Non-static plane symmetric inflationary Universe in scalar tensor theory

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Abstract Non-static plane symmetric cosmological solutions are presented in the presence of cosmic strings in the scalar-tensor theory of gravitation formulated by Sen & Dunn. It is shown that string cosmological models representing geometric strings ($\rho = \lambda$) and massive strings ($\rho + \lambda = 0$) do exist in this theory. Further, it is found that the Takabayasi string, i.e. $\rho = (1+\xi)\lambda$, does not exist. Some physical and geometrical features of these models are discussed.

Key words: string fields — inflation — plane symmetry — acceleration of particles — gravitation

1 INTRODUCTION

One of the outstanding problems in cosmology today is developing a more precise understanding of structure formation in the Universe, that is, the origin of galaxies and other large-scale structures. Existing theories for the structure formation of the Universe fall into two categories, based either upon the amplification of quantum fluctuations in a scalar field during inflation, or upon symmetry breaking of phase transitions in the early Universe, which leads to the formation of topological defects such as domain walls, cosmic strings, monopoles, textures and other "hybrid" creatures. Cosmic strings play an important role in the study of the early Universe. Cosmic strings are topologically stable objects, which might be found during a phase transition in the early Universe (Kibble 1976). The general relativistic treatment of strings was initiated by Letelier (1979, 1983). Letelier (1979) obtained the solution to Einstein's field equations for a cloud of strings with spherical, plane and cylindrical symmetry. Then, in 1983, he solved Einstein's field equations for a cloud of massive strings and obtained cosmological models in Bianchi I and Kantowski-Sachs space-times. It is also believed that strings may be one of the sources of density perturbations that are required for the formation of large scale structures in the Universe. String theory and higher dimensional unifying theories, in their low energy limit, predict the existence of a scalar partner of the tensor graviton - the dilation. These derived scalar-tensor theories are considered as the most natural generalization of general relativity (Damour & Polyakov 1994). A lot of efforts have been devoted to finding and analyzing exact scalar-tensor solutions in order to more deeply understand the physics behind these

theories, in particular their relevance to cosmology and astrophysics. The scalar-tensor gravity equations are much more complicated than the Einstein equations and their solution in the presence of a source is a very difficult task. That is why one should assume some simplifications in order to solve these scalar-tensor equations. In this way, many homogeneous cosmological solutions with a perfect fluid have been obtained. Some inhomogeneous scalar-tensor cosmologies have also been found and a method for generating general classes of exact scalar-tensor solutions with a stiff perfect fluid has been given by Yazadjiev (2002). However, the known exact scalar-tensor solutions cover only a small part of the physical content of the scalar-tensor equations. The search for new exact solutions is therefore necessary if further progress is to be made in understanding the scalar-tensor theories.

The existence of an inflationary period of expansion of the Universe is necessary for solving the horizon (homogeneity), flatness, and relic-monopole problems, which have been discussed in detail by Guth (1981). A somewhat different approach to the construction of exact solutions in cosmological inflation models was presented by Barrow (1994). In his approach, the evolution of the scalar field is first specified, and then the evolution of the scale factor and the potential, which depends explicitly on the scalar field, is determined. The work of Maartens et al. (1995) used a method similar to fine tune the potentials, but with a special parameter introduced in place of time. The approach might well be practical for the solution of some problems, but it complicates the process of obtaining and analyzing solutions.

The study of cosmic strings in a relativistic framework was initiated by authors including Stachel (1980); Krori et al. (1990, 1994); Bali & Dave (2001); Bhattacharjee & Baruah (2001); Mahanta & Mukherjee (2001); Rahaman et al. (2003); Reddy (2003); Pant & Oli (2003); and Venkateswarlu et al. (2008, 2011), who have studied various aspects of string cosmologies in the theory of general relativity as well as in alternative theories of gravitation.

In this paper, we intend to study the non-static plane symmetric cosmological solutions in the context of cosmic strings in a new scalar-tensor theory of gravitation proposed by Sen & Dunn (1971). Section 2 contains a non-static plane symmetric metric and the field equations of this theory. In Section 3, the solutions of the field equations are obtained in the context of cosmic strings and some properties of the models are also discussed. Conclusions are given in the last section.

2 FIELD EQUATIONS AND THE METRIC

Modified theories of gravity have been the subject of study for the last few decades. As an alternative to Einstein's theory of gravitation, Sen & Dunn (1971) proposed a new scalar-tensor theory of gravitation in which both the scalar and tensor fields have intrinsic geometrical significance. The scalar field in this theory is characterized by the function $\phi = \phi(x^i)$, where x^i are coordinates in the four-dimensional Lyra manifold and the tensor field is identified with the metric tensor g_{ij} of the manifold. The field equations given by Sen & Dunn (1971) for the combined scalar and tensor fields are

$$R_{ij} - \frac{1}{2}g_{ij}R = \omega\phi^{-2}(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}) - \phi^{-2}T_{ij}, \tag{1}$$

where $\omega = \frac{3}{2}$; R_{ij} and R are respectively the usual Ricci-tensor and Riemaan-curvature scalar (in our units $C = 8\pi G = 1$). Jeavons et al. (1975) pointed out that the field equations proposed by Sen and Dunn are heuristically useful even though they are not derived from the usual variational principle. The energy momentum tensor for a cloud of strings is given by

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j. \tag{2}$$

Here ρ, the proper energy density, and λ, the string tension density, are related by $\rho = \rho_{\mathrm{p}} + \lambda$, where ρ_{p} is the particle density of the configuration. The velocity u^i describes the 4-velocity which has components (1, 0, 0, 0) for a cloud of particles and x^i represents the direction of the string which will satisfy

$$u^i u_i = -x^i x_i = 1 \ \text{ and } \ u^i x_j = 0. \tag{3}$$

We consider the non-static plane symmetric metric as

$$ds^2 = e^{2h}(dt^2 - dr^2 - r^2 d\theta^2 - S^2 dz^2), \tag{4}$$

where $h = h(t)$, $s = s(t)$ and $x^1 = r$, $x^2 = \theta$, $x^3 = z$, $x^4 = t$. We now consider x^i to be along the z-axis so that $x^i = (0, 0, 0, \frac{e^{-h}}{S})$. Plane symmetric models are important because they constitute plane wave solutions of the Universe. It is known that this line element is plane-symmetric. The plane symmetry assumed implies that the scalar field ϕ shares the same symmetry. The field Equation (1) with the help of Equations (2) and (3) can be written as

$$2\ddot{h} + \frac{\ddot{S}}{S} + \dot{h}^2 + 2\dot{h}\frac{\dot{S}}{S} = \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2, \tag{5}$$

$$2\ddot{h} + \dot{h}^2 = \frac{\lambda}{\phi^2}e^{2h} + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2, \tag{6}$$

$$3\dot{h}^2 + 2\dot{h}\frac{\dot{S}}{S} = \frac{\rho}{\phi^2}e^{2h} - \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2, \tag{7}$$

where the dot above the variables denotes partial derivative with respect to t. From all the three energy conditions (weak, strong and dominant) for the string model, one can find that $\rho > 0$ and $\rho_{\mathrm{p}} \geq 0$, and the sign of λ is unrestricted.

3 SOLUTION OF THE FIELD EQUATIONS

The field Equations (5)–(7) are a system of three equations with five unknown parameters: h, S, ϕ, ρ and λ. In order to obtain explicit solutions of the system, we must impose two additional conditions. We assume that

(i) $\rho = \beta\lambda$, where β is a proportionality constant which gives rise to the following three cases:
  (a) For a cloud of geometric strings or Nambu strings, we have $\beta = 1$.
  (b) For massive strings we have $\beta = -1$.
  (c) For $\beta = (1 + \xi)$, $\xi \geq 0$, we get p-string or Takabayasi strings.
(ii) power law and exponent law assumptions exist between metric potentials h and S. We find the solutions of the field Equations (5)–(7) in the following three cases.

3.1 Geometric Strings or Nambu Strings ($\rho = \lambda$) i.e. when $\beta = 1$

Now the field Equations (5)–(7) together with $\rho = \lambda$ reduce to

$$\ddot{\lambda} + 2\dot{h}^2 + 3\dot{h}\frac{\dot{S}}{S} + \frac{\ddot{S}}{S} = 0, \tag{8}$$

which indicates that S and h are connected by

$$Se^h = c_1, \tag{9}$$

where c_1 is a constant of integration. This assumes

$$e^h = t^{-m_0}, \tag{10}$$

where m_0 is a constant. The general solution of the field Equations (5)–(7) is obtained as

$$S = t^{m_0},$$
$$e^h = t^{-m_0}, \tag{11}$$
$$\phi = \phi_0 t^k,$$

where ϕ_0 is an integration constant, and $k = \sqrt{\frac{2m_0}{\omega}}$, $m_0 > 0$.

The non-static plane symmetric model for a cloud of geometric strings can be expressed as

$$ds^2 = t^{-2m_0}(dt^2 - dr^2 - r^2 d\theta^2 - t^{2m_0} dz^2). \tag{12}$$

The energy density and tension density of stings are given by

$$\rho = \lambda = \phi_0^2 m_0 (m_0 + 1) t^{2(k+m_0+1)}, \tag{13}$$

and physical and kinematical parameters are given by

$$\text{Scalar expansion } \theta = -3m_0 t^{m_0-1},$$
$$\text{Shear scalar } \sigma = \frac{\sqrt{5}m_0 t^{m_0-1}}{\sqrt{2}},$$
$$\text{Spatial volume } V = \sqrt{-g} = rt^{-2m_0}, \tag{14}$$
$$\text{Deceleration parameter } q = \frac{-a\ddot{a}}{\dot{a}^2} = -\left(\frac{2m_0+3}{2m_0}\right).$$

The energy density ρ and string tension λ satisfy all the energy conditions for $t > 0$. At the initial singularity, (i.e. $t \to 0$) the spatial volume V tends to infinity while other parameters θ and σ tend to zero. However, all these parameters remain finite and physically significant for all $m_0 > 1$ and $t > 0$. Hence we see that space-time admits a big-bang singularity but the rate of expansion of the Universe decreases with increasing time. Since $\frac{\sigma}{\theta} = $ constant, the anisotropy is maintained throughout.

We have $\phi = $ constant, and $\lambda = 0 = \rho$ when $m_0 = 0$. Thus geometric strings or Nambu strings do not exist for $m_0 = 0$. The values of the deceleration parameter separate decelerating $(q > 0)$ from accelerating $(q < 0)$ periods in the evolution of the Universe. Here for all m_0, the model accelerates at any stage. Determination of the deceleration parameter from the count magnitude relation for galaxies is a difficult task due to evolutionary effects. The present value q of the deceleration parameter obtained from observations (Schuecker et al. 1998) is in the range $-1.27 \leq q \leq 2$. Studies of galaxy counts from redshift surveys provide a value of $q = 0.1$, with an upper limit of $q \leq 0.75$ (Schuecker et al. 1998). Recent observations by Perlmutter et al. (1998, 1999) and Riess et al. (1998) show that the deceleration parameter of the Universe is in the range $-1 \leq q \leq 0$ and the present day Universe is undergoing accelerated expansion. It may be noted that though the current observations of SNe Ia and the CMBR favor accelerating models $(q < 0)$, they do not altogether rule out the existence of the decelerating phase in the early history of our Universe, which is also consistent with these observations (Vishwakarma 2003).

3.2 Massive Strings ($\rho + \lambda = 0$) i.e. when $\beta = -1$

In this case, the field Equations (5)–(7) together with $\rho + \lambda = 0$ yield

$$\ddot{\lambda} + 2\dot{h}^2 + 2\dot{h}\frac{\dot{S}}{S} = 0, \tag{15}$$

which admits a solution

$$S\dot{h}\,\mathrm{e}^{2h} = c_2 \tag{16}$$

where c_2 is a constant of integration. Now the solution of the field Equations (5)–(7) is given by

$$h = \mathrm{e}^{m_1 t},$$
$$S = \frac{c_2 \mathrm{e}^{-2h}}{\dot{h}} \tag{17}$$

where m_1 is a constant. The scalar field is given by

$$\phi = \phi_1 \exp\left(\sqrt{\frac{2}{\omega}}(m_1 t + e^{m_1 t})\right), \tag{18}$$

where ϕ_1 is an integration constant. The non-static plane symmetric model for a cloud of massive strings can be expressed as

$$ds^2 = \exp(\exp(2m_1 t))\left[dt^2 - dr^2 - r^2 d\theta^2 - \frac{(\exp(\exp(-4m_1 t))}{m_1^2 \exp(2m_1 t)}dz^2\right]. \tag{19}$$

The energy and tension densities of strings are given by

$$\rho = -\lambda = m_1^2 \phi_1^2 \exp 2\left[\sqrt{\frac{2}{\omega}}(m_1 t + e^{m_1 t})\right]. \tag{20}$$

The particle density of the model is

$$\rho_{\mathrm{p}} = 2m_1^2 \phi_1^2 \exp 2\left[\sqrt{\frac{2}{\omega}}(m_1 t + e^{m_1 t})\right]. \tag{21}$$

The dominant energy conditions imply that $\rho > 0$ and $\rho^2 \geq \lambda^2$. These energy conditions do not restrict the sign of λ, and accordingly the expressions given by Equation (21) satisfy all these conditions. Here $\frac{\rho_{\mathrm{p}}}{|\lambda|} = 2$. Since $\frac{\rho_{\mathrm{p}}}{|\lambda|} > 1$, we may conclude that the particles dominate over the strings in this model. Physical and kinematical parameters are given by

$$\begin{aligned}
\theta &= \frac{m_1(2\exp(m_1 t) - 1)}{\exp(\exp(m_1 t))}, \\
\sigma &= \left[\frac{m_1}{3\sqrt{2}\exp(\exp(m_1 t))}\sqrt{171\exp(2m_1 t) + 72\exp(m_1 t) + 10}\right], \\
V &= \sqrt{-g} = \frac{r\exp(\exp(m_1 t))}{m_1 \exp(m_1 t)}, \\
q &= -\frac{3m_1^4}{\exp(\exp(4m_1 t))}\left[1 + m_1^2 \exp(2m_1 t)\frac{(3 - 2\exp(m_1 t))^2}{(2\exp(m_1 t) - 1)^2}\right].
\end{aligned} \tag{22}$$

The non static metric (19) shows that the proper volume never vanishes for any value of t. Thus the space-time is non singular with respect to time. We observe that the spatial volume increases with time t for $r > 0$. Thus inflation is possible in plane symmetric space-time with a massless scalar field. Also, the model inflates because of the fact that the deceleration parameter is negative. Since $\lim_{t \to \infty}\left(\frac{\sigma}{\theta}\right) \neq 0$, the model does not approach isotropy for large values of t.

3.3 P-string or Takabayasi Strings $\rho = (1 + \xi)\lambda$ i.e. when $\beta = (1 + \xi)$

Here the equation of state $\rho = (1 + \xi)\lambda$, where $\xi(> 0)$ is a constant and it is small for the string dominant era but large for the particle dominant era. Now the field Equations (5)–(7) together with $\rho = (1 + \xi)\lambda$ reduce to

$$2\ddot{\lambda} + 4\dot{h}^2 + (6 + 2\xi)\dot{h}\frac{\dot{S}}{S} + (2 + \xi)\frac{\ddot{S}}{S} = 0, \tag{23}$$

which on integration yields

$$\begin{aligned}
e^h &= k_1(at + b)^{\frac{1}{k_1}}, \\
S &= k_1(at + b)^{\frac{-m}{k_1}},
\end{aligned} \tag{24}$$

and

$$\phi = \phi_2(at + b)^{\sqrt{\frac{2}{\omega}}\frac{k_2}{k_1}}, \tag{25}$$

where

$$k_1 = \frac{(2 + \xi)m^2 - (6 + 2\xi)m + 4}{(2 - 2m - \xi m)},$$
$$k_2 = \sqrt{m(3\xi + 10) - 2m^2(\xi + 2) - 6}$$

and ϕ_2, a, b are constants of integration. By making use of Equations (23) and (24) together with Equations (6) and (7), the string energy density ρ and tension density λ vanish identically. This shows that the p-strings or Takabayasi strings do not co-exist in the Sen-Dunn Theory of gravitation.

4 CONCLUSIONS

In this paper we have obtained exact solutions of non-static plane symmetric field equations in the Sen-Dunn theory. The natures of string tension density λ, string energy density ρ and particle density ρ_{p} have been examined for three cases; geometric strings ($\rho = \lambda$) and massive strings ($\rho + \lambda = 0$) do exist in this theory. Further it is found that the Takabayasi string, i.e. $\rho = (1+\xi)\lambda$, does not exist. For some particular cases, the models of a Universe evolve from a pure geometric string-dominated era to a massive string-dominated era, and then continue to a particle-dominated area with a remnant of strings. In the recent past, there has been an upsurge of interest in scalar fields in the context of inflationary cosmology. Therefore the study of cosmological models in the Sen-Dunn theory may be relevant for inflationary models. The classical scalar fields are essential in the study of the present day cosmological models. In view of the fact that there is an increasing intersection of these models, in recent years, scalar fields in general relativity and alternative theories of gravitation in the context of an inflationary Universe help us to describe the early stages of evolution of the Universe. We hope that our model obtained here will be useful for a better understanding of inflationary cosmology in the non-static plane symmetric model.

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