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Calculation of the structural properties of a strange quark star in the presence of a strong magnetic field using a density dependent bag constant

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Abstract We have calculated the structural properties of a strange quark star with a static model in the presence of a strong magnetic field. To this end, we use the MIT bag model with a density dependent bag constant. To parameterize the density dependence of the bag constant, we have used our results for the lowest order constrained variational calculation of the asymmetric nuclear matter. By calculating the equation of state of strange quark matter, we have shown that the pressure of this system increases by increasing both density and magnetic field. Finally, we have investigated the effect of density dependence of the bag constant on the structural properties of a strange quark star.

Key words: stars: dense matter — equation of state — ISM: structure — supernova remnants

1 INTRODUCTION

The core of a neutron star forms from nuclear matter, composed of neutrons, protons, electrons (to ensure negation of electric charge) and other particles like pions, mesons, etc (Lattimer & Prakash 2004). It is known that nuclear matter is meta-stable, and, after releasing a lot of energy, can convert into strange quark matter (SQM) to achieve stability. This type of quark matter is the most stable state of matter currently known. Thus, there is a new class of compact stars that comes from the collapse of neutron stars, and is more stable compared to neutron stars (Farhi & Jaffe 1984). The best candidates for this conversion are neutron stars with masses of $1.5 - 1.8\ M_{\odot}$ and fast spins (Drake et al. 2002; Li et al. 1999; Weber 2005).

The collapse of a neutron star may lead to a strange quark star (SQS) or a hybrid star. Also under special conditions, an SQS may be directly born from the core collapse of a type II supernova. An SQS, from its center to surface, is made from SQM, and on its surface there may exist a layer of nuclear matter (Glendenning & Weber 1992). Hybrid stars are the ones with cores composed of SQM (Bhattacharyya et al. 2006). Here, we just consider the structural properties of an SQS.

The mass and density of an SQS is between the mass and density of a neutron star and that of a black hole. However, the mass-radius relation for an SQS is $M \propto R^3$, which is different from that of a neutron star. This star does not have a minimum value for mass. For an SQS with $1M_\odot \leq M \leq 2M_\odot$, the radius is about 10 km (Alcock et al. 1986; Shapiro & Teukolsky 1983).

Recent observations indicate that the object SWIFT J1749.4–2807 may be an SQS (Yu & Xu 2010). The results given by Chandra observations also show that the objects RX J185635–3754 and 3C 58 may be bare strange stars (Prakash et al. 2003). It is known that compact objects such as neutron stars, pulsars, magnetars and SQSs are under the influence of strong magnetic fields which are typically about $10^{15} - 10^{19}$ G (Kouveliotou et al. 1998, 1999; Haensel et al. 2007; Glendenning 2000; Weber 1999; Camenzind 2007). Therefore, in astrophysics, it is of special interest to study the effect of a strong magnetic field on the properties of SQM. We note that in the presence of a magnetic field, the conversion of neutron stars to bare quark stars cannot take place unless the value of the magnetic field exceeds 10^{20} G (Ghosh & Chakrabarty 2001).

In recent years, we have calculated the maximum gravitational mass and other structural properties of a neutron star with a quark core at zero (Bordbar et al. 2006) and finite temperatures (Yazdizadeh & Bordbar 2011). We have also computed the structural properties of an SQS at zero temperature (Bordbar et al. 2009) and finite temperature (Bordbar et al. 2011), as well as calculated the structure of a magnetized SQS using the MIT bag model with a fixed bag constant ($90\frac{\mathrm{MeV}}{\mathrm{fm}^3}$) (Bordbar & Peivand 2011). In the present work, we investigate the effect of density dependence of the bag constant on the structure of an SQS in the presence of a strong magnetic field.

2 COMPUTATION OF THE EQUATION OF STATE FOR STRANGE QUARK MATTER IN THE PRESENCE OF A MAGNETIC FIELD

The equation of state (EOS) of SQM plays an important role in determining the structure of stars at high densities. To obtain the EOS of SQM, there are different models based on Quantum Chromodynamics (QCD). At present, it is not possible to derive an exact EOS of SQM from first principles of QCD. Therefore, scientists have tried to find approximate methods by combining the basic features of QCD, for example, the MIT bag model (Chodos et al. 1974; Weber 1999; Peshier et al. 2000; Alford et al. 2005), the NJL model (Rehberg et al. 1996; Hanauske et al. 2001; Rüster & Rischke 2004; Menezes et al. 2006), and the perturbative QCD model (Baluni 1978; Fraga et al. 2001; Farhi & Jaffe 1984).

In the MIT bag model, the quarks in the bag are considered to be a free Fermi gas, and the energy per volume for SQM is equal to the kinetic energy of the free quarks plus a bag constant ($\mathcal{B}$) (Chodos et al. 1974). The bag constant $\mathcal{B}$ can be interpreted as the difference between the energy densities of the noninteracting quarks and the interacting ones. Dynamically, its role is to maintain the pressure that keeps the quark gas at a constant density and potential. In the first MIT bag model, different values such as 55 and $90\,\mathrm{MeV\ fm^{-3}}$ were considered for the bag constant. As far as we know, the density of SQM increases from the surface to the core of an SQS, therefore using a density dependent bag constant instead of a fixed bag constant is more suitable.

2.1 Density Dependent Bag Constant

The analysis of the experimental data obtained at CERN shows that the quark-hadron transition happens at a density about seven times the normal nuclear matter energy density ($156\,\mathrm{MeV\ fm^{-3}}$) (Heinz 2001; Heinz & Jacob 2000; Farhi & Jaffe 1984). However theoretically, there is no density-independent value of the bag constant for the hadron to quark matter transition to occur (Burgio et al. 2002). Therefore, it is essential to use a density dependent bag constant. Recently, a density dependent form has also been considered for $\mathcal{B}$ (Adami & Brown 1993; Jin & Jennings 1997; Blaschke et al. 1999; Burgio et al. 2002). The density dependence of $\mathcal{B}$ is highly model dependent. According to the hypothesis of a constant energy density along the transition line, Burgio et al. tried to determine a range of possible values for $\mathcal{B}$ by exploiting the experimental data obtained at the CERN SPS

(Burgio et al. 2002). By assuming that the transition to quark-gluon plasma is only determined by the value of the energy density, they estimated the value of the bag constant and its possible density dependence. They attempted to provide effective parameterizations for this density dependence, trying to cover a wide range by considering some extreme choices in such a way that at asymptotic densities, the bag constant has some finite value. They employed a Gaussian form as follows

$$\mathcal{B}(\rho) = \mathcal{B}_\infty + (\mathcal{B}_0 - \mathcal{B}_\infty)\mathrm{e}^{-\gamma(\rho/\rho_0)^2}. \tag{1}$$

The parameter $\mathcal{B}_0 = \mathcal{B}(\rho = 0)$ is constant and equal to $\mathcal{B}_0$ = 400 MeV fm^{-3}. In the above equation, γ is a numerical parameter which is usually equal to $\rho_0 \approx 0.17\ \mathrm{fm}^{-3}$, the normal nuclear matter density. $\mathcal{B}_\infty$ depends only on the free parameter $\mathcal{B}_0$.

The value of the bag constant ($\mathcal{B}$) should be compatible with the experimental data. The experimental results at CERN SPS confirm a proton fraction $x_{\mathrm{pt}} = 0.4$ (Heinz 2001; Heinz & Jacob 2000; Burgio et al. 2002). Therefore, we use the EOS of asymmetric nuclear matter to evaluate $\mathcal{B}_\infty$. We use the lowest order constrained variational (LOCV) many-body method based on the cluster expansion of the energy for calculating the EOS of asymmetric nuclear matter as follows (Bordbar & Modarres 1997, 1998; Modarres & Bordbar 1998; Bordbar & Bigdeli 2007a,b, 2008a,b; Bigdeli et al. 2009, 2010).

The asymmetric nuclear matter is defined as a system consisting of Z protons (pt) and N neutrons (nt) with the total number density $\rho = \rho_{\mathrm{pt}} + \rho_{\mathrm{nt}}$ and proton fraction $x_{\mathrm{pt}} = \frac{\rho_{\mathrm{pt}}}{\rho}$, where ρ_{pt} and ρ_{nt} are the number densities of protons and neutrons, respectively. For this system, we consider a trial wave function of the form,

$$\psi = F\phi, \tag{2}$$

where ϕ is the slater determinant of the single-particle wave function, and F is the A-body correlation operator ($A = Z + N$) which is given by

$$F = \mathcal{S}\prod_{i>j} f(ij). \tag{3}$$

In the above equation, $\mathcal{S}$ is a symmetrizing operator.

For asymmetric nuclear matter, the energy per nucleon up to the two-body term in the cluster expansion is as follows

$$E([f]) = \frac{1}{A}\frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle} = E_1 + E_2. \tag{4}$$

The one-body energy, E_1, is

$$E_1 = \sum_{i=1}^{2}\sum_{k_i}\frac{\hbar^2 k_i^2}{2m}, \tag{5}$$

where labels 1 and 2 are used for proton and neutron respectively, and k_i is the momentum of particle i. The two-body energy, E_2, is given by

$$E_2 = \frac{1}{2A}\sum_{ij}\langle ij|\mathcal{V}(12)|ij - ji\rangle, \tag{6}$$

where

$$\mathcal{V}(12) = -\frac{\hbar^2}{2m}[f(12), [\nabla_{12}^2, f(12)]] + f(12)V(12)f(12). \tag{7}$$

In Equation (7), $f(12)$ and $V(12)$ are the two-body correlation and nucleon-nucleon potential, respectively. In our calculations, we use $UV_{14} + TNI$ nucleon-nucleon potential (Lagaris &

Pandharipande 1981a,b). We minimize the two-body energy with respect to the variations in the correlation functions subject to the normalization constraint. From the minimization of the two-body energy, we get a set of differential equations. By numerically solving these differential equations, we can calculate the correlation functions. The two-body energy is obtained using these correlation functions and then we can calculate the energy of asymmetric nuclear matter. The procedure of these calculations has been fully discussed in Bordbar & Modarres (1998).

The experimental results at CERN SPS confirm a proton fraction $x_{\rm pt} = 0.4$ (Heinz 2001; Heinz & Jacob 2000; Burgio et al. 2002). Therefore, to calculate $\mathcal{B}_\infty$, we use our results of the above formalism for the asymmetric nuclear matter characterized by a proton fraction $x_{\rm pt} = 0.4$. By assuming that the hadron-quark transition takes place at the energy density equal to 1100 MeV fm^{-3} (Heinz 2001; Burgio et al. 2002), we find that the baryonic density of nuclear matter corresponding to this value of the energy density is $\rho_{\mathcal{B}} = 0.98\,{\rm fm}^{-3}$ (transition density). At densities lower than this value, the energy density of SQM is higher than that of the nuclear matter. By increasing the baryonic density, these two energy densities become equal at the transition density, and above this value, the nuclear matter energy density always remains higher. Later, we determine $\mathcal{B}_\infty$ = 8.99 MeV fm^{-3} by setting the energy density of SQM and that of the nuclear matter equal to each other.

2.2 Energy Density Calculation of Strange Quark Matter in the Presence of a Magnetic Field

We consider SQM composed of u, d and s quarks with spin up (+) and down (−). We denote the number density of quark i with spin up by $\rho_i^{(+)}$ and spin down by $\rho_i^{(-)}$. We introduce the polarization parameter ξ_i by

$$\xi_i = \frac{\rho_i^{(+)} - \rho_i^{(-)}}{\rho_i}, \tag{8}$$

where $0 \leq \xi_i \leq 1$ and $\rho_i = \rho_i^{(+)} + \rho_i^{(-)}$. Under the conditions of beta-equilibrium and charge neutrality, we get the following relation for the number density,

$$\rho = \rho_u = \rho_d = \rho_s, \tag{9}$$

where ρ is the total baryonic density of the system.

Within the MIT bag model, the total energy of SQM in the presence of magnetic field (B) can be written as

$$E_{\rm tot} = E_{\rm K} + \mathcal{B} + E_{\rm M}, \tag{10}$$

where $E_{\rm M}$ is the contribution of magnetic energy, $\mathcal{B}$ is the bag constant (in this article, we use a density dependent bag constant (Eq. (1)), and $E_{\rm K}$ is the total kinetic energy of SQM. The total kinetic energy of SQM is as follows,

$$E_{\rm K} = \sum_{i=u,d,s} E_i, \tag{11}$$

where E_i is the kinetic energy of quark i,

$$E_i = \sum_{p=\pm} \sum_{k^{(p)}} \sqrt{m_i^2 c^4 + \hbar^2 k^{(p)2} c^2}. \tag{12}$$

We ignore the masses of u and d quarks, while we assume $m_s = 150$ MeV for s quarks. After performing some algebra, supposing that $\xi_s = \xi_u = \xi_d = \xi$, we obtain the following relation for the total kinetic energy density ($\varepsilon_{\rm K}=\frac{E_{\rm K}}{V}$) of SQM,

$$\varepsilon_{\rm K} = \frac{3}{16\pi^2\hbar^3}\sum_{p=\pm}\left[\frac{\hbar}{c^2}k_F^{(p)}E_F^{(p)}(2\hbar^2 k_F^{(p)^2}c^2+m_s^2c^4)-m_s^4c^5\ln\left(\frac{\hbar k_F^{(p)}+E_F^{(p)}/c}{m_s c}\right)\right] + \frac{3\hbar c\pi^{2/3}}{4}\rho^{4/3}[(1+\xi)^{4/3}+(1-\xi)^{4/3}], \quad (13)$$

where

$$k_F^{(\pm)} = (\pi^2\rho)^{1/3}(1\pm\xi)^{1/3}, \quad (14)$$

and

$$E_F^{(\pm)} = (\hbar^2 k_F^{(\pm)^2}c^2+m_s^2c^4)^{1/2}. \quad (15)$$

For SQM, the contribution of magnetic energy is $E_{\rm M}=-\boldsymbol{M}\cdot\boldsymbol{B}$. If we assume that the magnetic field is along the z direction, the contribution of the magnetic energy of SQM is given by

$$E_{\rm M} = -\sum_{i=u,d,s}M_z^{(i)}B, \quad (16)$$

where $M_z^{(i)}$ is the magnetization of the system corresponding to particle i which is given by

$$M_z^{(i)} = N_i\mu_i\xi_i. \quad (17)$$

In the above equation, N_i and μ_i are the number and magnetic moment of particle i, respectively. By some simplification, the contribution of the magnetic energy density ($\varepsilon_{\rm M}=\frac{E_{\rm M}}{V}$) of SQM can be obtained as follows

$$\varepsilon_{\rm M} = -\sum_{i=u,d,s}\rho_i\mu_i\xi_i B. \quad (18)$$

Using the above equation and $\rho=\rho_u=\rho_d=\rho_s$, along with the assumption that $\xi=\xi_u=\xi_d=\xi_s$, we have

$$\varepsilon_{\rm M} = -(\rho B\xi\mu_s+\rho B\xi\mu_u+\rho B\xi\mu_d). \quad (19)$$

Now, we take advantage of numerical values of the magnetic moment for quarks (Wong 2007) : $\mu_s=-0.581\,\mu_N$, $\mu_u=1.852\,\mu_N$, $\mu_d=-0.972\,\mu_N$.

Using Equation (19) and the above values, we conclude that

$$\varepsilon_{\rm M} = -0.299\,\rho\,\xi\,\mu_{\rm N}B, \quad (20)$$

where $\mu_{\rm N}=5.05\times10^{-27}\ (J/T)$ is the magneton of the nucleus.

2.3 The Results of Energy Calculations of Strange Quark Matter in the Presence of a Magnetic Field

We have calculated the properties of SQM in the presence of a magnetic field with the density dependent bag constant (Eq. (1)). Our results are as follows.

Our results for the total energy density ($\varepsilon_{\rm tot}=E_{\rm tot}/V$) of SQM in the presence of a magnetic field have been plotted versus the polarization parameter in Figure 1 for various densities at $B=5\times10^{18}$ G. We see that there is a minimum point in the energy curve for each density which shows a meta-stable state for this system. As the density increases, the minimum point of energy shifts to lower values of the polarization, and finally it disappears at high densities in which the system becomes nearly unpolarized.

In Figure 2, we have plotted the polarization parameter corresponding to the minimum point of energy versus density for two magnetic fields $B=5\times10^{18}$ G and $B=5\times10^{19}$ G. We can see that

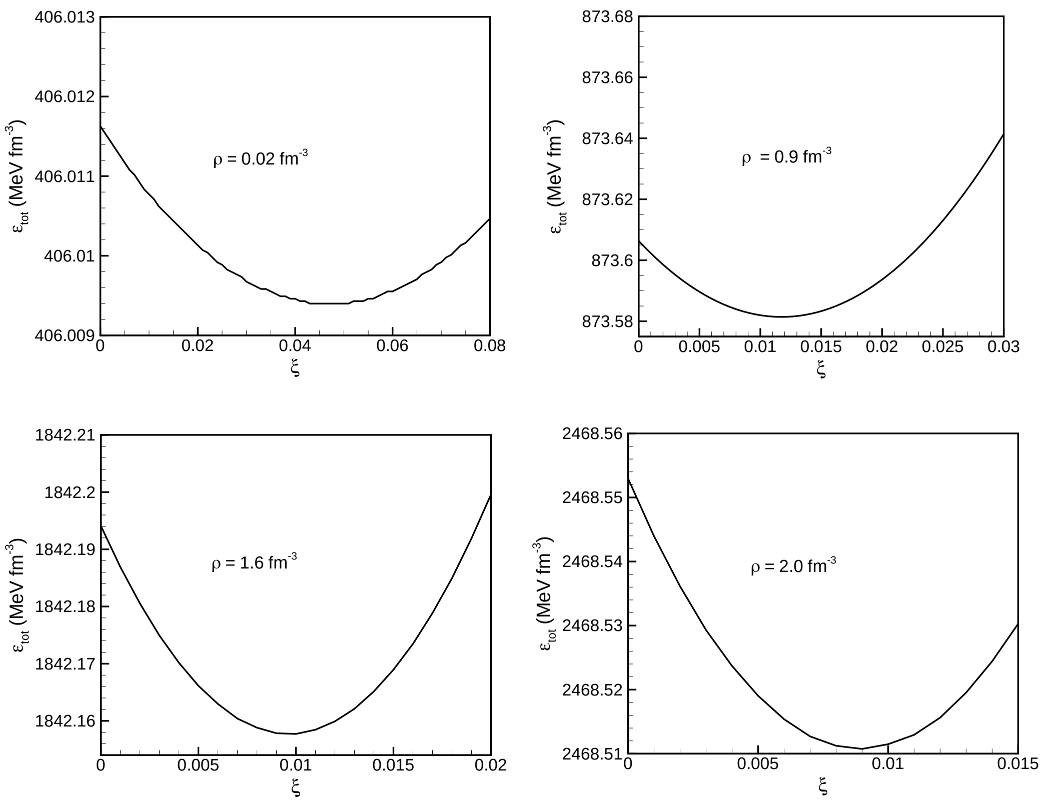

Fig. 1 Total energy density ($\varepsilon_{\rm tot}$) as a function of the polarization parameter (ξ) for different densities (ρ).

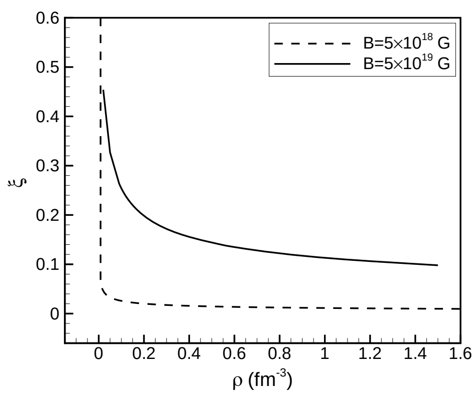

Fig. 2 Polarization parameter (ξ) versus density (ρ) for $B = 5 \times 10^{18}$ and 5×10^{19} G.

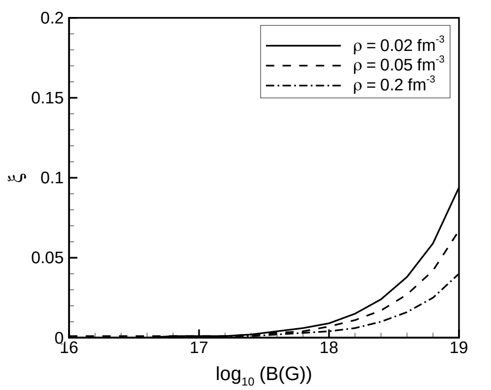

Fig. 3 Polarization parameter (ξ) corresponding to the minimum points of energy density versus the magnetic field (B) for different values of density (ρ).

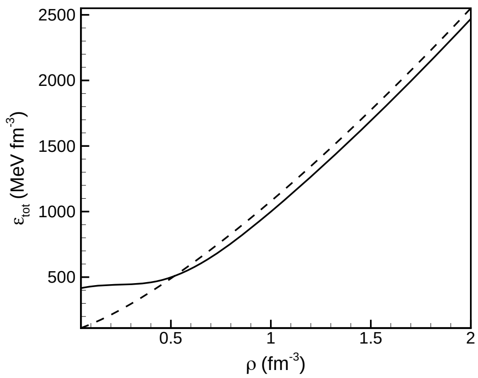

Fig. 4 Total energy density versus density (ρ) calculated by a density dependent bag constant (*solid curve*) at $B = 5 \times 10^{18}$ G. The results for $\mathcal{B}$ = 90 MeV fm^{-3} (*dashed curve*) at $B = 5 \times 10^{18}$ G have also been given for comparison.

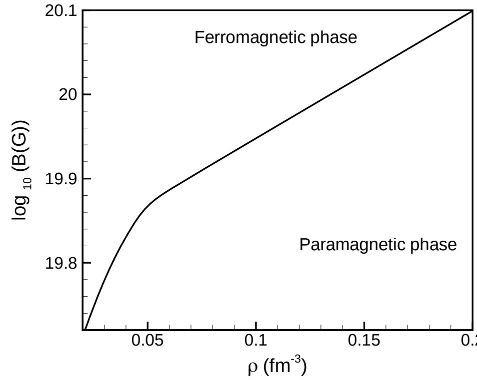

Fig. 5 Phase diagram for SQM in the presence of a strong magnetic field.

the value of ξ decreases by increasing the density, and it becomes nearly zero at high densities. We have also drawn the polarization parameter as a function of the magnetic field at different densities in Figure 3. As this figure shows, the polarization parameter increases by increasing the magnetic field for all densities.

For the SQM, our results for the total energy density at $B = 5 \times 10^{18}$ G, which were calculated with the density dependent bag constant, have been shown as a function of density in Figure 4. The results for $\mathcal{B}$ = 90 MeV fm^{-3} at $B = 5 \times 10^{18}$ G (Bordbar & Peivand 2011) are also given for comparison. It can be observed that the total energy density has an increasing rate by increasing the density. Also, it can be found that for ρ greater (lower) than about $0.6\ \mathrm{fm}^{-3}$, the energy of SQM with the density dependent bag constant is lower (greater) than that with the fixed bag constant. From Figure 4, it is seen that for $\rho < 0.6\ \mathrm{fm}^{-3}$, the increasing energy has a relatively flat slope, whereas for $\rho > 0.6\ \mathrm{fm}^{-3}$ this increase shows a steeper slope.

Figure 5 shows the phase diagram for the SQM. We can see that as the density increases, the magnetic field grows monotonically. This explicitly shows that at higher densities, the ferromagnetic phase transition occurs at higher values of the magnetic field.

2.4 The Equation of State for Strange Quark Matter in the Presence of a Magnetic Field

In this section, we calculate the EOS of SQM in the presence of a magnetic field with a density dependent bag constant. Generally, we can calculate the EOS using the following relation,

$$P(\rho) = \rho \frac{\partial \varepsilon_{\mathrm{tot}}}{\partial \rho} - \varepsilon_{\mathrm{tot}}, \tag{21}$$

where P is the pressure and $\varepsilon_{\mathrm{tot}}$ is the energy density, which in the presence of a magnetic field, is obtained from Equation (10).

In Figure 6, we have compared our results for the EOS of SQM at different magnetic fields. This shows that for all magnetic fields, by increasing the density, pressure has an increasing rate. Also, we can see that with an increasing magnetic field, the pressure increases.

In Figure 7, we have drawn the EOS of SQM for the density dependent bag constant at $B = 5 \times 10^{18}$ G. The results for $\mathcal{B}$ = 90 MeV fm^{-3} at $B = 5 \times 10^{18}$ G (Bordbar & Peivand 2011) are also given for comparison. This figure indicates that for ρ greater than about $0.52\ \mathrm{fm}^{-3}$,

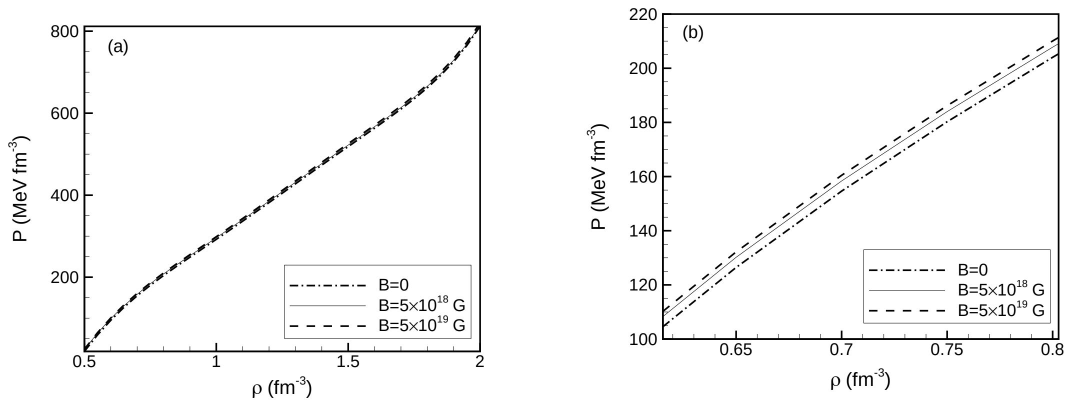

Fig. 6 The EOS of SQM at $B = 0,\ 5\times 10^{18}$ and 5×10^{19} G.

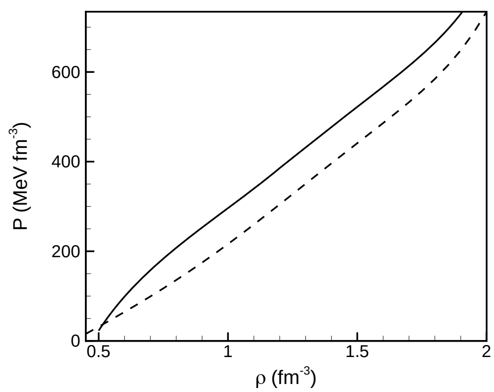

Fig. 7 The EOS of SQM in the case of a density dependent bag constant (*solid curve*) at $B = 5\times 10^{18}$ G. The results for the case of a fixed bag constant ($\mathcal{B}$ = 90 MeV fm^{-3}) (*dashed curve*) at $B = 5\times 10^{18}$ G have also been given for comparison.

when the bag constant is density dependent, the pressure of SQM is greater than that of the density independent case.

3 STRUCTURE OF A STRANGE QUARK STAR

Quark stars are relativistic objects, therefore we used general relativity for calculation of their structures. Since most of the massive general relativistic objects have some form of rotation (very rapid in the case of pulsars), in these calculations, we are interested in investigating effects of the strong magnetic field on the structure of a static SQS. Using the EOS of SQM, we can obtain the structure of these stars by numerically integrating the general relativistic equations of hydrostatic equilibrium, the Tolman-Oppenheimer-Volkoff (TOV) equation (Shapiro & Teukolsky 1983),

$$\begin{aligned}\frac{dP}{dr} &= -\frac{G(\frac{\varepsilon(r)m(r)}{r^2})(1+\frac{P(r)}{c^2\varepsilon(r)})(1+\frac{4\pi r^3 P(r)}{m(r)c^2})}{(1-\frac{2Gm(r)}{c^2})},\\ \frac{dm}{dr} &= 4\pi r^2\varepsilon(r).\end{aligned} \tag{22}$$

In the above equations, P is pressure and $\varepsilon(r)$ is the energy density, G is the gravitational constant and $m(r)$ is the mass inside radius r which is calculated as follows

$$m(r) = \int_0^r 4\pi r'^2 \varepsilon(r') dr'. \tag{23}$$

Now, by selecting a central energy density ε_c, under the boundary conditions $P(0) = P_c$ and $m(0) = 0$, we integrate the TOV equations outwards to a radius $r = R$ at which P vanishes (Shapiro & Teukolsky 1983).

In this section, we calculate the structure of the SQS with the density dependent bag constant in the presence of a magnetic field. We should note that a strong magnetic field changes the spherical symmetry of the system and for magnetic fields less than 10^{19} G, this effect is ignorable (Felipe & Martínez 2009; González Felipe et al. 2011). Considering the anisotropy of the pressure from SQM in the presence of a magnetic field, it has been shown that for vanishing anomalous magnetic moments, the perpendicular component of the pressure $P_\perp$ goes to zero at about 2×10^{19} G (González Felipe et al. 2008). Thus in the case of SQM, for $B < 10^{19}$ G, the anisotropy in the pressures is relatively small, i.e. $P_\perp = P_\parallel$.

In Figure 8, we have drawn the gravitational mass versus the central density (ε_c) for an SQS in the magnetic fields with strengths $B = 0$ and 5×10^{18} G. We see that as the central density increases, the gravitational mass of an SQS increases, and finally it reaches a limiting value which is called the maximum gravitational mass.

Figure 8 shows that when the magnetic field is present, the gravitational mass decreases. The results for $\mathcal{B}$ = 90 MeV fm^{-3} at $B = 5 \times 10^{18}$ G (Bordbar & Peivand 2011) are also given in Figure 8 for the sake of comparison. This indicates that the density dependence of the bag constant leads to substantially higher values for the gravitational mass of an SQS. With the density dependent bag constant, we have found that the maximum gravitational mass of an SQS is about $1.62\ M_\odot$, but with the fixed bag constant, it is about $1.33\ M_\odot$.

We have plotted the gravitational mass of an SQS as a function of the radius (mass-radius relation) for the magnetic fields $B = 0$ and 5×10^{18} G in Figure 9. It is seen that for all cases of SQSs, the gravitational mass increases by increasing the radius.

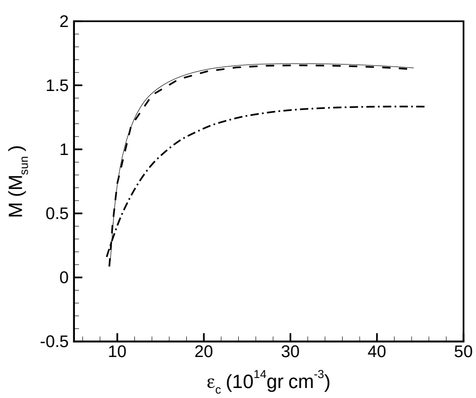

Fig. 8 Gravitational mass versus the central energy density (ε_c) at $B = 0$ (*solid curve*) and $B = 5 \times 10^{18}$ G (*dashed curve*). The results for $\mathcal{B}$ = 90 MeV fm^{-3} (*dashed dotted curve*) at $B = 5 \times 10^{18}$ G have also been given for comparison.

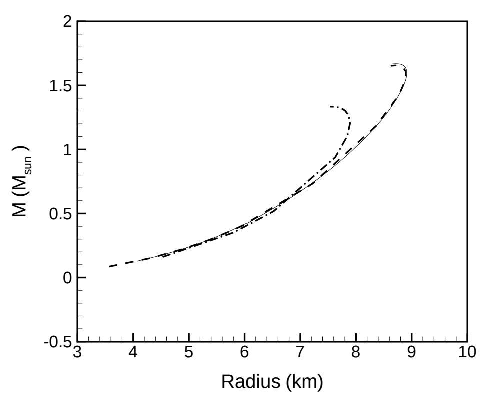

Fig. 9 The gravitational mass versus radius at $B = 0$ (*solid curve*) and $B = 5 \times 10^{18}$ G (*dashed curve*). The results for $\mathcal{B}$ = 90 MeV fm^{-3} (*dashed dotted curve*) at $B = 5 \times 10^{18}$ G have also been given for comparison.

In Figure 9, we have also compared our results for the density dependent case of the bag constant with those of the density independent case. We can see that for the case of the fixed bag constant, the increasing rate of gravitational mass versus radius is higher than that of the density dependent case. However, it will be more constructive to consider the effects of rotation on the properties of the star, which is beyond our present investigation. Some authors have shown that considering the rotation of the star leads to a larger maximum mass for SQSs (Shen et al. 2005).

4 SUMMARY AND CONCLUSIONS

We have investigated a cold static SQS in the presence of a strong magnetic field. For this purpose, some of the bulk properties of the SQM, such as the energy density and EOS, have been computed using the MIT bag model with the density dependent bag constant. Calculations of the energy for different magnetic polarizations in the presence of a magnetic field demonstrated that as the density increases, the minimum point of energy shifts to lower values of the polarization. We have shown that the value of the polarization parameter decreases by increasing the density, and it also increases by increasing the magnetic field. Our results at $B = 5 \times 10^{18}$ G show that for both the cases of a density dependent bag constant and fixed bag constant, the total energy density has the effect of an increasing rate from increasing the density. We have shown that there is a ferromagnetic phase transition at high magnetic fields. Our computations indicate that the pressure increases by increasing the density and magnetic field. In this work, we have also studied the structural properties of SQSs. Our results show that the gravitational mass of the SQS increases by increasing the central energy density. It was shown that this gravitational mass reaches a limiting value at higher values of the central energy density. We have shown that the maximum mass of the SQS reduces in the presence of the magnetic field. Finally, a comparison has also been made between the results of the density dependent bag constant and those of a fixed bag constant. Our calculation with the density dependent bag constant shows a higher maximum mass with respect to that of the fixed bag constant.

One of the possible astrophysical implications of our results is calculation of the surface redshift $(z_{\rm s})$ of an SQS. This parameter is of special interest in astrophysics and can be obtained from the mass and radius of the star using the following relation (Camenzind 2007),

$$z_{\rm s} = \left(1 - \frac{2GM}{Rc^2}\right)^{-\frac{1}{2}} - 1. \tag{24}$$

Our results corresponding to the maximum mass and radius of an SQS calculated by the density dependent bag constant lead to $z_s = 0.534\,{\rm ms}^{-1}$ for the magnetic field of $B = 5 \times 10^{18}$ G.

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