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The effect of $f(T)$ gravity on an interplanetary clock and its time transfer link *

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Abstract As an extension of the "teleparallel" equivalent of general relativity, $f(T)$ gravity is proposed to explain some puzzling cosmological behaviors, such as accelerating expansion of the Universe. Given the fact that modified gravity also has impacts on the Solar System, we might test it during future interplanetary missions with ultra-stable clocks. In this work, we investigate the effects of $f(T)$ gravity on the dynamics of the clock and its time transfer link. Under these influences, the Λ-term and the α-term of $f(T)$ gravity play important roles. Here, Λ is the cosmological constant and α represents a model parameter in $f(T)$ gravity that determines the divergence from teleparallel gravity at the first order approximation. We find that the signal of $f(T)$ gravity in the time transfer is much more difficult to detect with the current state of development for clocks than those effects on dynamics of an interplanetary spacecraft with a bounded orbit with parameters $0.5\,\mathrm{au} \leq a \leq 5.5\,\mathrm{au}$ and $0 \leq e \leq 0.1$.

Key words: gravitation — celestial mechanics — space vehicles: clock — time

1 INTRODUCTION

With tremendous advances in the accuracy of observations, Einstein's general relativity (GR) has passed nearly all tests in the Solar System. However, some big problems on the cosmological scale are still challenging GR. Thus, in order to find the possible solutions to them, modified theories of gravity have been proposed and tested (e.g., Brans & Dicke 1961; Jacobson & Mattingly 2001; Bekenstein 2004; Moffat 2006; Deng et al. 2009; Deng 2011). Among them, as an extension of the "teleparallel" equivalent of GR, which was originally proposed by Albert Einstein around 1928 as an attempt to unify gravitation and electromagnetism, $f(T)$ gravity (Ferraro & Fiorini 2007; Bengochea & Ferraro 2009; Linder 2010; Iorio & Saridakis 2012) is intended to explain some puzzling cosmological behaviors, such as accelerating expansion of the Universe.

It might also have an impact on the scale of the Solar System. For example, in $f(T)$ gravity, a spherically symmetric spacetime can be described by the metric (Iorio & Saridakis 2012),

$$ds^2 = N(r)^2 dt^2 - K(r)^{-2} dr^2 - R(r)^2 d\Omega^2 \,, \tag{1}$$

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where $R(r) = r$, $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ and

$$N(r)^2 = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda}{3}r^2 + \alpha\left[-6\Lambda - \frac{6}{r^2} - \frac{4GM\Lambda}{c^2 r}\right], \quad (2)$$

$$K(r)^2 = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda}{3}r^2 + \alpha\left[\frac{8\Lambda}{3} - \frac{14}{r^2} - 2\Lambda^2 r^2 - \frac{2GM}{c^2 r}\left(8\Lambda - \frac{8}{r^2}\right)\right]. \quad (3)$$

Here, Λ denotes the cosmological constant and α denotes a parameter that determines the divergence of $f(T)$ gravity from teleparallel gravity at the first approximation. From the above equation, we can see that it yields the usual Schwarzschild spacetime in GR if Λ and α are equal to zero. We can obtain the corrected Newtonian gravitational potential caused by $f(T)$ gravity as (Iorio & Saridakis 2012)

$$V = V_N(r) + V_{f(T)}(r), \quad (4)$$

where

$$V_N(r) = \frac{\mu}{r}, \quad (5)$$

$$V_{f(T)}(r) = c^2\frac{\Lambda}{6}r^2 + \alpha c^2\frac{3}{r^2}. \quad (6)$$

It is worth mentioning that, as pointed out by a previous work (Iorio & Saridakis 2012), solution (1) would not be applicable on a smaller length scale, such as in the vicinity of the Earth, because terms on the order of α^2/r^4 and higher are neglected, but it is still valid on an interplanetary scale and planetary orbital motions are used to constrain Λ and α (Iorio & Saridakis 2012).

Inspired by the ideas of probing theories of gravity using interplanetary ultra-stable clocks and laser links, such as Télémétrie InterPlanétaire Optique (TIPO) (Samain 2002), Solar System Odyssey (Christophe et al. 2009) or Outer Solar System (OSS) (Christophe et al. 2012), we would like to investigate the effect of $f(T)$ gravity on a clock onboard an interplanetary spacecraft and its observability within the spacetime (1) which is sufficient to evaluate its leading effects. The advantage of using a space-borne clock is that the effect of $f(T)$ gravity, in principle, will not only appear in the dynamics of the spacecraft but also in the time transfer signals.

In Section 2, treating the effect of $f(T)$ gravity as a small correction, we will follow the standard procedure of perturbation in celestial mechanics and obtain the Lagrange planetary equations of an interplanetary spacecraft which has a bounded orbit around the Sun. We will study its effects on the time transfer of an onboard clock in Section 3. In Section 4, the observability of these effects is discussed. Conclusions and a discussion will be given in Section 5.

2 EFFECTS ON THE ORBITAL DYNAMICS OF A SPACE-BORNE CLOCK

For some specific scientific purposes, a spacecraft in the Solar System may have quite different orbits. In the following descriptions, we focus on the case of the interplanetary spacecraft with a bounded orbit that is only around the Sun. (The escaping trajectories are also astronomically and physically interesting and we will return to this issue in our future work.) The evolution of an interplanetary clock's orbit under a perturbing potential R is (Danby 1962)

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2}{na}\frac{\partial R}{\partial M}, \quad (7)$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{(1-e^2)}{na^2 e}\frac{\partial R}{\partial M} - \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial \omega}, \quad (8)$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{\cos i}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial \omega} - \frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial \Omega}, \quad (9)$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = \frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial i}\,, \tag{10}$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial e} - \frac{\cos i}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial i}\,, \tag{11}$$

$$\frac{\mathrm{d}M}{\mathrm{d}t} = n - \frac{(1-e^2)}{na^2 e}\frac{\partial R}{\partial e} - \frac{2}{na}\frac{\partial R}{\partial a}\,. \tag{12}$$

In the case of $f(T)$ gravity, the correction to the Newtonian potential according to Equation (6) yields,

$$R = c^2\frac{\Lambda}{6}r^2 + \alpha c^2\frac{3}{r^2}\,. \tag{13}$$

The above expression shows that R only depends on r, which means the two-body problem under the framework of Newtonian gravitation with the correction of $f(T)$ gravity is a special case of the central force problem. Evaluating the partial derivatives involving R, we have the Lagrange planetary equations for the space-borne clock as

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2e}{n\sqrt{1-e^2}}\left(c^2\frac{\Lambda}{3}r - \alpha c^2\frac{6}{r^3}\right)\sin f, \tag{14}$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{\sqrt{1-e^2}}{na}\left(c^2\frac{\Lambda}{3}r - \alpha c^2\frac{6}{r^3}\right)\sin f, \tag{15}$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} = 0\,, \tag{16}$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = 0\,, \tag{17}$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = -\frac{\sqrt{1-e^2}}{nae}\left(c^2\frac{\Lambda}{3}r - \alpha c^2\frac{6}{r^3}\right)\cos f\,, \tag{18}$$

$$\frac{\mathrm{d}M}{\mathrm{d}t} = n + \frac{(1-e^2)}{nae}\left(c^2\frac{\Lambda}{3}r - \alpha c^2\frac{6}{r^3}\right)\cos f - \frac{2}{na^2}\left(c^2\frac{\Lambda}{3}r^2 - \alpha c^2\frac{6}{r^2}\right). \tag{19}$$

With the aid of the averaging method (Kozai 1959), we can express the long term variations of the elements as

$$\left\langle\frac{\mathrm{d}a}{\mathrm{d}t}\right\rangle = 0\,, \tag{20}$$

$$\left\langle\frac{\mathrm{d}e}{\mathrm{d}t}\right\rangle = 0\,, \tag{21}$$

$$\left\langle\frac{\mathrm{d}i}{\mathrm{d}t}\right\rangle = 0\,, \tag{22}$$

$$\left\langle\frac{d\Omega}{dt}\right\rangle = 0\,, \tag{23}$$

$$\left\langle\frac{\mathrm{d}\omega}{\mathrm{d}t}\right\rangle = \Lambda c^2\frac{\sqrt{1-e^2}}{2n} + \alpha c^2\frac{3}{na^4(1-e^2)}\,, \tag{24}$$

$$\left\langle\frac{\mathrm{d}M}{\mathrm{d}t}\right\rangle = n - \Lambda c^2\frac{7+3e^2}{6n} + \alpha c^2\frac{9}{na^4\sqrt{1-e^2}}\,. \tag{25}$$

Equation (24) identically matches the one given by a previous work (Iorio & Saridakis 2012). With the dynamics of the clock under the perturbation described by $f(T)$ gravity, we can study its effects on the time transfer of an onboard clock.

3 EFFECTS ON TIME TRANSFER LINK

For an interplanetary clock in the Solar System, the Barycentric Coordinate Time (TCB) is the coordinate time that acts as a reference for dynamics and the time that varies in the equations of motion (Soffel et al. 2003). However, the actual reading of a clock onboard a spacecraft is the proper time τ (Soffel et al. 2003). They diverge with increasing time. To compare the readings from an interplanetary clock with a clock on Earth or other time scales, the transformation between TCB and τ is usually taken as an intermediate step (Deng 2012). According to the relativistic theory of time scales, the transformation between τ and $t \equiv \mathrm{TCB}$ is

$$\Delta t = \int_A^B \left(1 + \frac{1}{c^2}V + \frac{1}{2}\frac{v^2}{c^2}\right) d\tau + \mathcal{O}(c^{-4})\,, \tag{26}$$

in which, if taking the $f(T)$ correction into account,

$$V = \frac{\mu}{r} + c^2\frac{\Lambda}{6}r^2 + \alpha c^2\frac{3}{r^2}\,, \tag{27}$$

where $\mu \equiv GM_{\odot}$. Following the procedures used by Nelson (2011), we can obtain

$$\begin{aligned}\Delta t \;=\; & \left[1 + \frac{3}{2}\frac{1}{c^2}\frac{\mu}{a} + \frac{\Lambda}{6}a^2 + \alpha\frac{3}{a^2}\right]\Delta\tau \\ & + \frac{2}{c^2}\sqrt{\mu a}\, e\sin E - \frac{\Lambda}{3}\sqrt{\frac{a}{\mu}}a^3 e\sin E \\ & + \alpha\frac{6}{\sqrt{\mu a}}e\sin E + \mathcal{O}(e^2\alpha, e^2\Lambda, c^{-4})\,,\end{aligned} \tag{28}$$

where E is the eccentric anomaly. When $\alpha = 0$ and $\Lambda = 0$, it returns the result of Nelson (2011). From Equation (28), we can see that the effect of $f(T)$ gravity on the time transfer has two components: a secular term and a periodic term. The secular term might not be observable because it is mixed with the intrinsic frequency drift of the clock. The periodic term, whose frequency is the same as the leading term due to GR, might show up, but its observability depends on amplitude

$$A_{\mathrm{T}} = -\frac{\Lambda}{3}\sqrt{\frac{a}{\mu}}a^3 e + \alpha\frac{6}{\sqrt{\mu a}}e\,. \tag{29}$$

4 OBSERVABILITY OF THE EFFECTS OF $f(T)$ GRAVITY

In a previous work (Iorio & Saridakis 2012; Kagramanova et al. 2006), because of equations describing planetary orbital motions, Λ and α are constrained as: $|\Lambda| \leq 6.1 \times 10^{-42}$ m^{-2} and $|\alpha| \leq 1.8 \times 10^4$ m^2. Thus, in this section, we will take these values to investigate the observability of $f(T)$ gravity for an interplanetary spacecraft with orbits: $0.5\,\mathrm{au} \leq a \leq 5.5\,\mathrm{au}$ and $0 \leq e \leq 0.1$. This range is based on the different goals of proposed missions (TIPO, Solar System Odyssey and OSS), the possible capabilities of interplanetary time transfer and laser ranging, and the strength of effects caused by $f(T)$ gravity.

According to Equation (24), we can plot the secular change of ω, $\langle\dot{\omega}\rangle$, ranging from a few tens of micro-arcseconds per year to several thousands of micro-arcseconds per year (see color indexed Figure 1). They are just marginally below the current threshold of measurements (Fienga et al. 2011).

For the time transfer, the amplitude A_{T} [Eq. (29)] of the periodic term due to $f(T)$ gravity holds the value from about -8 picoseconds to 4 picoseconds (see color indexed Fig. 2). Even if a current optical clock with a frequency inaccuracy of 8.6×10^{-18} (Chou et al. 2010) is carried onboard the spacecraft, it would hardly be able to detect the $f(T)$ gravity signal in the duration of one orbit. Since picking up a periodic term usually requires sampling the data in a time interval comparable with its period, which is $\sim 10^7$ s in this case, such a long time will make the uncertainty in the clock's frequency exceed the signal.

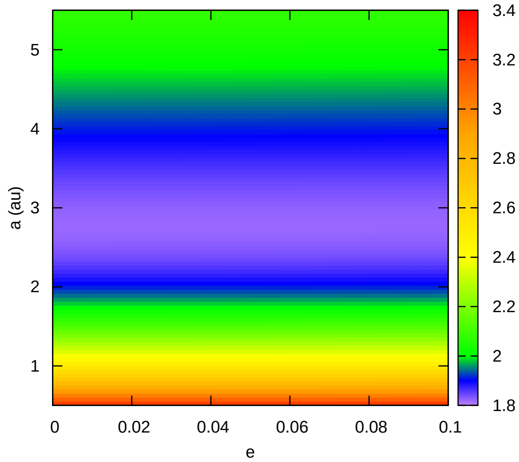

Fig. 1 A color index representing the secular change in ω according to the clock's orbit with $0.5\,\mathrm{au} \leq a \leq 5.5\,\mathrm{au}$ and $0 \leq e \leq 0.1$.

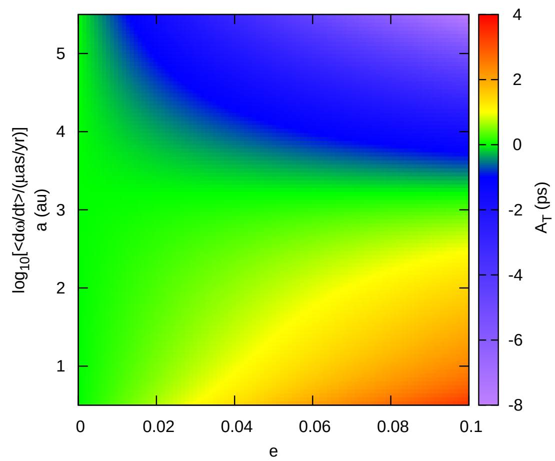

Fig. 2 A color index representing amplitude A_{T} according to the clock's orbit with $0.5\,\mathrm{au} \leq a \leq 5.5\,\mathrm{au}$ and $0 \leq e \leq 0.1$.

5 CONCLUSIONS AND DISCUSSION

In this work, we investigate the effects of $f(T)$ gravity on an interplanetary clock, including the dynamics of the space vehicle and the time transfer of the clock. We find that the signal from $f(T)$ gravity in the time transfer will reach a level of a few picoseconds with the clock's orbital range at $0.5\,\mathrm{au} \leq a \leq 5.5\,\mathrm{au}$ and $0 \leq e \leq 0.1$, which may be very difficult to detect with the current state of development for clocks. However, the signal of $f(T)$ gravity arising from dynamics of the clock, namely $\langle\dot{\omega}\rangle$, perhaps will be detectable in the future by using the technique of interplanetary laser ranging. Like lunar laser ranging experiments, interplanetary laser ranging will be able to very precisely measure the distance between the spacecraft and the ground station. These observables can be used to determine the orbits of the spacecraft and the unexplained variations from GR.

It is possible to go beyond the range of orbits we focus on here. From a theoretical point of view, getting closer to the Sun can make the spacecraft feel a larger influence of $f(T)$ gravity and make the dynamical effects, such as its perihelion advance, detectable according to the current threshold of measurements. However, it is required to launch the spacecraft into an orbit inside that of Mercury. At such a short distance, noises caused by the Sun in the time transfer link and laser ranging, and difficulties in thermal control of the spacecraft are expected to be more significant. On the other hand, a spacecraft further from the Sun will cause the radio and laser signals connecting the ground station and the spacecraft to become weaker, which will need larger antennae and more powerful laser devices for interplanetary communication. This will also cause the effects of $f(T)$ gravity to be smaller. Therefore, it is necessary to find a sweet spot in the orbital design for testing $f(T)$ gravity and other scientific goals.

In addition, some open issues remain. One of them is whether the effect of $f(T)$ gravity will reach the realm of detectability for high frequency in time transfer. After all, Equation (28) is truncated and other terms do exist but perhaps with much lower amplitudes.

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