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Parametrized post-Newtonian secular transit timing variations for exoplanets *

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Abstract Ground-based and space-borne observatories used for studying exoplanet transits now and in the future will considerably increase the number of exoplanets known from transit data and the precision of the measured times of transit minima. Variations in the transit times can not only be used to infer the presence of additional planets, but might also provide opportunities to test the general theory of relativity in these systems. To build a framework for these possible tests, we extend previous studies on the observability of the general relativistic precessions of periastron in transiting exoplanets to variations in secular transit timing under parametrized post-Newtonian formalism. We find that if one can measure the difference between observed and predicted variations of general relativistic secular transit timing to $1\,\mathrm{s\,yr^{-1}}$ in a transiting exoplanet system with a Sun-like mass, a period of ~ 1 day and a relatively small eccentricity of ~ 0.1, general relativity will be tested to the level of $\sim 6\%$.

Key words: gravitation — celestial mechanics — planetary systems

1 INTRODUCTION

Currently, more than 880 exoplanets have been discovered and about 300 of them are in transiting systems[1]. Now and in the future, ground-based and space-borne observatories used for studying exoplanet transits will considerably increase the number exoplanets discovered through transit data and the precision of observed times of transit minima[2]. The measured transit timing variations (TTVs) can be used to infer the presence of additional planets (e.g. Holman & Murray 2005; Agol et al. 2005; Heyl & Gladman 2007; Nesvorný et al. 2012) and study the dynamics of multiple planet systems (e.g. Holman et al. 2010; Lissauer et al. 2011; Fabrycky et al. 2012; Steffen et al. 2012; Nesvorný

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[1] *http://exoplanet.eu/catalog/*

[2] As mentioned in appendix A of Kipping (2011), the so-called "mid-transit time" in the exoplanet literature is highly ambiguous. Following the terminology used by Kipping (2011), we will use "transit minimum" and "time of transit minimum" in this paper. Because, for a limb-darkened star, the transit minimum occurs when the apparent sky-projected separation between the exoplanet and the star reaches a minimum has a completely unambiguous definition, "transit timing variations" also refers to "changes of times of transit minimum."

et al. 2013). Recently, the *Kepler* mission (Basri et al. 2005) released a catalog of transit timing measurements for the first twelve quarters, which identifies the Kepler objects of interest with significant TTVs (Mazeh et al. 2013).

Theoretically, the contribution due to general relativity (GR), especially the general relativistic periastron advance (GRPA), is among the causes of secular TTVs. Its observability in exoplanets has been investigated in several works (e.g. Miralda-Escudé 2002; Adams & Laughlin 2006a,b,c; Iorio 2006a; Heyl & Gladman 2007; Jordán & Bakos 2008; Pál & Kocsis 2008; Li 2010; Iorio 2011a,b; Li 2012b). It is found that GRPA can be detectable on timescales of less than about 10 years with current observational capabilities by observing the times of transits in exoplanets (Jordán & Bakos 2008).

This means that, like the well-known phenomenon in the Solar System of the anomaly in the perihelion shift of Mercury (Nobili & Will 1986) that gave a hint at new physics about GR and the dynamics of planets could be used to test fundamental laws of physics (e.g. Iorio 2005a,b; Folkner 2010; Pitjeva 2010; Fienga et al. 2011; Iorio et al. 2011; Iorio 2012a; Pitjeva 2012; Pitjev & Pitjeva 2013; Pitjeva & Pitjev 2013), the observation of secular TTVs can also serve as a test-bed for GR with the help of high-precision measurements which might be available in the not-so-distant future. It will also provide opportunities to test the fundamental theories of gravity, such as modified theories of gravity and alternative relativistic theories of gravity, in quite a large number of different locations beyond the Solar System. This will make transiting exoplanets very similar to binary pulsars in testing physical laws describing gravity (e.g. Bell et al. 1996; Damour & Esposito-Farèse 1996; Kramer et al. 2006; Iorio 2007b; Deng et al. 2009; Li 2010; Deng 2011; Li 2011; De Laurentis et al. 2012; Ragos et al. 2013; Xie 2013). Therefore, this inspires us to extend previous works within GR and to build a framework under the parametrized post-Newtonian (PPN) formalism (see Will 1993, 2006, for reviews) for modeling and evaluating these tests via secular TTVs. In this formalism, the values of its PPN parameters represent deviations caused by theories of gravity that are alternative to GR. For example, two Eddington-Robertson-Schiff parameters, γ and β, are both equal to 1 in GR, but might have different values in other cases (see Will 1993, 2006, for details). Our goal is to set up a PPN theory for measuring $\bar{\gamma} \equiv \gamma - 1$ and $\bar{\beta} \equiv \beta - 1$ and testing fundamental laws of gravity in transiting exoplanets.

The rest of the paper is organized as follows. Section 2 is devoted to describing TTVs under PPN formalism. In Section 3, we present an analysis about the observability of $\bar{\gamma}$ and $\bar{\beta}$ via secular TTVs. Finally, in Section 4, we summarize our results.

2 TTVS UNDER PPN FORMALISM

To describe the dynamics of a transiting exoplanetary system, understand its transit light curve and represent the observables, we adopt the coordinate systems defined and applied in Kipping (2011). The plane of $\hat{X}$-$\hat{Y}$ is defined as the plane of the sky, where the star is at the origin O and the observer is located at $(X, Y, Z) = (0, 0, +\infty)$. Then, in the $\hat{X}$-$\hat{Y}$-$\hat{Z}$ system, the inclination of a transiting exoplanet i is close to 90°[3]. The normalized apparent (sky-projected) separation between the planet and the star is defined as (Kipping 2011)

$$\begin{aligned} S &\equiv \frac{1}{R_*}\sqrt{X^2+Y^2} \\ &= \frac{a}{R_*}\varrho(f)\sqrt{1-\sin^2(\omega+f)\sin^2 i}\,, \end{aligned} \tag{1}$$

[3] We take widely used notations in celestial mechanics: a is the semi-major axis, e is the eccentricity, i is the inclination, Ω is the longitude of the ascending node, ω is the argument of periastron, M is the mean anomaly and f is the true anomaly.

where $\varrho(f) \equiv (1-e^2)/(1+e\cos f)$ and R_* is the radius of the star. For mathematical convenience in the following parts of this paper, we will also use the expression for S^2

$$S^2 = \frac{a^2}{R_*^2}\varrho^2(f)[1-\sin^2(\omega+f)\sin^2 i]\,. \tag{2}$$

2.1 Transit Minima

For a Kelperian transiting exoplanet, the instants of transit minima (and maxima) occur when $\mathrm{d}S/\mathrm{d}t = 0$ (Kipping 2011), which leads to

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{\mathrm{d}S}{\mathrm{d}f}\frac{\mathrm{d}f}{\mathrm{d}t} = 0\,. \tag{3}$$

It is worth mentioning that the condition defined by Equation (3) is a pure geometric criterion without any ambiguity. Because $\mathrm{d}f/\mathrm{d}t \neq 0$ for planetary orbital motions, the condition $\mathrm{d}S/\mathrm{d}t = 0$ is equivalent to $\mathrm{d}S/\mathrm{d}f = 0$. An easier way to handle the mathematics is to make use of $\mathrm{d}S^2/\mathrm{d}f = 0$, and such a condition can be proven to be equivalent to $\mathrm{d}S/\mathrm{d}f = 0$ (Kipping 2011). To obtain the true anomaly at the transit minima f_T (the subscript "T" denotes transit), one needs to solve a quartic equation that involves $\cos f$ [see eq. (4.5) given by Kipping (2011)]. Although solving it is mathematically possible, the solutions are pretty lengthy and impractical. Treating $\cos^2 i$ as a small quantity which is very close to zero for transiting exoplanets and using the Newton-Raphson iteration method, Kipping (2011) shows the series expansion solution for f_T can be written as

$$f_T = \left[\frac{\pi}{2} - \omega\right] - \sum_{j=1}^{n} \eta_j^T\,, \tag{4}$$

where, by $h \equiv e\sin\omega$ and $k \equiv e\cos\omega$,

$$\eta_1^T = \left(\frac{k}{1+h}\right)(\cos^2 i)^1, \tag{5}$$

$$\eta_2^T = \left(\frac{k}{1+h}\right)\left(\frac{1}{1+h}\right)(\cos^2 i)^2, \tag{6}$$

$$\eta_3^T = -\left(\frac{k}{1+h}\right)\left[\frac{-6(1+h)+k^2(-1+2h)}{6(1+h)^3}\right](\cos^2 i)^3\,. \tag{7}$$

It is demonstrated by Kipping (2011) that using a solution expanded to first-order can reduce the error to less than a millisecond for a highly eccentric planet with a short period. Solutions expanded to high order (up to $n = 6$) can be found in Kipping (2011).

2.2 PPN Secular TTVs

With the same approach used by Iorio (2011a) to work out long-term time variations of some observables for transiting exoplanets, for a given observable Γ that is a function of Keplerian orbital elements, i.e. $\Gamma = \Gamma(\{\sigma\})$, where $\{\sigma\} = \{a, e, i, \Omega, \omega, M\}$, if perturbations on the Keperlian orbital motion are taken into account, we can calculate its secular variation by averaging

$$\left\langle\frac{\mathrm{d}\Gamma}{\mathrm{d}t}\right\rangle = \frac{1}{P}\int_0^P \frac{\mathrm{d}\Gamma}{\mathrm{d}t}\mathrm{d}t = \frac{1}{P}\int_0^P \sum_{\kappa\in\{\sigma\}} \frac{\partial\Gamma}{\partial\kappa}\frac{\mathrm{d}\kappa}{\mathrm{d}t}\mathrm{d}t\,, \tag{8}$$

where P is the Keplerian period of the orbit. Applying this approach to f_T, we can obtain its secular changes as

$$\left\langle \frac{\mathrm{d}f_T}{\mathrm{d}t} \right\rangle = -\left\langle \frac{\mathrm{d}\omega}{\mathrm{d}t} \right\rangle - \sum_{j=1}^{n} \left\langle \frac{\mathrm{d}\eta_j^T}{\mathrm{d}t} \right\rangle . \tag{9}$$

If a PPN two-body problem is considered, we can find that

$$\left\langle \frac{\mathrm{d}\eta_1^T}{\mathrm{d}t} \right\rangle = -\frac{h+e^2}{(1+h)^2}\cos^2 i \left\langle \frac{\mathrm{d}\omega}{\mathrm{d}t} \right\rangle, \tag{10}$$

$$\left\langle \frac{\mathrm{d}\eta_2^T}{\mathrm{d}t} \right\rangle = -\frac{k^2+h+e^2}{(1+h)^3}\cos^4 i \left\langle \frac{\mathrm{d}\omega}{\mathrm{d}t} \right\rangle, \tag{11}$$

$$\left\langle \frac{\mathrm{d}\eta_3^T}{\mathrm{d}t} \right\rangle = \frac{1}{2}\frac{(2h^3k^2+2hk^4+h^2k^2-2k^4-2h^3-7hk^2-4h^2-6k^2-2h)}{(1+h)^5}\cos^6 i \left\langle \frac{\mathrm{d}\omega}{\mathrm{d}t} \right\rangle , \tag{12}$$

where only $\langle \mathrm{d}\omega/\mathrm{d}t \rangle$ contributes in the above expression because the secular variations of a, e, i and Ω are zero (see Appendix A for details).

However, the secular variation of f_T is not practically observable so that, for realistic measurements, it needs to be converted to the secular variation of time of transit minimum t_T, i.e. secular TTV,

$$\left\langle \frac{\mathrm{d}t_T}{\mathrm{d}t} \right\rangle = \left\langle \frac{\mathrm{d}t_T}{\mathrm{d}f_T}\frac{\mathrm{d}f_T}{\mathrm{d}t} \right\rangle = \frac{1}{n\sqrt{1-e^2}}\varrho_T^2 \left\langle \frac{\mathrm{d}f_T}{\mathrm{d}t} \right\rangle , \tag{13}$$

where $\varrho_T \equiv \varrho(f_T)$ and $n = 2\pi/P$. If we consider the PPN 2-body problem, by substituting Equations (9), (4) and (A.19) into Equation (13), we can derive the PPN secular TTV as

$$\left\langle \frac{\mathrm{d}t_T}{\mathrm{d}t} \right\rangle = -(3+2\bar{\gamma}-\bar{\beta})\frac{\sqrt{1-e^2}}{(1+h)^2}\frac{Gm}{c^2 a} + \mathcal{O}(\cos^2 i) , \tag{14}$$

where G is the gravitational constant, c is the speed of light and m is the total mass of the transiting exoplanet system. For a time duration Δt, the PPN secular TTV Δt_T is

$$\frac{\Delta t_T}{\Delta t} = -3\frac{Gm}{c^2 a}(1-2h) - (2\bar{\gamma}-\bar{\beta})\frac{Gm}{c^2 a}(1-2h) + \mathcal{O}\left(\cos^2 i, \frac{e^2}{c^2}\right) . \tag{15}$$

It is obvious that $\Delta t_T/\Delta t$ has two parts: a contribution caused by GR and one due to the deviation from GR. They respectively are, in more convenient expressions,

$$\left.\frac{\Delta t_T}{\Delta t}\right|_{\mathrm{GR}} = -\left(\frac{47.7\,\mathrm{s}}{1\,\mathrm{yr}}\right)\left(\frac{m}{m_\odot}\right)^{2/3}\left(\frac{P}{1\,\mathrm{day}}\right)^{-2/3}(1-2h) + \mathcal{O}\left(\cos^2 i, \frac{e^2}{c^2}\right) , \tag{16}$$

and

$$\left.\frac{\Delta t_T}{\Delta t}\right|_{\bar{\gamma},\bar{\beta}} = -\left(\frac{15.9\,\mathrm{s}}{1\,\mathrm{yr}}\right)(2\bar{\gamma}-\bar{\beta})\left(\frac{m}{m_\odot}\right)^{2/3}\left(\frac{P}{1\,\mathrm{day}}\right)^{-2/3}(1-2h) + \mathcal{O}\left(\cos^2 i, \frac{e^2}{c^2}\right) . \tag{17}$$

Equation (16) shows that, in a transiting exoplanet system with a Sun-like mass, a period of ~ 1 day and a relatively small eccentricity of ~ 0.1, the variation in times of transit minima can reach ~ 40 seconds in a year.

For future observation on relativistic secular TTVs, if we define the difference between the measured TTVs and its prediction by GR as

$$\delta^{\mathrm{GR}}_{\Delta t_T/\Delta t} \equiv \left.\frac{\Delta t_T}{\Delta t}\right|_{\mathrm{obs}} - \left.\frac{\Delta t_T}{\Delta t}\right|_{\mathrm{GR}} , \tag{18}$$

then $2\bar{\gamma} - \bar{\beta}$ via TTVs can be determined to be

$$2\bar{\gamma} - \bar{\beta} = -6.28 \times 10^{-2} \left(\frac{m}{m_\odot}\right)^{-2/3} \left(\frac{P}{1\,\mathrm{day}}\right)^{2/3} (1+2h) \left(\frac{\delta^{\mathrm{GR}}_{\Delta t_T/\Delta t}}{1\,\mathrm{s\,yr}^{-1}}\right) + \mathcal{O}\left(\cos^2 i, \frac{e^2}{c^2}\right). \quad (19)$$

According to the above equation, $\bar{\gamma}$ and $\bar{\beta}$ cannot be separately constrained by only using TTVs, but their combination $\bar{\zeta}_T \equiv 2\bar{\gamma} - \bar{\beta}$ is accessible. Since $\bar{\gamma}$ and $\bar{\beta}$ have been measured to be very close to zero in various experiments and observations (Will 1993, 2006), $\bar{\zeta}_T$ is expected to also be nearly zero in well-determined systems with transiting exoplanets. Equation (19) tells us that if one can measure the difference between observed and predicted general relativistic secular TTVs to $1\,\mathrm{s\,yr}^{-1}$ in a transiting system with a Sun-like mass, a period of ~ 1 day and a relatively small eccentricity of ~ 0.1, then GR will be tested to the level of $\sim 6\%$.

3 OBSERVABILITY OF $\bar{\zeta}_T$ VIA TTVS

This section will be dedicated to an important issue: the observability of $\bar{\zeta}_T$ via TTVs. Based on the catalog of confirmed transiting exoplanets[4], we will focus on the space of parameters describing transiting exoplanets and associated measurements, given as

$$\mathcal{D} = \left\{ \left(m, P, h, \delta^{\mathrm{GR}}_{\Delta t_T/\Delta t}\right) \;\middle|\; 0.1\,m_\odot \le m \le 5\,m_\odot,\ 10^{-1}\,\mathrm{day} \le P \le 10^3\,\mathrm{day}, \right.$$
$$\left. 0 \le h \le 0.4,\ 10^{-2}\,\mathrm{s\,yr}^{-1} \le \delta^{\mathrm{GR}}_{\Delta t_T/\Delta t} \le 10\,\mathrm{s\,yr}^{-1} \right\}. \quad (20)$$

According to Equation (19), the space $\mathcal{D}$ can generate $|\bar{\zeta}_T|$ ranging from $\sim 10^{-4}$ to $\sim 10^2$ (see color indexed Figs. 1 and 2 with identical logarithmic color bars). Figure 1(a) shows the color indexed $|\bar{\zeta}_T|$ when $h = 0.1$ and $\delta^{\mathrm{GR}}_{\Delta t_T/\Delta t} = 1.0\,\mathrm{s\,yr}^{-1}$. It suggests that, under this specific condition, only the transiting systems with $P \lesssim 10$ days can determine $|\bar{\zeta}_T|$ down to $\sim 10^{-1}$. If $\delta^{\mathrm{GR}}_{\Delta t_T/\Delta t}$ can be determined to the level of $0.1\,\mathrm{s\,yr}^{-1}$, the transiting systems can be used to extend those cases with $P \lesssim 10^2$ days as shown in Figure 1(b). Figure 2(a) represents the color indexed $|\bar{\zeta}_T|$ when $m = 1.0\,m_\odot$ and $h = 0.1$. It can be seen that, for a system with periods even as long as $10^2 \lesssim P \lesssim 10^3$ days, improvement of $\delta^{\mathrm{GR}}_{\Delta t_T/\Delta t}$ will make it available for detecting $|\bar{\zeta}_T|$. Finally, Figure 2(b) indicates that determination of $|\bar{\zeta}_T|$ is not sensitive to h. All of these figures show that, even if $\delta^{\mathrm{GR}}_{\Delta t_T/\Delta t}$ might not be determined very precisely, short-period transiting systems can serve as good test-beds.

However, like in the Solar System, the story of testing GR in transiting exoplanets is not so simple. Many other sources might cause secular TTVs, such as additional planets or tidal deformations (e.g. Holman & Murray 2005; Agol et al. 2005; Heyl & Gladman 2007; Jordán & Bakos 2008; Iorio 2012b; Nesvorný et al. 2012; Iorio 2013). For example, Jordán & Bakos (2008) show that the precession caused by tidal deformations may dominate the total precession in cases where GRPA is detectable. Thus, theoretically and numerically modeling the full dynamics and observables of transiting exoplanets up to a level compatible with observational datasets will be an important step to separate and extract information.

4 CONCLUSIONS AND DISCUSSION

In the context of a potential, considerable increase in the number of exoplanets discovered through transiting and the precision of measured times of transit minima by both ground-based and space-borne observatories used for studying exoplanet transits now and in the future, we investigate the

[4] *http://exoplanet.eu/catalog/*

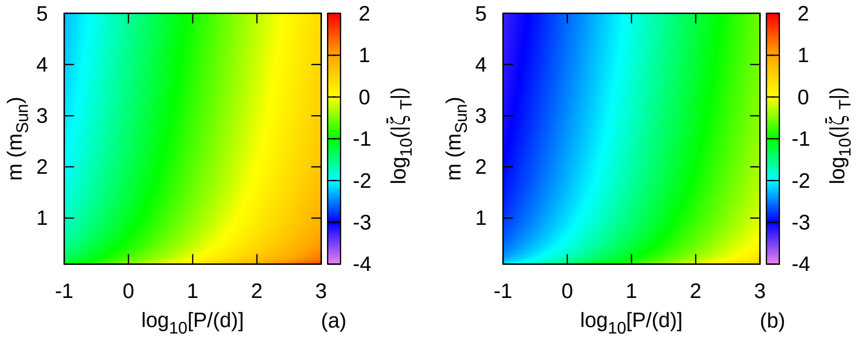

Fig. 1 Panel (a) shows the color indexed $|\bar{\zeta}_T|$ according to Eq. (19) when $h = 0.1$ and $\delta^{\rm GR}_{\Delta t_T/\Delta t} = 1.0\,{\rm s\,yr}^{-1}$. Panel (b) is similar to (a) except that $\delta^{\rm GR}_{\Delta t_T/\Delta t} = 0.1\,{\rm s\,yr}^{-1}$.

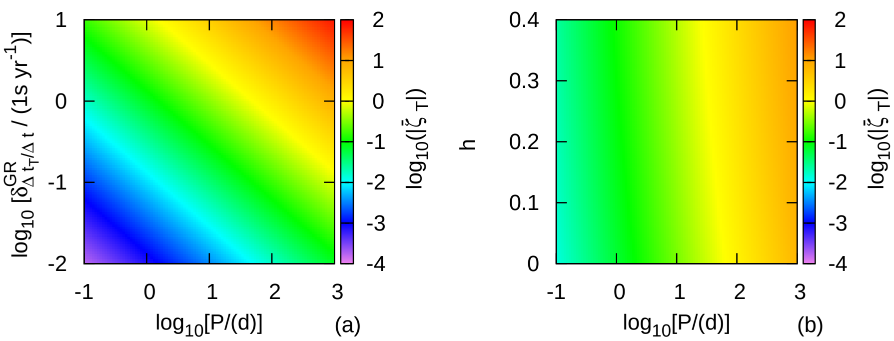

Fig. 2 Panel (a) shows the color indexed $|\bar{\zeta}_T|$ according to Eq. (19) when $m = 0.1m_\odot$ and $h = 0.1$ and Panel (b) shows the color indexed $|\bar{\zeta}_T|$ when $m = 0.1m_\odot$ and $\delta^{\rm GR}_{\Delta t_T/\Delta t} = 1.0\,{\rm s\,yr}^{-1}$.

PPN secular TTVs to test fundamental laws of gravity in transiting systems. We find that if one can measure the difference between observed and predicted general relativistic secular TTVs to $1\,{\rm s\,yr}^{-1}$ in a transiting system with a Sun-like mass, a period of ~ 1 day and a relatively small eccentricity of ~ 0.1, GR will be tested to the level of $\sim 6\%$.

However, exoplanetary systems are full of complexity, so many sources can trigger secular TTVs, such as additional planets and tidal deformations. Separating and discriminating them need theoretical and numerical models of dynamics and observables up to a level compatible with astronomical observations. Other important timings, like variations in transit duration, transit to occultation time and transit to occultation duration ratio, will be our next goal for studying their possibilities to test GR. Another interesting and promising direction for this line of research is to combine various observational datasets from astrometry, radial velocity and transit timing, which will be analogous to constructing ephemerides of the Solar System and using them to test GR (Pitjeva 2005; Folkner 2010; Fienga et al. 2011).

Furthermore, transiting exoplanets might also be used to detect other non-standard effects which may show up through TTVs and other observables. These interesting features are the effect of a steady mass loss in the host star (or of a change in G) (e.g. Iorio 2010a,c; Li 2012a,c, 2013; Pitjeva & Pitjev 2012), dark matter in exoplanetary systems (e.g. Iorio 2006b, 2010b,d; Griest et al. 2011;

Hooper & Steffen 2012; Iorio 2013) and fifth force-like Yukawa-type effects (e.g. Iorio 2002, 2007a; Haranas et al. 2011; Deng & Xie 2013).

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Appendix A: PPN 2-BODY PROBLEM

Under the conditions of weak gravitational fields and low velocities, the two-body problem in the PPN formalism can be treated as Keplerian motion with PPN perturbations. The Gaussian perturbation equations read as

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2}{n\sqrt{1-e^2}}[Se\sin f + T(1+e\cos f)], \tag{A.1}$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{\sqrt{1-e^2}}{na}[S\sin f + T(\cos f + \cos E)], \tag{A.2}$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{r\cos u}{na^2\sqrt{1-e^2}}W, \tag{A.3}$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = \frac{r\sin u}{na^2\sqrt{1-e^2}\sin i}W, \tag{A.4}$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\sqrt{1-e^2}}{nae}\left[-S\cos f + T\left(1+\frac{r}{p}\right)\sin f\right] - \cos i\frac{\mathrm{d}\Omega}{\mathrm{d}t}, \tag{A.5}$$

$$\frac{\mathrm{d}M}{\mathrm{d}t} = n - \sqrt{1-e^2}\frac{\mathrm{d}\omega}{\mathrm{d}t} - \sqrt{1-e^2}\cos i\frac{\mathrm{d}\Omega}{\mathrm{d}t} - \frac{2}{na^2}Sr\,, \tag{A.6}$$

in which $u = f + \omega$, $p = a(1-e^2)$ and S, T and W are the radial, transverse and out-of-plane components of the perturbing acceleration respectively (Soffel et al. 1987).

$$S = \frac{Gm}{c^2r^2}\left[2(\beta+\gamma+\nu)\frac{Gm}{r} - (\gamma+3\nu)\boldsymbol{v}^2 + \left(2\gamma+2-\frac{1}{2}\nu\right)\dot{r}^2\right], \tag{A.7}$$

$$T = \frac{Gm}{c^2r^2}(2\gamma+2-2\nu)\frac{n^2a^3}{r}e\sin f\,, \tag{A.8}$$

$$W = 0\,, \tag{A.9}$$

where G is the gravitational constant, c is the speed of light, $m = m_1 + m_2$ and $\nu = m_1m_2/m^2$. With the help of Keplerian relations

$$v^2 = m\left(\frac{2}{r} - \frac{1}{a}\right), \tag{A.10}$$

$$\dot{r}^2 = m\left(\frac{2}{r} - \frac{1}{a}\right) - m\frac{p}{r^2}\,, \tag{A.11}$$

we can have

$$S = -\left(\gamma - \frac{7}{2}\nu + 2\right)\frac{1}{a}\frac{Gm^2}{c^2r^2} + (2\beta + 4\gamma - 5\nu + 4)\frac{Gm^2}{c^2r^3}$$
$$-\left(2\gamma + 2 - \frac{1}{2}\nu\right)\frac{Gm^2p}{c^2r^4}\,, \tag{A.12}$$
$$T = 2(\gamma + 1 - \nu)\frac{Gm}{c^2}n^2e\frac{a^3}{r^3}\sin f\,, \tag{A.13}$$
$$W = 0\,. \tag{A.14}$$

After averaging over a period, we obtain the secular evolution of the orbital elements as

$$\left\langle\frac{\mathrm{d}a}{\mathrm{d}t}\right\rangle = 0, \tag{A.15}$$
$$\left\langle\frac{\mathrm{d}e}{\mathrm{d}t}\right\rangle = 0, \tag{A.16}$$
$$\left\langle\frac{\mathrm{d}i}{\mathrm{d}t}\right\rangle = 0, \tag{A.17}$$
$$\left\langle\frac{\mathrm{d}\Omega}{\mathrm{d}t}\right\rangle = 0, \tag{A.18}$$
$$\left\langle\frac{\mathrm{d}\omega}{\mathrm{d}t}\right\rangle = (2\gamma + 2 - \beta)\frac{Gm}{c^2}\frac{n}{p}\,, \tag{A.19}$$
$$\left\langle\frac{\mathrm{d}M}{\mathrm{d}t}\right\rangle = n - (3\beta + 6\gamma - 9\nu + 6)\sqrt{1-e^2}\frac{Gmn}{c^2p}$$
$$+(2\gamma - 7\nu + 4)(1-e^2)\frac{Gmn}{c^2p}\,, \tag{A.20}$$

which identically match those given by Soffel et al. (1987).

References

Adams, F. C., & Laughlin, G. 2006a, ApJ, 649, 992

Adams, F. C., & Laughlin, G. 2006b, ApJ, 649, 1004

Adams, F. C., & Laughlin, G. 2006c, International Journal of Modern Physics D, 15, 2133

Agol, E., Steffen, J., Sari, R., & Clarkson, W. 2005, MNRAS, 359, 567

Basri, G., Borucki, W. J., & Koch, D. 2005, New Astron. Rev., 49, 478

Bell, J. F., Camilo, F., & Damour, T. 1996, ApJ, 464, 857

Damour, T., & Esposito-Farèse, G. 1996, Phys. Rev. D, 53, 5541

De Laurentis, M., De Rosa, R., Garufi, F., & Milano, L. 2012, MNRAS, 424, 2371

Deng, X.-M. 2011, Science in China G: Physics and Astronomy, 54, 2071

Deng, X.-M., & Xie, Y. 2013, MNRAS, 431, 3236

Deng, X.-M., Xie, Y., & Huang, T.-Y. 2009, Phys. Rev. D, 79, 044014

Fabrycky, D. C., Ford, E. B., Steffen, J. H., et al. 2012, ApJ, 750, 114

Fienga, A., Laskar, J., Kuchynka, P., et al. 2011, Celestial Mechanics and Dynamical Astronomy, 111, 363

Folkner, W. M. 2010, in IAU Symposium, vol. 261, eds. S. A. Klioner, P. K. Seidelmann, & M. H. Soffel, 155

Griest, K., Lehner, M. J., Cieplak, A. M., & Jain, B. 2011, Physical Review Letters, 107, 231101

Haranas, I., Ragos, O., & Mioc, V. 2011, Ap&SS, 332, 107

Heyl, J. S., & Gladman, B. J. 2007, MNRAS, 377, 1511

Holman, M. J., Fabrycky, D. C., Ragozzine, D., et al. 2010, Science, 330, 51
Holman, M. J., & Murray, N. W. 2005, Science, 307, 1288
Hooper, D., & Steffen, J. H. 2012, J. Cosmol. Astropart. Phys., 7, 046
Iorio, L. 2002, Physics Letters A, 298, 315
Iorio, L. 2005a, A&A, 431, 385
Iorio, L. 2005b, A&A, 433, 385
Iorio, L. 2006a, New Astron., 11, 490
Iorio, L. 2006b, J. Cosmol. Astropart. Phys., 5, 002
Iorio, L. 2007a, Journal of High Energy Physics, 10, 041
Iorio, L. 2007b, Ap&SS, 312, 331
Iorio, L. 2010a, SRX Physics, 2010, 1249
Iorio, L. 2010b, J. Cosmol. Astropart. Phys., 5, 018
Iorio, L. 2010c, Natural Science, 2, 329
Iorio, L. 2010d, J. Cosmol. Astropart. Phys., 11, 046
Iorio, L. 2011a, MNRAS, 411, 167
Iorio, L. 2011b, Ap&SS, 331, 485
Iorio, L. 2012a, Sol. Phys., 281, 815
Iorio, L. 2012b, Celestial Mechanics and Dynamical Astronomy, 112, 117
Iorio, L. 2013, Galaxies, 1, 6
Iorio, L. 2013, Celestial Mechanics and Dynamical Astronomy, doi:10.1007/s10569-013-9491-x
Iorio, L., Lichtenegger, H. I. M., Ruggiero, M. L., & Corda, C. 2011, Ap&SS, 331, 351
Jordán, A., & Bakos, G. Á. 2008, ApJ, 685, 543
Kipping, D. 2011, The Transits of Extrasolar Planets with Moons (Berlin: Springer-Verlag)
Kramer, M., Stairs, I. H., Manchester, R. N., et al. 2006, Science, 314, 97
Li, L.-S. 2010, Ap&SS, 327, 59
Li, L.-S. 2011, Ap&SS, 334, 125
Li, L.-S. 2012a, MNRAS, 419, 1825
Li, L.-S. 2012b, Ap&SS, 341, 323
Li, L.-S. 2012c, Chinese Astronomy and Astrophysics, 36, 63
Li, L.-S. 2013, MNRAS, 431, 2971
Lissauer, J. J., Fabrycky, D. C., Ford, E. B., et al. 2011, Nature, 470, 53
Mazeh, T., Nachmani, G., Holczer, T., et al. 2013, ArXiv:1301.5499
Miralda-Escudé, J. 2002, ApJ, 564, 1019
Nesvorný, D., Kipping, D., Terrell, D., et al. 2013, arXiv:1304.4283
Nesvorný, D., Kipping, D. M., Buchhave, L. A., et al. 2012, Science, 336, 1133
Nobili, A. M., & Will, C. M. 1986, Nature, 320, 39
Pál, A., & Kocsis, B. 2008, MNRAS, 389, 191
Pitjev, N. P., & Pitjeva, E. V. 2013, Astronomy Letters, 39, 141
Pitjeva, E. 2012, in IAU Joint Discussion, vol. 7
Pitjeva, E. V. 2005, Astronomy Letters, 31, 340
Pitjeva, E. V. 2010, in IAU Symposium, vol. 261, eds. S. A. Klioner, P. K. Seidelmann, & M. H. Soffel, 170
Pitjeva, E. V., & Pitjev, N. P. 2012, Solar System Research, 46, 78
Pitjeva, E. V., & Pitjev, N. P. 2013, MNRAS, accepted
Ragos, O., Haranas, I., & Gkigkitzis, I. 2013, Ap&SS, 345, 67
Soffel, M., Ruder, H., & Schneider, M. 1987, Celestial Mechanics, 40, 77
Steffen, J. H., Fabrycky, D. C., Ford, E. B., et al. 2012, MNRAS, 421, 2342
Will, C. M. 1993, Theory and Experiment in Gravitational Physics (Cambridge, UK: Cambridge Univ. Press)
Will, C. M. 2006, Living Reviews in Relativity, 9, 3
Xie, Y. 2013, RAA (Research in Astronomy and Astrophysics), 13, 1