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Relativistic transformation between τ and TCB for Mars missions: Fourier analysis on its accessibility with clock offset *

Jun-Yang Pan[1] and Yi Xie[1,2]

[1] Department of Astronomy, Nanjing University, Nanjing 210093, China; *yixie@nju.edu.cn*
[2] Key Laboratory of Modern Astronomy and Astrophysics, Nanjing University, Ministry of Education, Nanjing 210093, China

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Abstract In the context of the fact that Einstein's general relativity has become an inevitable part of deep space missions, we will extend previous works on relativistic transformation between the proper time τ of a clock onboard a spacecraft orbiting Mars and the Barycentric Coordinate Time (TCB) by taking the clock offset into account and investigate its accessibility by Fourier analysis on the residuals after fitting the $\tau-$TCB curve in terms of n-th order polynomials. We find that if the accuracy of a clock can achieve better than $\sim 10^{-5}$ s or $\sim 10^{-6}$ s (depending on the type of clock offset) in one year after calibration, the relativistic effects on the difference between τ and TCB will need to be carefully considered.

Key words: reference systems — time — methods: numerical — space vehicles

1 INTRODUCTION

Recent years have witnessed Einstein's general relativity (GR) becoming an inevitable part of deep space missions. This is driven by significant increases in measurement precisions with modern techniques. It also makes GR go far beyond the territory of theoretical astronomy and physics into the realm of practice and engineering. Relativistic effects obviously appear in the radio links with the Cassini spacecraft (Bertotti et al. 2003) and the New Horizons spacecraft (Jensen & Weaver 2008). The measurement of the frequency shift in the links connecting Cassini and Earth also yields the most stringent test for the validity of GR in the Solar System (Bertotti et al. 2003).

One fundamental point associated with GR is to distinguish between the proper time and coordinate time (Misner et al. 1973). The readings of an ideal clock are the proper time τ, which is in the reference frame of the clock. Although coordinate times *cannot* be measured directly, some of them can be taken as independent variables in the equations of motion of celestial and artificial bodies and photons. The coordinate time is related to the proper time through the invariant 4-dimensional space-time interval, which depends on the motion of the clock (effects of special relativity) and the gravitational fields (effects of GR). This dramatically changes the method of clock synchronization and time transfer (Nelson 2011).

For Mars and other exploration missions, the synchronization between the clock onboard the spacecraft and the clock on the ground is critical for control, navigation and scientific operation.

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As an intermediate step, the relativistic transformation between τ and the Barycentric Coordinate Time (TCB) is usually required because TCB is the temporal variable describing the propagation of the radio (or even laser) signal connecting the spacecraft with the station according to International Astronomical Union (IAU) Resolutions (Soffel et al. 2003). Taking the Yinghuo-1 mission (Ping et al. 2010a,b) as a technical example of future Chinese Mars explorations, Deng (2012) studies this transformation by analytic and numerical methods and finds two main effects associated with it: the gravitational field of the Sun and the velocity of the spacecraft in the barycentric reference system. The combined contribution of these two effects can reach a few sub-seconds in a year (Deng 2012).

In this paper, we will extend previous works by taking the clock offset into account and investigate its accessibility by Fourier analysis. In Section 2, we will describe the relativistic transformation between τ and TCB and derive its numerical relation based on a proposed clock onboard a spacecraft orbiting Mars. In Section 3, we will separate the components behaving like clock offset from the transformation and analyze the accessibility of the remaining "signals." Conclusions and discussion will be presented in Section 4.

2 TRANSFORMATION BETWEEN τ AND TCB

According to IAU Resolutions (Soffel et al. 2003), the proper time τ of a clock onboard a spacecraft and $t \equiv$TCB are related up to the first order post-Newtonian (PN) approximation as

$$\tau - t = -\epsilon^2 \int \left(\sum_A \frac{Gm_A}{r_{sA}} + \frac{1}{2} v_s^2 \right) \mathrm{d}t + \mathcal{O}(\epsilon^2 J_n^{(A)}, \epsilon^4) \,, \tag{1}$$

where $\epsilon \equiv 1/c$ and the non-spherically-symmetric parts of each body's gravitational field are omitted. The index "s" stands for the spacecraft and the index "A" enumerates each body whose gravitational effect needs to be considered.

To solve Equation (1), we integrate it numerically by Simpson's rule (Stoer & Bulirsch 2002) with the help of the ephemeris DE405; the positions and velocities of celestial bodies are taken from DE405; the orbit of the spacecraft is solved by numerically integrating the Einstein-Infeld-Hoffmann equation (Einstein et al. 1938) with the Runge-Kutta 7 method (Stoer & Bulirsch 2002) and the stepsize is taken as one-hundredth of its Keplerian period. In the calculation, we neglect the difference between TCB and the Barycentric Dynamical Time (TDB) which is the time variable of DE405 because $\mathrm{dTDB} = (1 - L_B)\mathrm{dTCB}$ where $L_B = 1.550519768 \times 10^{-8}$ (Petit et al. 2010) so that the contribution associated with L_B is less than 10 nanoseconds in a year (Deng 2012). Our calculation only covers the phase of the spacecraft's orbit around Mars. (Deng 2012 briefly discusses the case of the interplanetary cruise.)

We assume a spacecraft has been orbiting around Mars from 2017-Jan-01 00:00:00.0000 (TDB) to 2018-Jan-01 00:00:00.0000 (TDB) and we rescale all of the time variables from the starting point in the other parts of this paper. Its orbital inclination with respect to the Martian equator is set as 5°. The apoapsis altitude is 80 000 km and the periapsis altitude is 800 km, with a period of about 3.2 d. We find that, like the result of Deng (2012), the curve of $\tau - t$ is nearly linear (see the top panel of Fig. 1), which includes the gravitational contributions from the Sun, eight planets, the Moon and three large asteroids: Ceres, Pallas and Vesta.

However, this does not mean the curve is able to be accessed in realistic experiments because the theoretical calculation misses an important issue: clock offset.

3 ACCESSIBILITY WITH CLOCK OFFSET

A realistic clock has an offset from an ideal clock of $\Delta\tau$, depending on its intrinsic physical properties. In practice, the offset can often be fitted by n-th order polynomials (Audoin & Guinot 2001), i.e.

$$\Delta\tau = a_0 + a_1\tau^1 + \cdots a_n\tau^n, \tag{2}$$

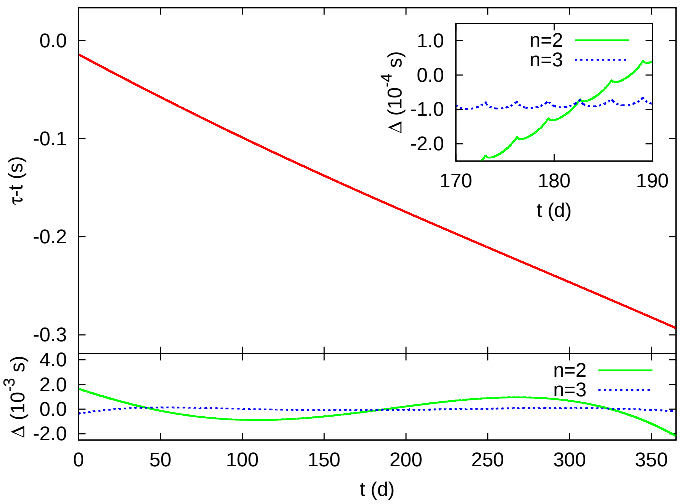

Fig. 1 *Top panel*: evolution of $\tau - t$ with respect to t. *Bottom panel*: the residuals of the fit for cases $n = 2$ (*solid line*) and $n = 3$ (*dashed line*).

where the time is rescaled from the starting time of the calculation. For the Chinese Beidou Navigation Satellite System and its time scales, the 2nd order polynomial is adopted from Han et al. (2011). Thus, to analyze the accessibility of the $\tau - t$ curve, we need to separate its components behaving like clock offset from "signals" by fitting the curve with 2nd and 3rd order polynomials which represent two possible types of the offset respectively. (Other types of clock offset can be processed with the same approach.) The reason is that any clock-offset-like components will be cleared out in the calibration of the clock, which means they are not accessible. The coefficients of fitted polynomials are given in Table 1 and the residuals of the fit (which we call "signals") are shown in the bottom panel of Figure 1. In the case of $n = 2$ (solid line), the residuals are at the level of milliseconds and have a low frequency oscillation. In the case of $n = 3$ (dashed line), the residuals are about 10 times less than those of $n = 2$. An important feature they both have is tiny oscillations roughly corresponding to the orbital period of the spacecraft (3.2 d) (see the inset of Fig. 1).

Since periodic variations can be effectively identified by a Fourier transformation, we employ this technique to process the residual "signals." The resulting spectrum is shown in Figure 2. Its top panel is for the case of $n = 2$ and the bottom one is for $n = 3$. Peaks in the top one are not as distinct as those in the bottom one because of its very small relative amplitudes (e.g., see the inset with the same scales and units as the big one). The 10 peaks with the largest amplitudes based on Fourier analysis are given in Table 2. (It is worth mentioning that the Keplerian frequency of the spacecraft orbiting Mars is about 2.2584×10^{-5} rad s^{-1}.) For the case $n = 2$, it shows "dual-peak" features (two peaks with close frequencies and amplitudes), but these features do not show up in the case of

Table 1 The Coefficients of Fitted Polynomials

$n = 2$	a_i	Value	$n = 3$	a_i	Value
	a_0	$-1.640111353146620 \times 10^{-3}$		a_0	$+3.549462221118650 \times 10^{-4}$
	a_1	$-8.898456910791267 \times 10^{-4}$		a_1	$-9.554397450110422 \times 10^{-4}$
	a_2	$+2.690895236548168 \times 10^{-7}$		a_2	$+7.183681395594635 \times 10^{-7}$
				a_3	$-8.206002116979848 \times 10^{-10}$

Table 2 The Parameters of the 10 Peaks with the Largest Amplitudes

	f (rad s^{-1})	Amplitude (s)
$n = 2$	$2.251395039673050 \times 10^{-5}$	$1.233630762883909 \times 10^{-5}$
	$2.331090439307495 \times 10^{-5}$	$1.005179670503308 \times 10^{-5}$
	$4.542637779163323 \times 10^{-5}$	$5.660790359807275 \times 10^{-6}$
	$4.602409328889156 \times 10^{-5}$	$5.176295331970426 \times 10^{-6}$
	$6.813956668744985 \times 10^{-5}$	$3.801385296939757 \times 10^{-6}$
	$6.873728218470818 \times 10^{-5}$	$3.494538194103410 \times 10^{-6}$
	$9.085275558326646 \times 10^{-5}$	$2.832293178828826 \times 10^{-6}$
	$9.125123258143868 \times 10^{-5}$	$2.639571402821048 \times 10^{-6}$
	$1.135659444790831 \times 10^{-4}$	$2.230736259459634 \times 10^{-6}$
	$1.362791333748997 \times 10^{-4}$	$1.822013183108982 \times 10^{-6}$
$n = 3$	$2.251395039673050 \times 10^{-5}$	$4.155663307883478 \times 10^{-6}$
	$4.522713929254712 \times 10^{-5}$	$1.722240619375779 \times 10^{-6}$
	$6.794032818836374 \times 10^{-5}$	$1.055705666726146 \times 10^{-6}$
	$9.065351708418035 \times 10^{-5}$	$7.488017620881390 \times 10^{-7}$
	$1.133667059799970 \times 10^{-4}$	$5.681957100403799 \times 10^{-7}$
	$1.360798948758136 \times 10^{-4}$	$4.452869900439114 \times 10^{-7}$
	$1.587930837716302 \times 10^{-4}$	$3.535376773038794 \times 10^{-7}$
	$1.815062726674468 \times 10^{-4}$	$2.808845501229303 \times 10^{-7}$
	$2.042194615632634 \times 10^{-4}$	$2.212453954239430 \times 10^{-7}$
	$2.269326504590800 \times 10^{-4}$	$1.713292734822900 \times 10^{-7}$

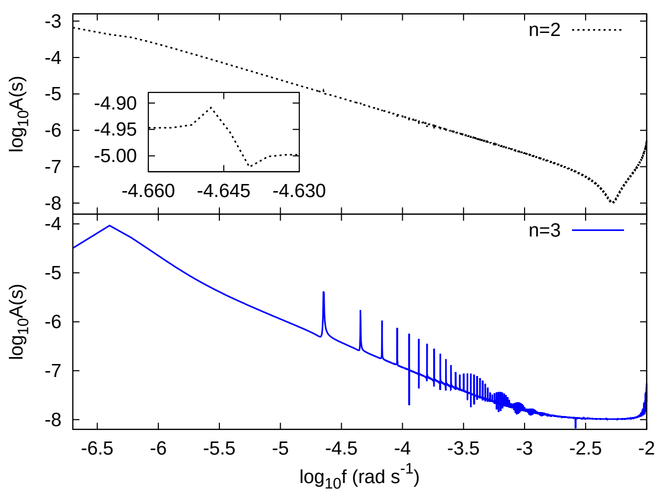

Fig. 2 The spectrum of residual "signals." The top panel is for the case $n = 2$ and the bottom one is for $n = 3$. The inset shows an enlarged region of the big one. The label "A" represents the amplitude in the unit of seconds and "f" denotes the frequency in the unit of rad s^{-1}.

$n = 3$. The amplitudes of the highest peaks in these cases are at the level of $\sim 10^{-5}$ s and $\sim 10^{-6}$ s respectively. This means the effects caused by relativistic transformation between τ and TCB will become significant if an onboard clock can reach an accuracy better than $\sim 10^{-5}$ s or $\sim 10^{-6}$ s (depending on the type of clock offset) within one year after calibration.

4 CONCLUSIONS AND DISCUSSION

In this work, we numerically integrate the relativistic transformation between the proper time τ of a clock onboard a spacecraft orbiting Mars and TCB. Its clock-offset-like behavior is separated by fitting the $\tau - t$ curve in terms of n-th order polynomials. Fourier analysis on the residual "signals" shows that if the accuracy of a clock can achieve better than $\sim 10^{-5}$ s or $\sim 10^{-6}$ s (depending on the type of clock offset) within one year after calibration then the relativistic effects on the difference between τ and TCB will need to be carefully considered.

Although, in terms of hardware, the realistic offset of an onboard clock in the environment of deep space and the strategy that should be taken to calibrate it are still very complicated issues, a complete analysis of the proper time onboard a spacecraft and time scales on ground stations needs to be established for practical purposes. Relativistic light propagation in the solar system under a realistic condition will be another important issue that needs to be investigated in detail for future theoretical works.

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