and again by Abraham Taub in 1951. The solution, which is a curved Bianchi type I metric, describes a model universe which is homogeneous but anisotropic. The solution is given by

$$ds^2 = c^2 dt^2 - t^{2p_1} dx^2 - t^{2p_2} dy^2 - t^{2p_3} dz^2, \tag{14}$$

where the constants  $p_1, p_2$  and  $p_3$  satisfy

$$p_1 + p_2 + p_3 = 1, \quad p_1 p_2 + p_2 p_3 + p_3 p_1 = 0.$$

It should be noted that the metric admits a singularity<sup>3</sup> at  $t = 0$  (note that all of the exponents  $p_1, p_2$  and  $p_3$  are not positive).

The usual interpretation provided to (14) is that it represents an empty homogeneous universe in which the space is expanding and contracting (anisotropically) at different rates in different directions (for example, for  $p_1 = p_2 = 2/3$  and  $p_3 = -1/3$ , the space is expanding in two directions and contracting in the third). As we have noted, the solution (14) contains a singularity at  $t = 0$  but not at any other time. However, a past singularity, which does not exist now, fueling the gravitational energy now without any other source, does not seem compatible with the understanding of the gravitational energy. Further, the curvature in (14) is expected to have contributions from the net non-zero momentum resulting from the anisotropic expansion/contraction of the homogeneous space. However, it does not make much sense to imagine momentum resulting from the expanding/contracting *empty* space with no matter. It does not make sense, in the first place, to think of expanding/contracting ‘homogeneous’ space *without* matter.

<sup>3</sup> The form of the Kasner metric discovered by Narlikar & Karmarkar (1946) is

$$ds^2 = c^2 dt^2 - (1 + nt)^{2p_1} dx^2 - (1 + nt)^{2p_2} dy^2 - (1 + nt)^{2p_3} dz^2, \tag{15}$$

where  $n$  is a constant. We note that by the use of the transformations  $nt = 1 + n\bar{t}$ ,  $x = n^{p_1} \bar{x}$ ,  $y = n^{p_2} \bar{y}$  and  $z = n^{p_3} \bar{z}$ , the metric (14) takes the form (15) in the new coordinates. As the singularity in (15) now appears at  $t = -1/n$ , it seems that it can be avoided with a positive  $n$ . However, as the average scale factor (= spatial volume<sup>1/3</sup> =  $1 + nt$ ) has the same behavior in the interval  $-1/n \leq t \leq 0$  as it has for  $0 \leq t$  (i.e., no bouncing behavior at  $t = 0$ ), this feature to push the singularity back before the time  $t = 0$  just appears as a rescaling of time.

As has been shown above, expressions from equation  $R^{\mu\nu} = 0$  do reveal gravitational energy, through geometry, in its curved solutions (12) and (13). Hence, the third curved solution (14) is also expected to contain the gravitational energy. However, unlike the solutions expressed in Equations (12) and (13) (which represent space outside the source mass), the cosmological solution (14) cannot be expected to have any ‘outside.’ Since the ultimate source of the gravitational field is matter, this implies that the source matter fields (together with the resulting gravitational field) must also be inherently contained in solution (14). Does it then mean that, like the gravitational field, the matter field is also inherently present in equation  $R^{\mu\nu} = 0$ ? Let us postpone this question until its answer emerges as a natural consequence from the following issue.

#### 4.3 On the Flatness of Solution (7)

It would be natural to ask why solution (7) is flat while the other solutions (12)–(14), of the same equation  $R^{\mu\nu} = 0$ , are curved. [It should be noted that, by the use of the transformations  $\bar{t} = t\sqrt{1+r^2}$  and  $\bar{r} = ctr$ , metric (7) can be brought to manifestly Minkowskian form in the coordinates  $\bar{t}, \bar{r}, \theta, \phi$  (Narlikar 2002).]

One may argue that solution (7) is Minkowskian simply because  $T^{\mu\nu}$  is zero in (2). However, if this is so, why do we get curved solutions (12)–(14) from the same equation  $R^{\mu\nu} = 0$ ? If expressions from equation  $R^{\mu\nu} = 0$  contain a gravitational field that acts as the source of curvature present in solutions (12)–(14), they must also do so in solution (7)<sup>4</sup>.

Obviously, solutions (12)–(14) have singularities to fuel the gravitational field, but solution (7) does not. However, what is there to stop the singularity from occurring in (7)? The only difference between the solutions (7) and (12)–(14) is that they have different types of symmetries in their spacetime structures. While metric (7) is homogeneous and isotropic, metrics (12)–(14) are either inhomogeneous or/and anisotropic. However, how a relaxation in the homogeneity and isotropy can result in a singularity cannot be answered by conventional wisdom. A possible explanation to the present situation leads us to the following two possibilities.

- (1) Equations arising from  $R^{\mu\nu} = 0$  represent an empty spacetime structure and can support curved as well as flat spacetime solutions. However, they are unable to explain how a solution acquires curvature or flatness. For instance, solutions (14) and (7) represent similar spacetime structures with the only difference being that while the homogeneous space in (14) is expanding and contracting in different directions at different rates, the same space is expanding or contracting isotropically in (7). How this difference accounts for their curved and flat states, and controls the appearance of the singularity, cannot be explained by equations from  $R^{\mu\nu} = 0$ .
- (2) The geometry of equation  $R^{\mu\nu} = 0$  does contain impressions of the gravitational as well as the matter fields. The structure of the geometry, of a chosen matter distribution, is determined by the net contribution from the material and the gravitational fields which, if non-zero, may be manifested in the guise of a singularity (which may be considered a general-relativistic analog of the ‘source’).

Though the second possibility appears baffling and orthogonal to usual understanding, it provides not only a sensible meaning to the Kasner solution (14) in the absence of the energy-stress tensors of the gravitational or the matter fields, but also a reasonable explanation to the flatness of solution (7), as we see in the following. If we believe that Equation (2) inherently contains material and gravitational fields, solution (7) would then represent homogeneously distributed matter

<sup>4</sup> Equations from  $R^{\mu\nu} = 0$  are not competent enough to decipher the source term  $m$  in the Schwarzschild and Kerr solutions (12) and (13), just from their symmetries. Rather, this is done through an additional constraint that GR should reduce to Newtonian gravitation in the case of a weak stationary gravitational field, as has been mentioned earlier. Hence, taken on face value, the other two solutions of  $R^{\mu\nu} = 0$ , viz. (7) and (14), must also have the same status and it is also quite probable to assign fields to them, in a manner consistent with their symmetries.

throughout the space at all times. As the positive energy of the matter field would be exactly balanced, point by point, by the negative energy of the resulting gravitational field (contrary to the case of the Schwarzschild solution where there is only the gravitational energy and no matter at the points represented by the metric), this would provide a net vanishing energy. There would also not be any momentum contribution from the isotropic expansion/contraction of the material system (contrary to the case of the Kasner solution). Hence, in the absence of any net non-zero energy, momentum or angular momentum, the spacetime of (7) would not have any curvature.

This implies that it is the symmetry of the chosen spacetime structure which determines whether a solution of  $R^{\mu\nu} = 0$  will be curved (may have a singularity) or flat (without a singularity). This is in perfect agreement with the appearance of different kinds of singularities, in accordance with the chosen symmetries in the solutions: while the Schwarzschild solution (describing the spacetime structure exterior to a point mass) has a point singularity, the Kerr solution (describing the spacetime structure exterior to a rotating mass) has a ring singularity; the Kasner solution (in which the  $t = \text{constant}$  hypersurfaces are expanding and contracting at different rates in different directions) presents an oscillating kind of singularity at  $t = 0$ .

The discovery of the net vanishing energy-momentum-angular momentum in a homogeneous distribution of matter expanding/contracting isotropically, appears to be consistent with several investigations and results which indicate that the total energy of the universe is zero. Thus, a flat spacetime solution, which has so far been a notion of special relativity (SR), can be achieved in the real universe in the presence of matter, which dynamically originates from the field equations, and is not assumed a priori (or just added by hand) as in SR. This new result also seems consistent with the theories of inflation which predict a nearly flat universe after expanding it by a factor of  $10^{78}$  in just  $10^{-36}$  seconds, leaving a nearly flat spacetime. Further, the appearance of a flat spacetime in the presence of matter is also not impossible in the conventional approach. For example, it has been shown by Ayón-Beato et al. (2005) that conformally coupled matter does not always curve spacetime.

It may be mentioned that solution (7) is generally considered as the Milne model, which is not quite correct when taken in the traditional context of the empty universe. Although the evolutionary dynamics of the Milne universe (based on Milne's kinematical relativity with foundations different from those of GR) are the same as those given by the empty Friedman model, it is not empty. Rather the Milne model assumes the Minkowskian spacetime filled with matter wherein the matter does not interact with the geometry due to some unknown reasons. The present theory, governed by the field equation  $R^{\mu\nu} = 0$ , not only explains why the homogeneous, isotropic universe is Minkowskian, but also predicts when a curved solution is also possible.

Thus the conclusion is that the spacetime already contains the 'field,' which must not be inserted again in the field equations of gravitation through the formulation of, for example,  $T^{\mu\nu}$ . It should be noted that being a geometric theory of gravitation, GR eliminates any possibility to represent gravitation in terms of a force. Rather the theory replaces the effects of the force through geometry. Similarly, the effects of stresses, momenta, angular momenta and energy are also revealed through geometry. Though this surprising discovery may appear too revolutionary to digest, it is in striking agreement with and corroborated by the recent studies on the relativistic formulation of matter which indicate that, like the energy-stress tensor of the gravitational field, a flawless energy-stress tensor of the matter fields also does not exist. This issue is discussed in brief in the next section.

#### 4.4 On the Formulation of Matter by $T^{\mu\nu}$

We have seen above that the appearance of the gravitational energy through the geometry of equation  $R^{\mu\nu} = 0$ , without adding any formulation of the energy-stress tensor of the gravitational field to this equation, is vindicated by the absence of a proper energy-stress tensor of the gravitational field. Do we have any similar evidence from the energy-stress tensor of the matter fields? Yes we

do. Recently, it has been shown (Vishwakarma 2012) that a critical analysis of the formulation of matter given by  $T^{\mu\nu}$  reveals some surprising inconsistencies and paradoxes. Corrections have been discovered to rectify the problems, which however render the theory incompatible with many observations (Vishwakarma 2012). This implies that the relativistic formulation of matter fields given by the energy-stress tensor  $T^{\mu\nu}$  is not consistent with the geometric description of GR. In fact, the relativistic formulation of matter in terms of  $T^{\mu\nu}$  has some more fundamental inconsistencies (besides those discovered in Vishwakarma 2012) which have not been realized earlier. We mention the following two.

- (1) The general expression of the energy-stress tensor  $T^{\mu\nu}$  is obtained by deriving it first in the absence of gravity, i.e., in SR. It is then imported to the actual case in the presence of gravity, by making use of an inertial observer, which exists admittedly at all points of spacetime (by courtesy of the principle of equivalence). Formulating a tensor representation of the fluid element in a small neighborhood of the observer, the expression of the tensor in the presence of gravity is imported, from SR to GR, through a coordinate transformation. This is the standard way to derive  $T^{\mu\nu}$  (Vishwakarma 2012). However, the simple point which has not been noticed, which makes this derivation questionable, is that an inertial coordinate system is valid only at a point, and *not* in a neighborhood, however small it is (Christoffel symbols can be made vanishing only at a point, and *not* in a neighborhood). However, because a fluid element cannot be defined at a point, we do need a neighborhood. Hence, at best, the tensor  $T^{\mu\nu}$  may approximate a dust (with vanishing pressure) but cannot represent a fluid (with non-zero pressure) which requires more than one particle to generate pressure.
- (2) As the derivation of  $T^{\mu\nu}$  assumes its validity in the absence of gravitation (in a flat spacetime), this becomes contradictory to the very notion of  $T^{\mu\nu}$  being the source of curvature. To exemplify this, let us note that in a flat spacetime, the left hand side of Equation (1) vanishes automatically, but not the right hand side which has to be made equal to zero by hand. That is, the source of curvature can exist there without producing any curvature. Although Equation (1), being a geometric formulation of gravitation, is expected to be valid in a curved spacetime, it must also consistently reduce to the no-gravitation case. This provides another reason why  $T^{\mu\nu}$  should not appear in the field equations of gravitation.

It should be noted that the relativistic description of the matter given by the energy-stress tensor  $T^{\mu\nu}$  has never been tested in any direct experiment. As has been mentioned earlier, the classical tests of GR consider  $T^{\mu\nu} = 0$  and test only the geometric aspect of GR given by equation  $R^{\mu\nu} = 0$ . The above-mentioned theoretical crisis in the relativistic formulation of matter acquires a new meaning in the present context and implies that the concept of the energy-stress tensor (be it of a matter field or a gravitational field) is not compatible with the geometric formulation of gravitation, simply because it is already inherently included in the geometry.

A similar view was expressed about four decades ago by J. L. Synge, one of the most distinguished mathematical physicists of the 20th Century: *“The concept of energy-momentum tensor is simply incompatible with general relativity.”*

## 5 DISCUSSION AND CONCLUSIONS

In spite of various observational verifications of GR, deep mysteries continue to haunt our theoretical understanding of the ingredients of the energy-stress tensor in the form of the dark sectors—inflaton, dark matter and dark energy, which do not have any non-gravitational or laboratory evidence and remain unidentified.

Motivated by this fact and guided by strong observational support for the so called ‘vacuum’ field equations expressed as  $R^{\mu\nu} = 0$ , we develop an understanding that the energy-stress tensor is perhaps a redundant part of the Einstein field equations and the source of gravitation is the geometry

itself. A critical analysis of the different solutions of equation  $R^{\mu\nu} = 0$  supports this view indicating that equation  $R^{\mu\nu} = 0$  *does not* represent an empty spacetime and impressions of the gravitational as well as material fields are inherently present in the equations (though they do not play a direct role in the dynamics), which are revealed through the geometry. Einstein believed that “*On the basis of the general theory of relativity, space as opposed to ‘what fills space’, has no separate existence*” (Einstein 1920). If this is true, considering a spacetime structure (conditioned by the equation  $R^{\mu\nu} = 0$ ) must be equivalent to considering the accompanying fields (material and gravitational) as well, and there should be no need to add any extra formulation thereof to the field equations.

The fact that the sources of gravitation are implicitly present in equations defined by  $R^{\mu\nu} = 0$  and must not be added again is vindicated by the failure to obtain a proper energy-stress tensor of the gravitational field. It is further supported by a number of paradoxes and inconsistencies discovered recently in the relativistic formulation of matter given by the energy-stress tensor  $T^{\mu\nu}$  implying that, as in the case of the gravitational field, a flawless proper energy-stress tensor of the matter fields also does not exist (Vishwakarma 2012). This, in fact, leaves equation  $R^{\mu\nu} = 0$  as the only possibility for a consistent field equation of gravitation in the existing framework of GR. One may argue that a consistent field equation of gravitation is expected to reduce to Poisson’s equation  $\nabla^2\psi = 4\pi G\rho$  in a weak stationary gravitational field. However, this requirement, which is already compromised in the concordance  $\Lambda$ CDM cosmology, no longer seems obligatory. It should be noted that the Einstein field equations with a non-zero<sup>5</sup>  $\Lambda$  do *not* fulfill this requirement (Weinberg 1972).

Although this entirely new insight about the geometry serving as the source of gravitation in the metric theories of gravity may appear orthogonal to the usual understanding, it is not only in striking agreement with the theory and observations, but also provides natural explanations to some unexplained puzzles. Additionally, it removes the long-standing problems of standard cosmology, viz., the horizon, the flatness and the cosmological constant.

It was Einstein’s obsession that the vibrant geometrical part of GR is ‘marble’ and matter is ‘wood,’ and that all attempts should be directed to turn wood into marble. It finally turns out that the ‘wood’ is a redundant part of the theory whose departure enhances the beauty of the ‘marble’ in the true field equation of gravitation  $R^{\mu\nu} = 0$  due to its extreme simplicity.

It may be mentioned that any proposed theory of gravitation, supplying a model of the universe, is expected to explain the observations of the CMB radiation and the baryon acoustic oscillations (BAO). Though a detailed discussion on this subject would require further study, it may be mentioned for the time being that, taken at face value, the only unanimous prediction of the CMB observations is a flat spatial geometry (Vishwakarma 2003; Blanchard 2005; Larson et al. 2011). As has been mentioned earlier, Equation (7) can be transformed to the Minkowskian form, by using suitable transformations, which does have a flat spatial geometry.

Additionally, we have shown that the universe is not empty in the present theory, though the matter fields do not play a direct role, hence providing full leverage for the parameters  $\Omega_m$ ,  $\Omega_b$ , etc., to fit the observations of CMB and BAO which indicate that  $\Omega_m \approx 0.3$  (Komatsu et al. 2011; Blake et al. 2011).

Finally, it would be worthwhile to mention an interesting interpretation (which is due to an anonymous referee) of the present findings. In terms of the standard picture, the gravitating sources can be viewed as a discrete distribution of point particles seen at a microscopic level (in a restricted sense). Although this interpretation would not hold in the case of the scalar fields or the cosmological constant, it would not create any real problem. The only obligatory scalar field required by the standard paradigm, to explain the horizon and flatness problems, is the inflaton field, which is however not required by the new theory, as we have seen earlier. While the horizon problem does not exist in the new theory as the whole universe is always causally connected, the flatness problem is averted due to the absence of the matter tensor. The cosmological constant or any other candidate

<sup>5</sup> It would not be correct to claim that  $\Lambda$  is negligible in the concordance cosmology, in order to give any advantage to this theory. In fact, the mass density associated with  $\Lambda$ , say,  $\rho_\Lambda = \Lambda c^2/8\pi G$ , is comparable with the matter density.

of dark energy is absent in the new theory, due to the same reason, as has been explained earlier. Thus the alternative interpretation appears promising and worth exploring further.

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## References

- Ayón-Beato, E., Martínez, C., Troncoso, R., & Zanelli, J. 2005, *Phys. Rev. D*, 71, 104037
- Banerjee, S. K., & Narlikar, J. V. 1999, *MNRAS*, 307, 73
- Blake, C., Kazin, E. A., Beutler, F., et al. 2011, *MNRAS*, 418, 1707
- Blanchard, A. 2005, *Dark Matter in Astro- and Particle Physics*, 34
- Cayrel, R., Hill, V., Beers, T. C., et al. 2001, *Nature*, 409, 691
- Einstein, A. 1920, *Relativity: The Special and the General Theory*
- Gnedin, O. Y., Lahav, O., & Rees, M. J. 2001, astro-ph/0108034
- Hawking, S. W., & Ellis, G. F. R. 1973, *The Large-scale Structure of Space-time* (Cambridge: Cambridge Univ. Press)
- Jackson, J. C., & Dodgson, M. 1997, *MNRAS*, 285, 806
- Kasner, E. 1921, *American Journal of Mathematics*, 43, 217
- Komatsu, E., Smith, K. M., Dunkley, J., et al. 2011, *ApJS*, 192, 18
- Larson, D., Dunkley, J., Hinshaw, G., et al. 2011, *ApJS*, 192, 16
- Mania, D., & Ratra, B. 2012, *Physics Letters B*, 715, 9
- Mould, J. R., Huchra, J. P., Freedman, W. L., et al. 2000, *ApJ*, 529, 786
- Narlikar, J. V. 2002, *An Introduction to Cosmology* (Cambridge: Cambridge Univ. Press), 140
- Narlikar, V. V., & Karamarkar, K. R. 1946, *Current Science*, 15, 69
- Perlmutter, S., Aldering, G., Goldhaber, G., et al. 1999, *ApJ*, 517, 565
- Riess, A. G., Strolger, L.-G., Casertano, S., et al. 2007, *ApJ*, 659, 98
- Siegel, E. R., Guzmán, R., Gallego, J. P., Orduña López, M., & Rodríguez Hidalgo, P. 2005, *MNRAS*, 356, 1117
- Taub, A. H. 1951, *Annals of Mathematics*, 53, 472
- Vishwakarma, R. G. 2000, *Classical and Quantum Gravity*, 17, 3833
- Vishwakarma, R. G. 2003, *MNRAS*, 345, 545
- Vishwakarma, R. G., & Singh, P. 2003, *Classical and Quantum Gravity*, 20, 2033
- Vishwakarma, R. G. 2007, *Nuovo Cimento B Serie*, 122, 113
- Vishwakarma, R. G., & Narlikar, J. V. 2010, *RAA (Research in Astronomy and Astrophysics)*, 10, 1195
- Vishwakarma, R. G. 2012, *Ap&SS*, 340, 373
- Weinberg, S. 1972, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (John Wiley & Sons), 155