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Cosmological perturbations in a noncommutative braneworld inflation

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Abstract We use the smeared, coherent state picture of noncommutativity to study evolution of perturbations in a noncommutative braneworld scenario. Within the standard procedure of studying braneworld cosmological perturbations, we study the evolution of the Bardeen metric potential and curvature perturbations in this model. We show that in this setup, the early stage of the universe's evolution has a transient phantom evolution with imaginary effective sound speed.

Key words: early universe cosmology — cosmology theory

1 INTRODUCTION

Inspired by some aspects of string theory and loop quantum gravity, the *fuzziness* of a spacetime manifold can be expressed through the following relation for non-commutativity of coordinate operators (Douglas & Nekrasov 2001; Szabo 2003; Seiberg & Witten 1999; Connes & Marcolli 2006; Connes 2000; Konechny & Schwarz 2002; Chaichian et al. 2003; Micu & Sheikh-Jabbari 2001; Chamseddine & Connes 2010; Veneziano 1986; Amati et al. 1987, 1988, 1989, 1990; Gross & Mende 1988)

$$[\hat{x}^i, \hat{x}^j] = i\theta^{ij}, \tag{1}$$

where θ^{ij} is a real, antisymmetric matrix, with the dimension of length squared which determines the fundamental cell discretization of the spacetime manifold. As a consequence of the above relation, the notion of a point in the spacetime manifold becomes obscure as there is a fundamental uncertainty in measuring the coordinates

$$\Delta x^i \Delta x^j \geq \frac{1}{2}|\theta^{ij}|. \tag{2}$$

This finite resolution of the spacetime points especially affects cosmological dynamics in early stages of the universe's evolution. On the other hand, inflation has been identified as a great opportunity to test theories of Planck scale physics including noncommutative geometry. Essentially, effects of trans-Planckian physics should be observable in the cosmic microwave background radiation (Easther et al. 2001, 2002, 2003; Kaloper et al. 2002; Bergström & Danielsson 2002; Martin & Brandenberger 2003; Maartens et al. 2000).

For this reason, various attempts to construct noncommutative inflationary models have been done by adopting different approaches. These approaches include using relation (1) for space-space (Chu et al. 2001; Lizzi et al. 2002; Hassan & Sloth 2003) and space-time (Brandenberger & Ho 2002) coordinates and constructing a noncommutative field theory on the spacetime manifold by replacing the ordinary product of fields by a Weyl-Wigner-Moyal $* - product$. Another way to incorporate effects of high energy physics in inflationary models is by using the generalized uncertainty principle (GUP) which is a manifestation of the existence of a fundamental length scale in the system (Alexander et al. 2003; Alexander & Magueijo 2001; Koh & Brandenberger 2007).

Recently a new approach to noncommutative inflation has been proposed by Rinaldi (2011) using the coherent state picture of noncommutativity introduced in Smailagic & Spallucci (2003a,b, 2004). This model is free from some of the problems that plagued models based on a $* - product$, such as unexpected divergences and UV/IR mixing (see Nicolini 2009 for a comprehensive review). The key idea in this model is that noncommutativity *smears* the initial singularity and as a result there will be an smooth transition between pre and post big bang eras via an accelerated expansion. It has been shown that noncommutativity eliminates point-like structures in favor of smeared objects in flat spacetime. As Nicolini et al. have shown (Nicolini 2005; Nicolini et al. 2006; Spallucci et al. 2006) (see also Rizzo 2006; Ansoldi et al. 2007; Spallucci et al. 2009; Nozari & Mehdipour 2008, 2009 for some other extensions), the effect of smearing is mathematically implemented as a substitution rule: the Dirac-delta function representing position is replaced everywhere with a Gaussian distribution of minimal width $\sqrt{\theta}$. In this framework, they have chosen the mass density of a static, spherically symmetric, smeared, particle-like gravitational source as follows

$$\rho_\theta(r) = \frac{M}{(2\pi\theta)^{\frac{3}{2}}} \exp\Big(- \frac{r^2}{4\theta}\Big) . \tag{3}$$

As they indicated, the particle mass M, instead of being perfectly localized at a point, is diffused throughout a region of linear size $\sqrt{\theta}$. This is due to the intrinsic uncertainty as has been shown in the coordinate commutators (1).

Before sketching the platform of our work in this paper, we emphasize two important issues: First the noncommutative relations (1) are written in the *comoving* coordinates. The commutators between *physical* spatial coordinates involve the scale factor. As a result, the physical noncommutative scale will become extremely small at earlier scales, and one would expect that this prevents the noncommutative effects from becoming too large at earlier times. This could be desirable, since such effects break rotational invariance. So, in the physical frame, all the subsequent equations are still valid but noncommutative parameter θ will be time dependant in this frame. Secondly, in the cosmology emerging from our model, time runs from $-\infty$ to $+\infty$ and the time $t = 0$ corresponds to when the typical scale of the universe coincides with the non-commutativity scale. As we will show (see also Nozari & Akhshabi 2010), it is the cosmological evolution around $t = 0$ which shows the features which differentiate this cosmology from the usual one. In particular, it is in this time region that the phantom divide is crossed. This may raise a question: why should the Friedmann equations be applicable on these scales where non-local physics is important? To address this problem and derive a solution in the absence of complete noncommutative Einstein field equations, we assume that at a semiclassical level, noncommutativity only alters the source side of the Einstein field equations and does not change the left hand side. In other words, we suppose that noncommutativity acts on the matter sector of the theory and leaves the geometric part unaltered. In this manner we can still use the usual Einstein (Friedmann) equations but with a noncommutative source smeared through a small region of spacetime.

Recently we have constructed a noncommutative braneworld inflation scenario (Nozari & Akhshabi 2010) based on the idea that initial singularity is smeared in a noncommutative background. Within the same streamline, the purpose of this paper is to study the time evolution of cosmological perturbations in a braneworld inflation scenario in the context of spacetime noncommutativity.

2 COSMOLOGICAL DYNAMICS IN THE NONCOMMUTATIVE RS II MODEL

The 5D field equations in the Randall-Sundrum (RS) II (Randall & Sundrum 1999) setup are

$$^{(5)}G_{AB} = -\Lambda_5 \, ^{(5)}g_{AB} + \delta(y)\, \frac{8\pi}{M_5^3} \left[-\lambda g_{AB} + T_{AB}\right], \tag{4}$$

where y is a Gaussian normal coordinate orthogonal to the brane (the brane is localized at $y = 0$), λ is the brane tension, and T_{AB} is the energy-momentum tensor of particles and fields confined to the brane. The effective field equations on the brane are derived from the Gauss-Codazzi equations and junction conditions (using Z_2-symmetry) (Shiromizu et al. 2000; Binétruy et al. 2000; Maartens 2004)

$$G_{ab} = -\Lambda g_{ab} + \kappa^2 T_{ab} + 6\frac{\kappa^2}{\lambda}\mathcal{S}_{ab} - \mathcal{E}_{ab} \,, \tag{5}$$

where $\mathcal{S}_{ab} \sim (T_{ab})^2$ is the high-energy correction term, which is negligible for $\rho \ll \lambda$, while $\mathcal{E}_{ab}$ is the projection of the bulk Weyl tensor on the brane. The general form of the brane energy-momentum tensor for any matter fields (scalar fields, perfect fluids, kinetic gases, dissipative fluids, etc.), including a combination of different fields, can be covariantly given in terms of a chosen 4-velocity u^{μ} as

$$T_{\mu\nu} = \rho u_{\mu} u_{\nu} + p h_{\mu\nu} + \pi_{\mu\nu} + q_{\mu} u_{\nu} + q_{\nu} u_{\mu} \,. \tag{6}$$

Here ρ and p are the energy density and isotropic pressure and q and π are the momentum density and anisotropic stress respectively. $h_{\mu\nu}$ defined as

$$h_{\mu\nu} = g_{\mu\nu} + u_{\mu} u_{\nu} = \, ^{(5)}g_{\mu\nu} - n_{\mu} n_{\nu} + u_{\mu} u_{\nu} \tag{7}$$

projects into the comoving rest space at each event where n_{ν} is the spacelike unit normal to the brane. The modified Friedmann and Raychaudhuri equations in the background are (Shiromizu et al. 2000; Binétruy et al. 2000)

$$H^2 = \frac{\kappa^2}{3}\rho\left(1 + \frac{\rho}{2\lambda}\right) + \frac{C}{a^4} + \frac{1}{3}\Lambda - \frac{K}{a^2}\,, \tag{8}$$

and

$$\dot{H} = -\frac{\kappa^2}{2}(\rho + p)\left(1 + \frac{\rho}{\lambda}\right) - 2\frac{C}{a^4} + \frac{K}{a^2}\,, \tag{9}$$

respectively. By definition, $C = \frac{\kappa^2}{3}\rho_{\varepsilon 0} a_0^4$ where $\rho_{\varepsilon 0}$ is the dark radiation energy density. For a matter content consisting of a perfect fluid or a minimally coupled scalar field, the total effective energy density, pressure, momentum density and anisotropic stress can be written as (Maartens 2004)

$$\rho^{\rm eff} = \rho\left(1 + \frac{\rho}{2\lambda} + \frac{\rho^{\varepsilon}}{\rho}\right)\,, \tag{10}$$

$$p^{\rm eff} = p + \frac{\rho}{2\lambda}(2p + \rho) + \frac{\rho^{\varepsilon}}{3}\,, \tag{11}$$

$$q_a^{\rm eff} = q_a^{\varepsilon}\,, \tag{12}$$

$$\pi_{ab}^{\rm eff} = \pi_{ab}^{\varepsilon}\,, \tag{13}$$

where superscript ε denotes the contribution of the bulk Weyl tensor which enters the modified Friedmann equation as a non-local dark radiation term. Using these definitions, the modified Friedmann and Raychaudhuri equations can be rewritten as

$$H^2 = \frac{\kappa^2}{3}\rho^{\rm eff} + \frac{1}{3}\Lambda + \frac{K}{a^2}\,, \tag{14}$$

$$\dot{H} = -\frac{\kappa^2}{2}(\rho^{\rm eff} + p^{\rm eff}) + \frac{K}{a^2}\,. \tag{15}$$

The tracefree property of $\mathcal{E}^{\mu}_{\nu}$ in Equation (5) implies that the pressure obeys $P^{\varepsilon} = \frac{1}{3}\rho^{\varepsilon}$.

The local conservation equations on the brane are (Maartens 2004)

$$\dot{\rho} + \Theta(\rho + p) = 0, \tag{16}$$

$$D_a p + (\rho + p)A_a = 0, \tag{17}$$

where Θ is the volume expansion rate, which reduces to $3H$ in the Friedmann-Robertson-Walker background (H is the background Hubble rate), A_a is the 4-acceleration, and D_a is the covariant derivative in the rest space. The non-local conservation equations for the dark radiation matter can be expressed as (Maartens 2004)

$$\dot{\rho}^{\varepsilon} + \frac{4}{3}\Theta\rho^{\varepsilon} + D^a q^{\varepsilon} = 0, \tag{18}$$

$$\dot{q}^{\varepsilon}_a + 4Hq^{\varepsilon}_a + \frac{1}{3}D_a\rho^{\varepsilon} + \frac{4}{3}\rho^{\varepsilon}A_a + D^b\pi^{\varepsilon}_{ab} = -\frac{(\rho+p)}{\lambda}D_a\rho\,. \tag{19}$$

We now suppose that the initial singularity that leads to RS II geometry afterwards is smeared due to spacetime noncommutativity. A newly proposed model for a similar scenario in the usual 4D universe suggests that one could write the energy density as (Rinaldi 2011; Nozari & Akhshabi 2010)

$$\rho(t) = \frac{1}{32\pi^2\theta^2}\mathrm{e}^{-t^2/4\theta}\,. \tag{20}$$

Note that we suppose that the universe enters the RS II geometry immediately after the initial smeared singularity which is a reasonable assumption (for instance, from an M-theory perspective of the cyclic universe this assumption seems to be reliable, see Steinhardt & Turok 2002, 2003; Khoury et al. 2004; Turok & Steinhardt 2005; Bojowald et al. 2004). Using Equation (20), and setting $\Lambda = 0 = K$, the Friedmann Equation (14) in noncommutative space could be rewritten as follows

$$H^2 = \frac{\kappa^2}{3}\rho^{\mathrm{eff}}(t), \tag{21}$$

where ρ^{eff} is given by Equation (10). From Equation (16) one can find the effective noncommutative pressure using Equation (20) as

$$p = -\rho + \frac{t}{6\theta}\mathrm{e}^{-t^2/8\theta}\,. \tag{22}$$

So, the equation of state parameter will be

$$\omega = -1 + \frac{16}{3}\pi^2\theta\, t\mathrm{e}^{-t^2/8\theta} \tag{23}$$

and the speed of sound is

$$c_s^2 = \frac{\dot{p}}{\dot{\rho}} = \frac{-3t - 64\theta^2\pi^2\mathrm{e}^{-t^2/8\theta} + 32\theta\pi^2t^2\mathrm{e}^{-t^2/8\theta}}{3t}\,. \tag{24}$$

Using Equations (10) and (11) we can find the *effective* equation of state and speed of sound. To this end, we note that there are constraints from nucleosynthesis on the value of ρ^{ε} so that $\frac{\rho^{\varepsilon}}{\rho} \leq 0.03$ at the time of nucleosynthesis (Burles et al. 1999; Langlois et al. 2001). In this respect, we can neglect this contribution to find

$$\begin{aligned}\omega^{\mathrm{eff}} \;=\; & \frac{1}{192}\,\mathrm{e}^{-\frac{1}{8}\,\frac{t^2}{\theta}}\Big[-192\,\pi^2\theta^2\lambda + 1024\,t\mathrm{e}^{\frac{-t^2}{8\theta}}\pi^4\theta^3\lambda - 3\,\mathrm{e}^{\frac{-t^2}{8\theta}} + 32\,t\mathrm{e}^{\frac{-t^2}{4\theta}}\pi^2\theta\Big] \\ & \times\Big\{\theta\,\Big[\frac{1}{64}\,\Big(64\,\pi^2\theta^2\lambda + \mathrm{e}^{\frac{-t^2}{8\theta}}\Big)\,\pi^{-2}\theta^{-2}\lambda^{-1}\Big]\Big\}^2 \\ & \times\pi^{-2}\theta^{-4}\lambda^{-1}\Big\{\mathrm{e}^{\frac{-t^2}{8\theta}}\,\Big[\frac{1}{64}\,\Big(64\,\pi^2\theta^2\lambda + \mathrm{e}^{\frac{-t^2}{8\theta}}\Big)\,\pi^{-2}\theta^{-2}\lambda^{-1}\Big]\Big\}^{-1},\end{aligned} \tag{25}$$

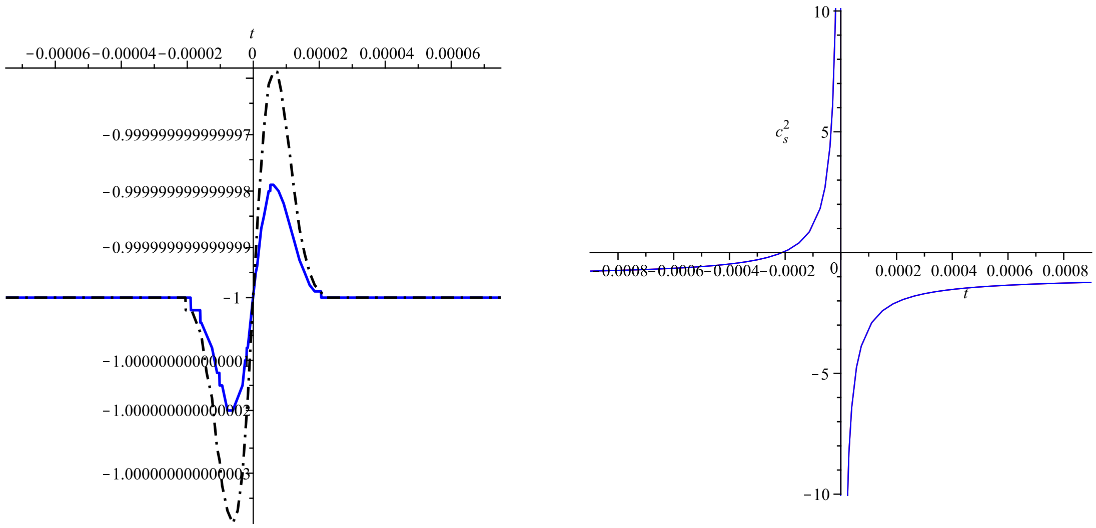

Fig. 1 (a) Evolution of the noncommutative equation of state parameter (*solid line*) and the noncommutative effective equation of state parameter (*dash-dotted line*) versus the cosmic time. (b) Evolution of the noncommutative effective speed of sound versus the cosmic time. For $t>0$ and in the high energy noncommutative regime, c_s is imaginary (a phantom evolution).

which simplifies to the following equation for the high energy regime ($\rho \gg \lambda$)

$$\omega^{\rm eff} \approx -1 + \frac{32}{3}\pi^2\theta\, t{\rm e}^{-t^2/8\theta}\,. \tag{26}$$

Similarly, the effective speed of sound in the high energy regime will be

$$(c_s^2)^{\rm eff} \approx \frac{16}{3}\pi^2\theta\, t{\rm e}^{-t^2/8\theta} + \frac{-3t - 64\theta^2\pi^2{\rm e}^{-t^2/8\theta} + 32\theta\pi^2 t^2{\rm e}^{-t^2/8\theta}}{3t}\,. \tag{27}$$

Figure 1(a) shows the evolution of the equation of state parameter and the effective equation of state parameter as given by Equations (23) and (25) respectively. As one can see from this figure, there is a small variation in ω and $\omega^{\rm eff}$ around the smeared singularity.

Figure 1(b) shows the evolution of the effective speed of sound. It is obvious from this figure that in $t>0$ and in the high energy noncommutative regime, c_s is imaginary. In this respect, the evolution of the universe in the early, inflationary stage is a phantom evolution.

We use these results in the next section to determine time evolution of the cosmological perturbations.

3 EVOLUTION OF LARGE SCALE SCALAR PERTURBATIONS

The evolution of cosmological perturbations in the Randall-Sundrum braneworld scenario has been studied extensively (see for instance Maartens 2000; Barrow & Maartens 2002; Maartens et al. 2000; Copeland et al. 2001; Langlois et al. 2000; Mukohyama 2000; Langlois 2001 and references therein). To analyze the scalar perturbations in our noncommutative setup, we define energy density and expansion perturbations following Maartens (2002) who used the covariant 3+1 analysis developed in Gordon & Maartens (2001)

$$\Delta = \frac{a^2}{\rho} D^2 \rho \,, \qquad Z = a^2 D^2 \Theta \,. \tag{28}$$

Similarly, the perturbations in nonlocal quantities associated with the dark radiation matter are defined as

$$U = \frac{a^2}{\rho} D^2 \rho^{\varepsilon} \,, \qquad Q = \frac{a}{\rho} D^2 q^{\varepsilon} \,, \qquad \Pi = \frac{1}{\rho} D^2 \pi^{\varepsilon} \,. \tag{29}$$

With these definitions, the equations governing the evolution of perturbations (Eqs. (16)–(19)) will take the following forms

$$\dot{\Delta} \;=\; 3wH\Delta - (1+w)Z \,, \tag{30}$$

$$\dot{Z} \;=\; -2HZ - \left(\frac{c_{\rm s}^2}{1+w}\right) D^2\Delta - \kappa^2 \rho U - \frac{1}{2}\kappa^2 \rho \left[1 + (4+3w)\frac{\rho}{\lambda} - \left(\frac{4c_{\rm s}^2}{1+w}\right)\frac{\rho^{\varepsilon}}{\rho}\right]\Delta \,, \tag{31}$$

$$\dot{U} \;=\; (3w-1)HU + \left(\frac{4c_{\rm s}^2}{1+w}\right)\left(\frac{\rho^{\varepsilon}}{\rho}\right) H\Delta - \left(\frac{4\rho^{\varepsilon}}{3\rho}\right) Z - aD^2 Q \,, \tag{32}$$

$$\dot{Q} \;=\; (3w-1)HQ - \frac{1}{3a}U - \frac{2}{3}aD^2\Pi + \frac{1}{3a}\left[\left(\frac{4c_{\rm s}^2}{1+w}\right)\frac{\rho^{\varepsilon}}{\rho} - 3(1+w)\frac{\rho}{\lambda}\right]\Delta \,, \tag{33}$$

where ρ, ω and c_s are given by Equations (20), (23) and (24), respectively.

In general, scalar perturbations on the brane cannot be predicted by brane observers without additional information from the bulk because there is no equation for $\dot{\Pi}$ in the above set of equations. Nevertheless, it has been shown that on large scales one can neglect the $D^2\Pi$ term in Equation (33). So, on large scales, the system of equations closes on the brane, and brane observers can predict scalar perturbations from initial conditions intrinsic to the brane without the need to solve the bulk perturbation equations (Maartens 2002; Gordon & Maartens 2001).

To solve the above system of equations using the simplification mentioned, we introduce two new variables; the first is a scalar covariant curvature perturbation variable

$$C \equiv a^4 D^2 R = -4a^2 HZ + 2\kappa^2 a^2 \rho \left(1 + \frac{\rho}{2\lambda}\right)\Delta + 2\kappa^2 a^2 \rho U \,, \tag{34}$$

where R is the Ricci curvature of the surfaces orthogonal to u^{μ}. The second variable is a covariant analog of the Bardeen metric potential Φ_H,

$$\Phi = \kappa^2 a^2 \rho \Delta \,. \tag{35}$$

Along each fundamental world-line, covariant curvature perturbation, C, is locally conserved

$$C = C_0 \,, \qquad \dot{C}_0 = 0 \,. \tag{36}$$

With these new variables, the system of equations reduces to

$$\dot{\Phi} \;=\; -H\left[1 + (1+w)\frac{\kappa^2\rho}{2H^2}\left(1+\frac{\rho}{\lambda}\right)\right]\Phi - \left[(1+w)\frac{a^2\kappa^4\rho^2}{2H}\right]U + \left[(1+w)\frac{\kappa^2\rho}{4H}\right]C_0, \tag{37}$$

$$\dot{U} \;=\; -H\left[1 - 3w + \frac{2\kappa^2\rho^{\varepsilon}}{3H^2}\right]U - \frac{2\rho^{\varepsilon}}{3a^2H\rho}\left[1 + \frac{\rho}{\lambda} - \frac{6c_{\rm s}^2 H^2}{(1+w)\kappa^2\rho}\right]\Phi + \left[\frac{\rho^{\varepsilon}}{3a^2H\rho}\right]C_0 \,. \tag{38}$$

If there is no dark radiation in the background, $\rho^{\varepsilon} = 0$, then

$$U = U_0 \exp\left[\int (3w-1)dN\right], \tag{39}$$

where N is the number of e-folds. In this case, the above system reduces to a single equation for Φ which is

$$\frac{d\Phi}{dN}+\left[1+\frac{(1+w)\kappa^2\rho}{2H^2}\left(1+\frac{\rho}{\lambda}\right)\right]\Phi=\left[\frac{(1+w)\kappa^2\rho}{4H^2}\right]C_o-\left[\frac{3(1+w)a_o^2\rho^2}{\lambda H^2}\right]\mathrm{e}^{2N}U, \quad (40)$$

where U is given by Equation (39). We use these results in the next section to study noncommutative modifications of the dynamics of scalar perturbations.

4 NONCOMMUTATIVE MODIFICATIONS

Now we want to solve Equation (40) using explicit noncommutative forms of ρ, H, ω and U given by (20), (21), (23) and (39) respectively. To this end, we need to specify the noncommutative form of N which has appeared in Equation (39). As we have shown in Nozari & Akhshabi (2010), the noncommutative number of e-folds is given by

$$N=\int_{t_i}^{t_f}H\,dt\simeq\frac{8}{3}\pi\kappa^2\,\rho_0\left[\sqrt{\pi\theta}\;\operatorname{erf}\Big(\frac{1}{2}\,\frac{t_f}{\sqrt{\theta}}\Big)+\frac{1}{2}\sqrt{2\pi\theta}\;\operatorname{erf}\Big(\frac{1}{2}\,\frac{\sqrt{2}t_f}{\sqrt{\theta}}\Big)\lambda^{-1}\right]$$
$$+\frac{8}{3}\pi\kappa^2\,\rho_0\left[\sqrt{\pi\theta}\;\operatorname{erf}\Big(\frac{1}{2}\,\frac{t_i}{\sqrt{\theta}}\Big)+\frac{1}{2}\sqrt{2\pi\theta}\;\operatorname{erf}\Big(\frac{1}{2}\,\frac{\sqrt{2}t_i}{\sqrt{\theta}}\Big)\lambda^{-1}\right], \quad (41)$$

where $\operatorname{erf}(x)$ denotes the error function. By expanding the error functions in Equation (41) in a series, the number of e-folds (supposing that the universe enters the inflationary phase immediately after the big bang, that is, $t_i=0$ and $t_f=t$) will be given by

$$N\simeq\frac{8}{3}\pi\kappa^2\,\rho_0\left[t-\frac{1}{12}\,\frac{t^3}{\sqrt{\pi}\theta^{\frac{3}{2}}}+\frac{1}{160}\,\frac{t^5}{\sqrt{\pi}\theta^{\frac{5}{2}}}+\frac{1}{2}\Big(2\,t-\frac{1}{6}\,\frac{\sqrt{2}t^3}{\sqrt{\pi}\theta^{\frac{3}{2}}}+\frac{1}{40}\,\frac{\sqrt{2}t^5}{\sqrt{\pi}\theta^{\frac{5}{2}}}\Big)\lambda^{-1}\right]. \quad (42)$$

Now we can integrate Equation (40) to find

$$\begin{aligned}\Phi\;=\;&\frac{1}{2}(1+\omega)\frac{\rho\,\lambda\,\kappa^2C_0}{2H^2\lambda+(1+\omega)(\kappa^2\rho\lambda+\kappa^2\rho^2)}\\&-6(1+\omega)\frac{H^2\rho\,\lambda\,{a_0}^2U\exp\left(3\,\omega\,a-3\,\omega\,a_0-a+a_0\right)}{6H^2\lambda+(1+\omega)(\kappa^2\rho\lambda+\kappa^2\rho^2)}\exp\Big(\frac{-t^2}{4\theta}\Big)\\&+\exp\left[-\frac{1}{2}\,\frac{2\,H^2\lambda+(1+\omega)(\kappa^2\rho\,\lambda+\kappa^2\rho^2)}{H^2\lambda}\,\frac{t^2}{8\theta}\right].\end{aligned} \quad (43)$$

Figure 2 shows the evolution of Φ for both the usual braneworld scenario and our noncommutative setup in the high energy inflation regime ($\rho\gg\lambda$). One should note that the subsequent evolution of the universe after times greater than a few $\sqrt{\theta}$ should be governed by a matter content[1] different than the one used in Equation (21) (i.e. energy density of the initial singularity smeared by noncommutativity). So, the evolution of ω, c_s^2 and Φ in the low energy regime should essentially be different. As Figure 2 shows, since for large times the length scale of the fluctuation mode is large, the noncommutativity should have no effect. In fact, due to the exponentially decaying profile of the source term, the right hand side of the Friedmann equation vanishes after a short time. In addition, note that at large times, the time dependences of the two curves are not the same. This is because the standard case contains an inflation field which essentially has a different cosmological evolution relative to our smeared initial singularity picture. Therefore, only around $t=0$ do we expect to have a similar behavior between the two scenarios.

[1] See for instance Parker (1969) for particle creation in an expanding universe.

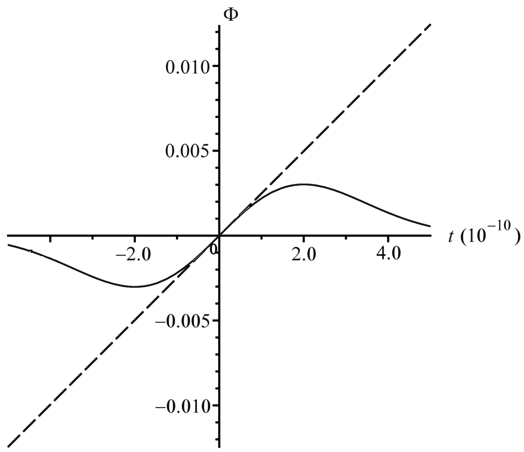

Fig. 2 Evolution of the parameter Φ which is an analog of the Bardeen metric potential as defined in (35) for both the usual braneworld scenario (*dashed line*) and our noncommutative setup (*solid line*) when $\frac{\rho_0}{\lambda} = 10^{10}$. We assumed that no dark energy is present in the background geometry.

The solution (43) is valid when there is no dark radiation in the background. If $\rho^{\varepsilon} \neq 0$, then one should solve the system of Equations (37) and (38) using the explicit form of ρ^{ε}. Generally the time dependence of ρ^{ε} for a brane observer is not determined. Here we introduce a possible candidate for this quantity: as we have mentioned previously, the constraint from nucleosynthesis on the value of ρ^{ε} is therefore $\frac{\rho^{\varepsilon}}{\rho} \leq 0.03$ at the time of nucleosynthesis. Based on this constraint, we can assume for instance that ρ^{ε} is a small fraction of ρ at a given time. Since the time evolution of ρ is determined by Equation (20), the time evolution of ρ^{ε} can be assumed to be

$$\rho^{\varepsilon}(t) = \frac{\delta}{32\pi^2\theta^2} \mathrm{e}^{-t^2/4\theta} \,,$$

where δ is a small constant less than 0.03. This form of $\rho^{\varepsilon}(t)$ can be used to explicitly solve the system of Equations (37) and (38). Nevertheless, this procedure needs a lot of calculations with very lengthy solutions, so we ignore their presentation here.

The curvature perturbation defined in a metric-based perturbation theory is

$$\xi = \mathcal{R} + \frac{\delta\rho}{3(\rho + p)} \,, \tag{44}$$

which reduces to $\mathcal{R}$ on hypersurfaces with uniform density ($\delta\rho = 0$). If there is no dark radiation in the background ($\rho^{\varepsilon} = 0$), the total curvature perturbation on a large scale is given by the following differential Equation (45)

$$\dot{\xi}^{\,\mathrm{eff}} = \dot{\xi}^{\,\mathrm{m}} + H \left[c_{\mathrm{s}}^2 - \frac{1}{3} + \left(\frac{\rho + p}{\rho + \lambda} \right) \right] \frac{\delta\rho^{\varepsilon}}{(\rho + p)(1 + \rho/\lambda)} \,, \tag{45}$$

where ξ^{m} is a matter perturbation which is zero for adiabatic perturbations. Since the time variations of ρ, H, p, c_s and $\delta\rho^{\varepsilon}$ are given by Equations (20), (21), (22), (24) and (39) respectively, we can

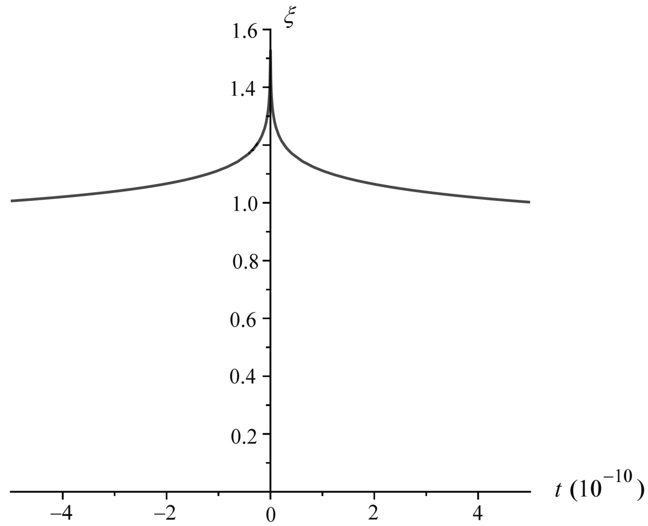

Fig. 3 Evolution of the parameter ξ defined in Equation (44) (with the same parameter values as in Fig. 2) when $\frac{\rho_0}{\lambda} = 10^{10}$ and no dark radiation is present in the background geometry.

obtain the time evolution of the curvature perturbation explicitly as follows

$$\xi^{\,\mathrm{eff}} = \frac{1}{96}\frac{\mathrm{Ei}\left(1, \frac{3t^2}{8\theta}\right)}{\pi^2\theta^2\lambda} - \frac{1}{48}\frac{\mathrm{e}^{-\frac{3t^2}{8\theta}}}{\pi^2\theta^2\lambda} + \frac{1}{3}\,\mathrm{Ei}\left(1, \frac{t^2}{4\theta}\right) - \frac{2}{3}\,\mathrm{e}^{-\frac{t^2}{4\theta}}$$
$$-\frac{1}{768}\,\mathrm{erf}\left(\frac{t}{2\sqrt{\theta}}\right)\lambda^{-1}\pi^{-3}\theta^{-3}\frac{1}{\sqrt{\theta\,\pi}} - \frac{1}{24}\,\mathrm{erf}\left(\frac{1}{4}\frac{\sqrt{2}t}{\sqrt{\theta}}\right)\sqrt{2}\theta^{-1}\pi^{-1}\frac{1}{\sqrt{\theta\,\pi}}\,, \quad (46)$$

where $\mathrm{Ei}(a, z)$ is the exponential integral defined as $\mathrm{Ei}(a, z) = z^{a-1}\Gamma(1 - a, z)$.

Figure 3 shows the evolution of $\xi^{\,\mathrm{eff}}$ versus the cosmic time in our noncommutative brane inflation. As the figure shows, ξ is a constant at late times.

5 CONCLUSIONS

Spacetime noncommutativity, as a trans-Planckian effect, could have some observable effects on the cosmic microwave background radiation. In this respect, it is desirable to study an inflation scenario within a noncommutative background. Recently we have shown the possibility of finding a non-singular, bouncing, early time cosmology in a noncommutative braneworld scenario (Nozari & Akhshabi 2010). In that work, using the smeared, coherent state picture of the spacetime noncommutativity, we have constructed a braneworld inflation that has the potential to support the scale invariant scalar perturbations. Here, following our previous work, we have studied the time evolution of the perturbations in this noncommutative braneworld setup. We have neglected the contribution of the dark radiation term (originating in the bulk Weyl tensor) in the background geometry to have a closed set of equations on the brane. However, the contributions of this term in the evolution of perturbations on the brane are taken into account. In this way, by studying the effective quantities (such as the effective equation of state and effective speed of sound on the brane) we have derived the possibility of a phantom evolution in the early, inflationary stage of the universe's history. We emphasize that, in general, one requires long wavelength perturbations to grow into large scale classical perturbations (around which structure eventually forms), and whose overall amplitude depends on the parameters of the setup and is consistent with what is observed (for instance by COBE normalization). As another important point, we note that one needs a smaller number of e-folds in the noncommutative regime to have a successful braneworld inflation (see also Nozari & Akhshabi 2010).

It should be noted that Equation (1) was defined with respect to the comoving coordinates x, which means we are considering a scenario in which noncommutativity parameter θ is constant in the comoving frame. In physical coordinates this means that θ is growing with the scale factor, something that seems to be reasonable and is suggested in string theory (Chu et al. 2001; Lizzi et al. 2002; Hassan & Sloth 2003, see also Stern 2010). If θ subsequently drops to zero, at the end of inflation, this should yield a viable cosmology. One may also consider the case in which θ is constant in the physical frame. In that case we have to use a time varying θ with an exponential decaying factor that causes the effect of noncommutativity to get redshifted away by inflation. However so long as θ is constant in the comoving frame and shuts down by the end of inflation, this can lead to a small amount of non-gaussianity (Chu et al. 2001; Lizzi et al. 2002; Hassan & Sloth 2003). We also note that in this noncommutative setup for large times, the noncommutativity should have no effect since the length scale of the fluctuation mode is large. This can be seen by constancy of ξ at late time in Figure 3. This is due to the exponentially decaying profile of the source term on the right hand side of the Friedmann equation which vanishes at late times.

Finally, in the cosmology emerging from this model, time runs from $-\infty$ to $+\infty$ and the time $t = 0$ plays a crucial role. In particular, it is during this time period that the phantom divide is crossed. One may doubt the validity of the standard Friedmann equation around this point. We note that in the absence of a complete formulation of the noncommutative Einstein field equations, we can assume that noncommutativity acts on the matter sector of the theory and leaves the geometric part unaltered. By this assumption, we still can use the usual Einstein (Friedmann) equations but with a noncommutative source smeared through a small region of spacetime.

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