Research in Astron. Astrophys. **2012** Vol. **12** No. **5**, 473–484
http://www.raa-journal.org *http://www.iop.org/journals/raa*

Research in Astronomy and Astrophysics

An interacting two-fluid scenario for dark energy in a Bianchi type-I cosmological model

Triloki Singh[1] and Raghavendra Chaubey[2]

[1] Department of Applied Mathematics, Institute of Technology, Banaras Hindu University, Varanasi, India; *drtrilokisingh@yahoo.co.in*

[2] DST-Centre for Interdisciplinary Mathematical Sciences, Faculty of Science, Banaras Hindu University, Varanasi, India; *rchaubey@bhu.ac.in*

Received 2011 July 23; accepted 2011 November 28

Abstract We study the evolution of the dark energy parameter within a Bianchi type-I cosmological model filled with barotropic fluid and dark energy. The solutions have been obtained for power law and exponential forms of the expansion parameter (they correspond to a constant deceleration parameter in general relativity). After a long time, the models tend to be isotropic under certain conditions.

Key words: cosmological model — dark energy parameter — barotropic fluid

1 INTRODUCTION

The nature of the dark energy component of the Universe (Riess et al. 1998; Perlmutter et al. 1997; Sahni 2004; Copeland et al. 2006; Amendola & Tsujikawa 2010) remains one of the deepest mysteries of cosmology. There is certainly no lack of candidates: cosmological constant, quintessence (Ratra & Peebles 1988; Caldwell et al. 1998; Barreiro et al. 2000), k-essence (Armendáriz-Picón et al. 1999; Armendariz-Picon et al. 2001; González-Díaz 2004), and phantom energy (Caldwell 2002; Carroll et al. 2003; Elizalde et al. 2004). Modifications of the Friedmann equation such as Cardassian expansion (Freese & Lewis 2002; Freese 2003; Gondolo & Freese 2003) and brane cosmology (Deffayet et al. 2002; Dvali et al. 2000; Dvali & Turner 2003) have also been used to explain the acceleration of the Universe. A particular case of the linear equation of state (EoS) has been used in a cosmological context by Xanthopoulos (1987). He considered space-times with two hypersurface orthogonal, spacelike, commuting Killing fields.

Observations of distant supernovae (SNe Ia) (Perlmutter et al. 1997, 1998, 1999; Riess et al. 1998; Garnavich et al. 1998; Schmidt et al. 1998; Tonry et al. 2003; Clocchiatti et al. 2006), fluctuation of Cosmic Microwave Background Radiation (CMBR) (de Bernardis et al. 2000; Hanany et al. 2000), the Large Scale Structure (LSS) (Spergel et al. 2003; Tegmark et al. 2004), Sloan Digital Sky Survey (SDSS) (Seljak et al. 2005; Adelman-McCarthy et al. 2006), Wilkinson Microwave Anisotropy Probe (WMAP) (Bennett et al. 2003) and *Chandra* X-ray observatory (Allen et al. 2004) by means of ground and altitudinal experiments have shown that our Universe is expanding with acceleration. The measurement of photometric distances to cosmological supernovae, supported by a number of independent observations, in particular by observational data on the angular temperature fluctuations of CMBR, shows that the lion's share of the energy density of matter belongs to non-baryonic matter. This form of matter cannot be detected in the laboratory and does not interact with electromagnetic radiation. Given the fact that almost three-quarters of the energy density of the Universe originated from dark energy and plays a crucial role in the accelerated mode of the expansion of the Universe, there appears to be a large number of models capable of describing this dark energy (Spergel et al. 2003).

The spatially homogeneous cosmological models allow extension of cosmological investigations to distorting and rotating Universes, giving estimates of effects of anisotropy on primordial element production and on the measured CMBR spectrum anisotropy (Ellis & van Elst 1999). Apart from observational reasons, there are various theoretical considerations that have motivated the study of anisotropic cosmologies. Among these are

(i) some kind of singularity in our "past" is strongly indicated if certain reasonable conditions hold (Hawking & Ellis 1973). However, it could differ greatly from the type found in FRW models (Belinskij et al. 1970).
(ii) The "Chaotic Cosmology" program of Misner (1968) sought a mechanism to explain why the observed isotropy and homogeneity should exist regardless of the initial conditions (MacCallum 1979; Ellis 1993; Kolb & Turner 1990).

There exists a wide class of anisotropic cosmological models, which are often studied in cosmology (Misner et al. 1973). There are theoretical arguments that sustain the existence of an anisotropic phase that approaches an isotropic case (Misner 1968) (Chaotic Cosmology). Also, anisotropic cosmological models have found a suitable candidate to avoid the assumption of specific initial conditions in FRW models. The early Universe could also be characterized by an irregular expansion mechanism. Therefore, it would be useful to explore cosmological models in which anisotropies, existing at an early stage of expansion, are damped out in the course of evolution. Such models have received some attention (Hu & Parker 1978).

In most of the models, the dark energy and dark matter components are considered to be non-interacting and are allowed to evolve independently. However, as the nature of these components is not completely known, the interaction between them will indeed provide a more general framework in which to work.

Zimdahl and Pavón (Zimdahl & Pavón 2004; Zimdahl et al. 2005) have shown that the interaction between dark energy and dark matter can be very useful in solving the coincidence problem (Pavón et al. 2004; Tsujikawa & Sami 2004; Gumjudpai et al. 2005).

Barrow & Clifton (2006) have investigated a wide range of homogeneous and isotropic cosmological models containing two fluids which are able to exchange energies and show non-equilibrium behavior.

The fact that the energy density of dark energy is of the same order as that of dark matter in the present Universe suggests that there may be some relation between them (Amendola & Tsujikawa 2010). Several different forms of coupling between dark energy and dark matter have been suggested (Wei 2010). One of the approaches is to introduce an interaction of the form $\Gamma\rho_m$ on the right hand side of the continuity equations (ρ_m is the dark matter energy density) with the normalization of Γ in terms of the Hubble parameter H, i.e. $\Gamma/H = Q$, where Q is a dimensionless coupling (Zimdahl et al. 2001; Guo et al. 2007; Caldera-Cabral et al. 2009).

Hassan et al. (2011) have studied the evolution of the dark energy parameter within the framework of an FRW cosmological model filled with two fluids.

In this paper, we study the evolution of the dark parameter within a Bianchi type-I cosmological model filled with barotropic fluid and dark energy. The solutions have been studied for power law and exponential forms of the expansion factor. The average volume expansion factor $V(t)$ has been used in the power-law and exponential forms based on these forms for FRW expanding models (when the deceleration parameter is constant). The behavior of the EoS has been analyzed.

2 THE METRIC AND FIELD EQUATIONS

We consider the homogeneous and anisotropic cosmological model, for the Bianchi type-I metric in the form (Ryan & Shepley 1975)

$$ds^2 = dt^2 - {a_1}^2 dx^2 - {a_2}^2 dy^2 - {a_3}^2 dz^2 \,, \tag{1}$$

where the metric functions a_1, a_2, a_3 are only a function of time (t).

Einstein's field equations (with $8\pi G = 1$ and $c = 1$) read as

$$R_\mu{}^\nu - \frac{1}{2} R \delta_\mu{}^\nu = -T_\mu{}^\nu \,, \tag{2}$$

where the symbols have their usual meaning and $T_\mu{}^\nu$ is the two fluid energy-momentum tensor consisting of a dark field and barotropic fluid.

In a co-moving coordinate system, Einstein's field Equations (2) for the line element (1) lead to

$$\frac{\dot a_1 \dot a_2}{a_1 a_2} + \frac{\dot a_2 \dot a_3}{a_2 a_3} + \frac{\dot a_3 \dot a_1}{a_3 a_1} = \rho_{\rm tot} \,, \tag{3}$$

$$\frac{\ddot a_2}{a_2} + \frac{\ddot a_3}{a_3} + \frac{\dot a_2 \dot a_3}{a_2 a_3} = -p_{\rm tot} \,, \tag{4}$$

$$\frac{\ddot a_1}{a_1} + \frac{\ddot a_3}{a_3} + \frac{\dot a_1 \dot a_3}{a_1 a_3} = -p_{\rm tot} \,, \tag{5}$$

$$\frac{\ddot a_1}{a_1} + \frac{\ddot a_2}{a_2} + \frac{\dot a_1 \dot a_2}{a_1 a_2} = -p_{\rm tot} \,, \tag{6}$$

where $p_{\rm tot} = p_m + p_{\rm D}$ and $\rho_{\rm tot} = \rho_m + \rho_{\rm D}$. Here p_m and ρ_m are pressure and energy density of the barotropic fluid and $p_{\rm D}$ and $\rho_{\rm D}$ are the pressure and energy density of dark fluid respectively.

Let the volume scale parameter V be a function of t defined by

$$V = a_1 a_2 a_3 \,. \tag{7}$$

We have followed the method of Singh and Chaubey (Singh & Chaubey 2009, 2007; Chaubey 2009). Briefly we derive the solution

$$a_1(t) = D_1 V^{1/3} \exp\left(X_1 \int \frac{dt}{V(t)} \right) \,, \tag{8}$$

$$a_2(t) = D_2 V^{1/3} \exp\left(X_2 \int \frac{dt}{V(t)} \right) \,, \tag{9}$$

$$a_3(t) = D_3 V^{1/3} \exp\left(X_3 \int \frac{dt}{V(t)} \right) \,, \tag{10}$$

where D_i $(i = 1, 2, 3)$ and X_i $(i = 1, 2, 3)$ satisfy the relations $D_1 D_2 D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

The Bianchi identity $G_{\mu\nu}{}^{;\nu} = 0$ leads to $T_{\mu\nu}{}^{;\nu} = 0$ which yields

$$\dot\rho_{\rm tot} + \frac{\dot V}{V}(\rho_{\rm tot} + p_{\rm tot}) = 0 \,. \tag{11}$$

The EoS of the barotropic fluid and dark field are given by

$$\omega_m = \frac{p_m}{\rho_m} \tag{12}$$

and

$$\omega_{\rm D} = \frac{p_{\rm D}}{\rho_{\rm D}} \,, \tag{13}$$

respectively.

In the following sections we deal with two cases, (i) the non-interacting two-fluid model and (ii) the interacting two-fluid model.

3 NON-INTERACTING TWO-FLUID MODEL

First, we consider the case that two-fluids do not interact with each other. Therefore, the general form of conservation Equation (11) leads us to separately write the conservation equation for the dark and barotropic fluid as

$$\dot{\rho}_m + \frac{\dot{V}}{V}(\rho_m + p_m) = 0 \tag{14}$$

and

$$\dot{\rho}_{\rm D} + \frac{\dot{V}}{V}(\rho_{\rm D} + p_{\rm D}) = 0\,. \tag{15}$$

From Equations (12) and (14), we have

$$\rho_m = \rho_0 V^{-(1+\omega_m)}\,. \tag{16}$$

Now we take two cases for the volume expansion factor, where increases in the terms representing time evolution are (i) the exponential type expansion in which $V = \alpha\, {\rm e}^{\beta t}$; where α and β are constants and (ii) the power law in which $V = at^b$; where a and b are constants.

3.1 Models with a Constant Deceleration Parameter

Case 1: Exponential-Type (When $V = \alpha\, {\rm e}^{\beta t}$)

Then Equations (8) – (10) reduce to

$$a_1(t) = D_1\alpha^{1/3}\exp\left(\frac{\beta t}{3} + \frac{X_1}{\alpha\beta}\,{\rm e}^{-\beta t}\right)\,, \tag{17}$$

$$a_2(t) = D_2\alpha^{1/3}\exp\left(\frac{\beta t}{3} + \frac{X_2}{\alpha\beta}\,{\rm e}^{-\beta t}\right)\,, \tag{18}$$

$$a_3(t) = D_3\alpha^{1/3}\exp\left(\frac{\beta t}{3} + \frac{X_3}{\alpha\beta}\,{\rm e}^{-\beta t}\right)\,. \tag{19}$$

By using Equation (13) in Equations (3) – (6), we first obtain the $\rho_{\rm D}$ and $p_{\rm D}$ in terms of scale factors.

$$\rho_{\rm D} = \frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1} - \rho_0(a_1a_2a_3)^{-(1+\omega_m)}\,, \tag{20}$$

and

$$p_{\rm D} = -\frac{2}{3}\left(\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3}\right) - \frac{1}{3}\left(\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1}\right) - \rho_0\omega_m(a_1a_2a_3)^{-(1+\omega_m)}\,. \tag{21}$$

Now from Equations (17)–(21), the $\rho_{\rm D}$ and $p_{\rm D}$ are obtained as

$$\rho_{\rm D} = \frac{\beta^2}{3} + \left(\frac{X_1X_2 + X_2X_3 + X_3X_1}{\alpha^2}\right){\rm e}^{-2\beta t} - \rho_0(\alpha\,{\rm e}^{\beta t})^{-(1+\omega_m)} \tag{22}$$

and

$$p_{\rm D} = -\left[\frac{\beta^2}{3} - \left(\frac{X_1X_2 + X_2X_3 + X_3X_1}{\alpha^2}\right){\rm e}^{-2\beta t} + \rho_0\omega_m(\alpha\,{\rm e}^{\beta t})^{-(1+\omega_m)}\right]\,, \tag{23}$$

respectively. By using Equations (22) and (23) in Equation (13), we find the EoS of the dark field in terms of time as

$$\omega_{\rm D} = -\left[\frac{\frac{\beta^2}{3} - \left(\frac{X_1X_2+X_2X_3+X_3X_1}{\alpha^2}\right){\rm e}^{-2\beta t} + \rho_0\omega_m(\alpha\,{\rm e}^{\beta t})^{-(1+\omega_m)}}{\frac{\beta^2}{3} + \left(\frac{X_1X_2+X_2X_3+X_3X_1}{\alpha^2}\right){\rm e}^{-2\beta t} - \rho_0(\alpha\,{\rm e}^{\beta t})^{-(1+\omega_m)}}\right]\,. \tag{24}$$

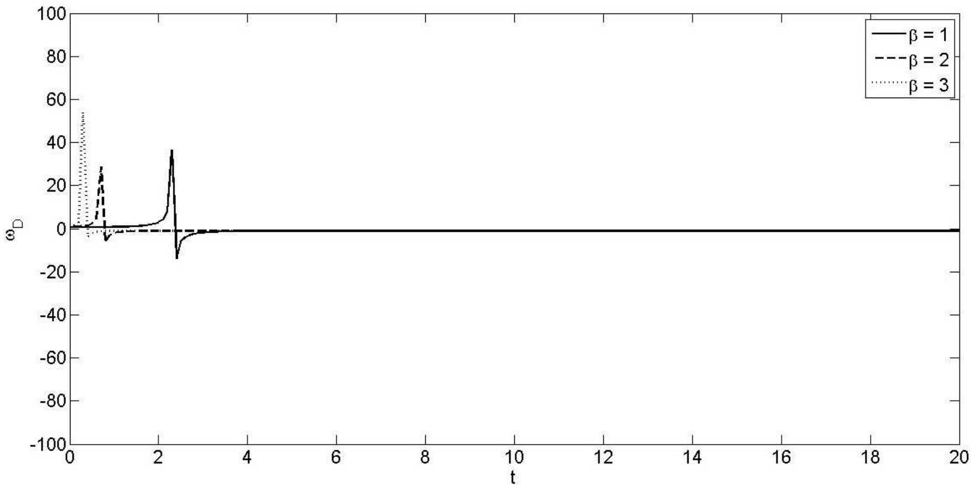

Fig. 1 Plot of EoS parameter '$\omega_{\rm D}$' with cosmic time 't' for β = 1, 2 and 3.

The behavior of the EoS for DE in terms of cosmic time t for an exponential expansion of the Universe is shown in Figure 1 for $X_1 = X_2 = 1, X_3 = -2, \alpha = 1$ and $\beta = 1, 2, 3$. It is observed that the EoS parameter is an increasing function of time, and the rapidity of its growth at the early stage depends on different values of β of the Universe. Later on it tends to the same constant value for different values of β.

The physical quantities of observational interest in cosmology are the expansion scalar θ, the mean anisotropy parameter A, the shear scalar σ^2 and the deceleration parameter q. They are defined as

$$\theta = 3H \,, \tag{25}$$

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 , \tag{26}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} {H_i}^2 - 3H^2\right) = \frac{3}{2}AH^2 \,, \tag{27}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \,. \tag{28}$$

Sahni et al. (2003) proposed a cosmological diagnostic pair $\{r, s\}$ called the statefinder parameter, which is defined for the isotropic cosmological model as

$$r = \frac{\dddot{a}}{aH^3} \qquad \text{and} \qquad s = \frac{r-1}{3(q-\frac{1}{2})} \,, \tag{29}$$

where $\dddot{a}$ means the triple dot of a, H is the Hubble parameter and q is the deceleration parameter.

Using Equations (7), (25) and (29), we have generalized the cosmological diagnostic pair $\{r, s\}$ for the anisotropic cosmological model as (Singh & Chaubey 2011)

$$r = \frac{9\left[\left(\frac{\dddot{V}}{V}\right) - 2\left(\frac{\dot{V}}{V}\right)\left(\frac{\ddot{V}}{V}\right) + 10\left(\frac{\dot{V}}{V}\right)^3\right]}{\left(\frac{\dot{V}}{V}\right)^3} \qquad \text{and} \qquad s = \frac{r-1}{3(q-\frac{1}{2})} \,, \tag{30}$$

where $\dddot{V}$ means the triple dot of V.

With the use of Equations (17) – (19) we can express the physical quantities as

$$\theta = \beta \,, \tag{31}$$

$$A = \frac{6X^2}{\alpha^2\beta^2}\, \mathrm{e}^{-2\beta t} \,, \tag{32}$$

$$\sigma^2 = \frac{X^2}{\alpha^2}\, \mathrm{e}^{-2\beta t} \,, \tag{33}$$

$$q = -1 \,, \tag{34}$$

where X is a constant. For large t, the model tends to be isotropic when $\beta > 0$. When $\beta = 0$ the anisotropy and shear both become constant.

By using Equation (30), the cosmological diagnostic pair $\{r, s\}$ is

$$r = 81 \,, \tag{35}$$

and

$$s = -\frac{160}{9} \,. \tag{36}$$

Case 2: Power-Law (When $V = at^b$)

Then Equations (8) – (10) are reduced to

$$a_1(t) = D_1 a^{1/3} t^{b/3} \exp\left(\frac{X_1}{a(1-b)} t^{1-b}\right) \,, \tag{37}$$

$$a_2(t) = D_2 a^{1/3} t^{b/3} \exp\left(\frac{X_2}{a(1-b)} t^{1-b}\right) \,, \tag{38}$$

$$a_3(t) = D_3 a^{1/3} t^{b/3} \exp\left(\frac{X_3}{a(1-b)} t^{1-b}\right) \,. \tag{39}$$

From Equations (37)–(39), (20) and (21), ρ_{D} and p_{D} are obtained as

$$\rho_{\mathrm{D}} = \frac{b^2}{3t^2} + \left(\frac{{X_1}^2 + {X_2}^2 + {X_3}^2}{a^2}\right) t^{-2b} - \rho_0 (at^b)^{-(1+\omega_m)} \tag{40}$$

and

$$p_{\mathrm{D}} = -\left[\frac{b(b-1)}{3t^2} - \left(\frac{{X_1}^2 + {X_2}^2 + {X_3}^2}{a^2}\right) t^{-2b} + \rho_0 \omega_m (at^b)^{-(1+\omega_m)}\right] \,, \tag{41}$$

respectively. By using Equations (40) and (41) in Equation (13), we find the EoS of the dark field in terms of time as

$$\omega_{\mathrm{D}} = -\left[\frac{\frac{b(b-1)}{3t^2} - \left(\frac{{X_1}^2+{X_2}^2+{X_3}^2}{a^2}\right) t^{-2b} + \rho_0\omega_m (at^b)^{-(1+\omega_m)}}{\frac{b^2}{3t^2} + \left(\frac{{X_1}^2+{X_2}^2+{X_3}^2}{a^2}\right) t^{-2b} - \rho_0 (at^b)^{-(1+\omega_m)}}\right] \,. \tag{42}$$

The behavior of the EoS for DE in terms of cosmic time t for power-law expansion of the Universe is shown in Figure 2 for $X_1 = X_2 = 1, X_3 = -2, a = 1$ and $b = 0, 1, 2, 3$. It is observed that the EoS parameter is a function of time. At the early stage, EoS has the same constant value which is independent of the values of b. Later on it increases for $b = 2$ and it attains the same constant value for different values of $b = 0, 1, 3$.

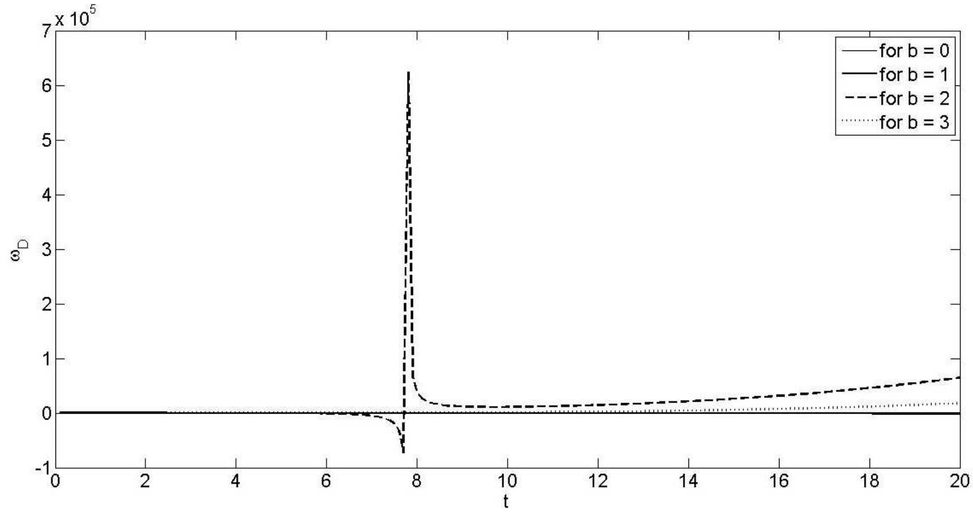

Fig. 2 Plot of EoS parameter '$\omega_{\rm D}$' with cosmic time 't' for $b = 0$, 1, 2 and 3.

With the use of Equations (37) – (39) we can express the physical quantities as

$$\theta = \frac{b}{t}\,, \tag{43}$$

$$A = \frac{6X^2}{a^2b^2}\frac{1}{t^{2(b-1)}}\,, \tag{44}$$

$$\sigma^2 = \frac{X^2}{a^2}\frac{1}{t^{2b}}\,, \tag{45}$$

$$q = \frac{3}{b} - 1, \tag{46}$$

where X is a constant. For large t, the model tends to be isotropic when $b > 1$. When $b = 1$, the anisotropy is constant and shear dies out. The expansion becomes zero.

By using Equation (30), the cosmological diagnostic pair $\{r, s\}$ is

$$r = \frac{9}{b^2}(9b^2 - b + 2)\,, \tag{47}$$

and

$$s = \frac{2(80b^2 - 9b + 18)}{9b(2-b)}\,. \tag{48}$$

4 INTERACTING TWO FLUID MODEL

Second, we consider the interaction between dark and barotropic fluids. For this purpose we can write the continuity equations for dark and barotropic fluids (Amendola & Tsujikawa 2010) as

$$\dot{\rho}_m + \frac{\dot{V}}{V}(\rho_m + p_m) = Q\,, \tag{49}$$

and

$$\dot{\rho}_{\rm D} + \frac{\dot{V}}{V}(\rho_{\rm D} + p_{\rm D}) = -Q\,. \tag{50}$$

The quantity Q expresses the interaction between the matter and dark energy components. Since we are interested in an energy transfer from the dark energy to matter, we consider $Q > 0$. $Q > 0$ ensures that the second law of thermodynamics stands fulfilled (Pavón & Wang 2009). Following Amendola et al. (2007), Zimdahl et al. (2001), Guo et al. (2007) and Caldera-Cabral et al. (2009), we consider

$$Q = 3H\sigma\rho_m \,, \tag{51}$$

where $H = \frac{1}{3}\frac{\dot{V}}{V}$ and σ is a coupling constant. Using Equation (51) in Equation (49) and after integrating the resulting equation, we obtain

$$\rho_m = \rho_0 V^{-(1+\omega_m-\sigma)} \,. \tag{52}$$

Using Equation (52) in Equations (3) – (6), we again obtain ρ_{D} and p_{D} in terms of scale factors.

$$\rho_{\mathrm{D}} = \frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1} - \rho_0(a_1a_2a_3)^{-(1+\omega_m-\sigma)} \,, \tag{53}$$

and

$$p_{\mathrm{D}} = -\frac{2}{3}\left(\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3}\right) - \frac{1}{3}\left(\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1}\right) - \rho_0\omega_m(a_1a_2a_3)^{-(1+\omega_m-\sigma)} \,. \tag{54}$$

Now we take two cases for the volume expansion factor, where there is an increase in terms of time evolution. (i) Exponential type expansion in which $V = \alpha\,\mathrm{e}^{\beta t}$ and (ii) Power law type in which $V = at^b$.

4.1 Models with a Constant Deceleration Parameter

Case 1: Exponential-Type (When $V = \alpha\,\mathrm{e}^{\beta t}$)

From Equations (17) – (19) and Equations (53) and (54), the ρ_{D} and p_{D} are obtained as

$$\rho_{\mathrm{D}} = \frac{\beta^2}{3} + \left(\frac{X_1X_2 + X_2X_3 + X_3X_1}{\alpha^2}\right)\mathrm{e}^{-2\beta t} - \rho_0(\alpha\,\mathrm{e}^{\beta t})^{-(1+\omega_m-\sigma)} \,, \tag{55}$$

and

$$p_{\mathrm{D}} = -\left[\frac{\beta^2}{3} - \left(\frac{X_1X_2 + X_2X_3 + X_3X_1}{\alpha^2}\right)\mathrm{e}^{-2\beta t} + \rho_0\omega_m(\alpha\,\mathrm{e}^{\beta t})^{-(1+\omega_m-\sigma)}\right] \,, \tag{56}$$

respectively. By using Equations (55) and (56) in Equation (13), we find the EoS of the dark field in terms of time as

$$\omega_{\mathrm{D}} = -\left[\frac{\frac{\beta^2}{3} - \left(\frac{X_1X_2+X_2X_3+X_3X_1}{\alpha^2}\right)\mathrm{e}^{-2\beta t} + \rho_0\omega_m(\alpha\,\mathrm{e}^{\beta t})^{-(1+\omega_m-\sigma)}}{\frac{\beta^2}{3} + \left(\frac{X_1X_2+X_2X_3+X_3X_1}{\alpha^2}\right)\mathrm{e}^{-2\beta t} - \rho_0(\alpha\,\mathrm{e}^{\beta t})^{-(1+\omega_m-\sigma)}}\right] \,. \tag{57}$$

The behavior of the EoS for DE in term of cosmic time t for the exponential expansion of the Universe is shown in Figure 3 for $X_1 = X_2 = 1, X_3 = -2, \alpha = 1$ and $\beta = 1, 2, 3$. It is observed that the EoS parameter is an increasing function of time, and the rapidity of its growth at the early stage depends on different values of β of the Universe. Later on it tends to the same constant value for different values of β.

With the use of Equations (25) – (28) we can express the physical quantities as

$$\theta = \beta \,, \tag{58}$$

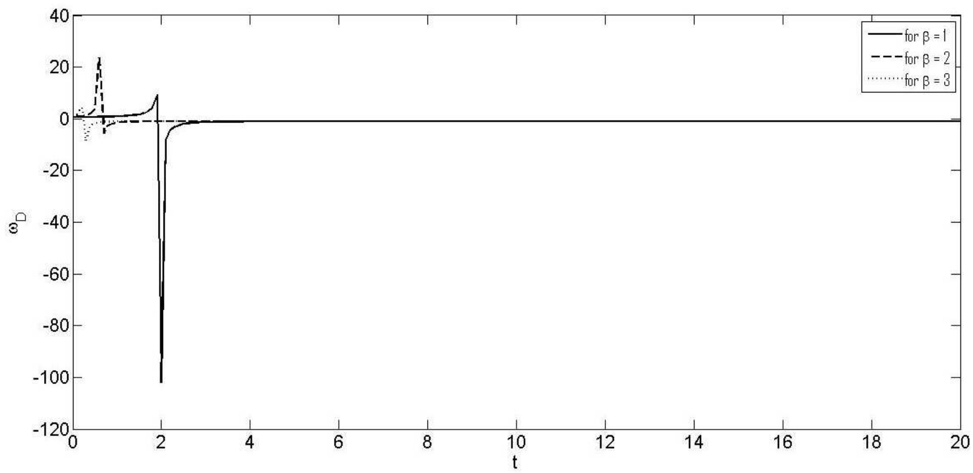

Fig. 3 Plot of EoS parameter '$\omega_{\rm D}$' for the two-fluid interacting case with cosmic time 't' for $\beta = 1$, 2 and 3.

$$A = \frac{6X^2}{\alpha^2\beta^2}{\rm e}^{-2\beta t}\,, \tag{59}$$

$$\sigma^2 = \frac{X^2}{\alpha^2}{\rm e}^{-2\beta t}\,, \tag{60}$$

$$q = -1\,, \tag{61}$$

where X is a constant. For large t, the model tends to be isotropic when $\beta > 0$. When $\beta = 0$ the anisotropy and shear both become constant; the expansion is also zero.

By using Equation (30), the cosmological diagnostic pair $\{r, s\}$ is

$$r = 81\,, \tag{62}$$

and

$$s = -\frac{160}{9}\,. \tag{63}$$

Case 2: Power-Law (When $V = at^b$)

From Equation (17) – (19) and Equation (53) and (54), the $\rho_{\rm D}$ and $p_{\rm D}$ are obtained as

$$\rho_{\rm D} = \frac{b^2}{3t^2} + \left(\frac{{X_1}^2 + {X_2}^2 + {X_3}^2}{a^2}\right) t^{-2b} - \rho_0(at^b)^{-(1+\omega_m-\sigma)}\,, \tag{64}$$

and

$$p_{\rm D} = -\left[\frac{b(b-1)}{3t^2} - \left(\frac{{X_1}^2 + {X_2}^2 + {X_3}^2}{a^2}\right) t^{-2b} + \rho_0\omega_m(at^b)^{-(1+\omega_m-\sigma)}\right]\,, \tag{65}$$

respectively. By using Equation (64) and (65) in Equation (13), we find the EoS of the dark field in terms of time as

$$\omega_{\rm D} = -\left[\frac{\frac{b(b-1)}{3t^2} - \left(\frac{{X_1}^2+{X_2}^2+{X_3}^2}{a^2}\right) t^{-2b} + \rho_0\omega_m(at^b)^{-(1+\omega_m-\sigma)}}{\frac{b^2}{3t^2} + \left(\frac{{X_1}^2+{X_2}^2+{X_3}^2}{a^2}\right) t^{-2b} - \rho_0(at^b)^{-(1+\omega_m-\sigma)}}\right]\,. \tag{66}$$

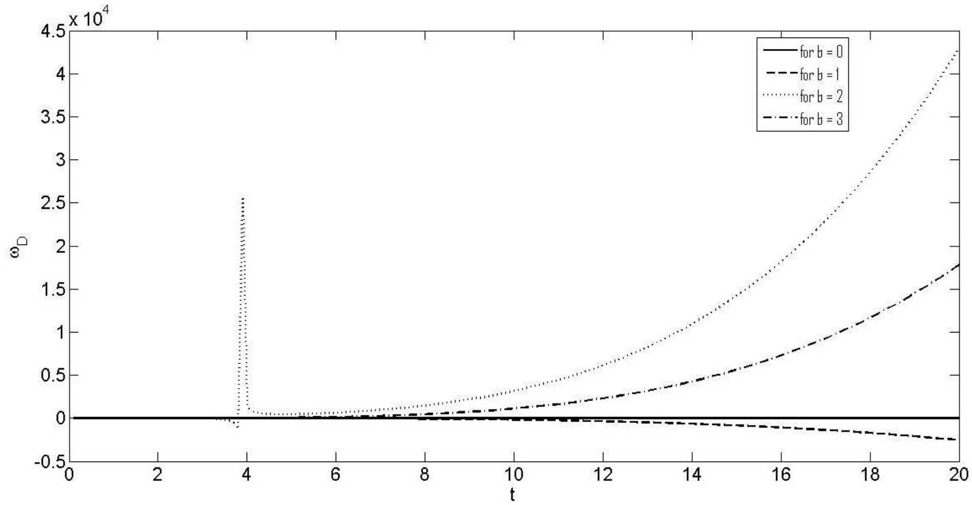

Fig. 4 Plot of EoS parameter '$\omega_{\rm D}$' for the two-fluid interacting case with cosmic time 't' for $b = 0$, 1, 2 and 3.

The behavior of EoS for DE in terms of cosmic time t for power-law expansion of the Universe is shown in Figure 4 for $X_1 = X_2 = 1, X_3 = -2, a = 1$ and $b = 0, 1, 2, 3$. It is observed that the EoS parameter is a function of time. At the early stage, EoS has the same constant value which is independent of the values of b. Later on it increases or decreases for different values of b.

With the use of Equations (25) – (28), we can express the physical quantities as

$$\theta = \frac{b}{t}\,, \tag{67}$$

$$A = \frac{6X^2}{a^2b^2}\frac{1}{t^{2(b-1)}}\,, \tag{68}$$

$$\sigma^2 = \frac{X^2}{a^2}\frac{1}{t^{2b}}\,, \tag{69}$$

$$q = \frac{3}{b} - 1\,, \tag{70}$$

where X is a constant. For large t, the model tends to be isotropic when $b > 1$. When $b = 1$, the anisotropy becomes constant and shear dies out. The expansion also becomes zero.

By using Equation (30), the cosmological diagnostic pair $\{r, s\}$ is

$$r = \frac{9}{b^2}(9b^2 - b + 2)\,, \tag{71}$$

and

$$s = \frac{2(80b^2 - 9b + 18)}{9b(2 - b)}\,. \tag{72}$$

5 CONCLUSIONS

We have studied the evolution of the dark energy parameter within an anisotropic cosmological Bianchi type-I model filled with barotropic fluid and dark energy. The solutions have been obtained for power law and exponential forms of the expansion factor. The behavior of these models has been analyzed, as well as the behavior of the EoS. With large t, the anisotropy and expansion become constant and the shear dies out. The models tend to be isotropic, and they give physically viable results under certain conditions.

Acknowledgements The authors wish to thank the referee for his valuable comments and suggestions.

References

Adelman-McCarthy, J. K., Agüeros, M. A., Allam, S. S., et al. 2006, ApJS, 162, 38
Allen, S. W., Schmidt, R. W., Ebeling, H., Fabian, A. C., & van Speybroeck, L. 2004, MNRAS, 353, 457
Amendola, L., Campos, G. C., & Rosenfeld, R. 2007, Phys. Rev. D, 75, 083506
Amendola, L., & Tsujikawa, S. 2010, Dark Energy: Theory and Observations by Luca Amendola and Shinji Tsujikawa (Cambridge Univ. Press), 2010
Armendáriz-Picón, C., Damour, T., & Mukhanov, V. 1999, Physics Letters B, 458, 209
Armendariz-Picon, C., Mukhanov, V., & Steinhardt, P. J. 2001, Phys. Rev. D, 63, 103510
Barreiro, T., Copeland, E. J., & Nunes, N. J. 2000, Phys. Rev. D, 61, 127301
Barrow, J. D., & Clifton, T. 2006, Phys. Rev. D, 73, 103520
Belinskij, V. A., Khalatnikov, I. M., & Lifshits, E. M. 1970, Advances in Physics, 19, 525
Bennett, C. L., Halpern, M., Hinshaw, G., et al. 2003, ApJS, 148, 1
Caldera-Cabral, G., Maartens, R., & Ureña-López, L. A. 2009, Phys. Rev. D, 79, 063518
Caldwell, R. R. 2002, Physics Letters B, 545, 23
Caldwell, R. R., Dave, R., & Steinhardt, P. J. 1998, Physical Review Letters, 80, 1582
Carroll, S. M., Hoffman, M., & Trodden, M. 2003, Phys. Rev. D, 68, 023509
Chaubey, R. 2009, International Journal of Theoretical Physics, 48, 952
Clocchiatti, A., Schmidt, B. P., Filippenko, A. V., et al. 2006, ApJ, 642, 1
Copeland, E. J., Sami, M., & Tsujikawa, S. 2006, International Journal of Modern Physics D, 15, 1753
de Bernardis, P., Ade, P. A. R., Bock, J. J., et al. 2000, Nature, 404, 955
Deffayet, C., Dvali, G., & Gabadadze, G. 2002, Phys. Rev. D, 65, 044023
Dvali, G., Gabadadze, G., & Porrati, M. 2000, Physics Letters B, 485, 208
Dvali, G., & Turner, M. S. 2003, arXiv: astro-ph/0301510
Elizalde, E., Nojiri, S., & Odintsov, S. D. 2004, Phys. Rev. D, 70, 043539
Ellis, G. F. R. 1993, in The Renaissance of General Relativity and Cosmology, Exact and Inexact Solutions of the Einstein Field Equations, eds. G. Ellis, A. Lanza, & J. Miller, 20
Ellis, G. F. R., & van Elst, H. 1999, in NATO ASIC Proc. 541, Theoretical and Observational Cosmology, ed. M. Lachièze-Rey, 1
Freese, K. 2003, Nuclear Physics B Proceedings Supplements, 124, 50
Freese, K., & Lewis, M. 2002, Physics Letters B, 540, 1
Garnavich, P. M., Kirshner, R. P., Challis, P., et al. 1998, ApJ, 493, L53
Gondolo, P., & Freese, K. 2003, Phys. Rev. D, 68, 063509
González-Díaz, P. F. 2004, Physics Letters B, 586, 1
Gumjudpai, B., Naskar, T., Sami, M., & Tsujikawa, S. 2005, J. Cosmol. Astropart. Phys., 6, 7
Guo, Z.-K., Ohta, N., & Tsujikawa, S. 2007, Phys. Rev. D, 76, 023508
Hanany, S., Ade, P., Balbi, A., et al. 2000, ApJ, 545, L5

Hassan, A., Anirudh, P., & Bijan, S. 2011, Chinese Physics Letters, 28, 039801
Hawking, S. W., & Ellis, G. F. R. 1973, The Large Scale Structure of Space-Time (U.K.: Cambridge Univ. Press)
Hu, B. L., & Parker, L. 1978, Phys. Rev. D, 17, 933
Kolb, E. W., & Turner, M. S. 1990, The Early Universe, Addison-Wesley
MacCallum, M. A. H. 1979, in General Relativity: An Einstein Centenary Survey, Anisotropic and Inhomogeneous Relativistic Cosmologies, eds. S. W. Hawking, & W. Israel (U.K.: Cambridge Univ. Press), Chapter 11
Misner, C. W. 1968, ApJ, 151, 431
Misner, C. W., Thorne, K. S., & Wheeler, J. A. 1973, Gravitation (W.H. Freeman: New York)
Pavón, D., Sen, S., & Zimdahl, W. 2004, J. Cosmol. Astropart. Phys., 5, 9
Pavón, D., & Wang, B. 2009, General Relativity and Gravitation, 41, 1
Perlmutter, S., Aldering, G., della Valle, M., et al. 1998, Nature, 391, 51
Perlmutter, S., Aldering, G., Goldhaber, G., et al. 1999, ApJ, 517, 565
Perlmutter, S., Gabi, S., Goldhaber, G., et al. 1997, ApJ, 483, 565
Ratra, B., & Peebles, P. J. E. 1988, Phys. Rev. D, 37, 3406
Riess, A. G., Filippenko, A. V., Challis, P., et al. 1998, AJ, 116, 1009
Ryan, M. P., & Shepley, L. C. 1975, Homogeneous Relativistic Cosmologies (Princeton Univ. Press)
Sahni, V. 2004, in Lecture Notes in Physics, Berlin Springer Verlag, 653, ed. E. Papantonopoulos, 141
Sahni, V., Saini, T. D., Starobinsky, A. A., & Alam, U. 2003, Soviet Journal of Experimental and Theoretical Physics Letters, 77, 201
Schmidt, B. P., Suntzeff, N. B., Phillips, M. M., et al. 1998, ApJ, 507, 46
Seljak, U., Makarov, A., McDonald, P., et al. 2005, Phys. Rev. D, 71, 103515
Singh, T., & Chaubey, R. 2007, Pramana, 68, 721
Singh, T., & Chaubey, R. 2009, Ap&SS, 321, 5
Singh, T., & Chaubey, R. 2011, International Journal of Theoretical Physics, DOI 10.1007/s10773-011-0881-0
Spergel, D. N., Verde, L., Peiris, H. V., et al. 2003, ApJS, 148, 175
Tegmark, M., Strauss, M. A., Blanton, M. R., et al. 2004, Phys. Rev. D, 69, 103501
Tonry, J. L., Schmidt, B. P., Barris, B., et al. 2003, ApJ, 594, 1
Tsujikawa, S., & Sami, M. 2004, Physics Letters B, 603, 113
Wei, H. 2010, Physics Letters B, 691, 173
Xanthopoulos, B. C. 1987, Journal of Mathematical Physics, 28, 905
Zimdahl, W., & Pavón, D. 2004, General Relativity and Gravitation, 36, 1483
Zimdahl, W., Pavón, D., & Chimento, L. P. 2001, Physics Letters B, 521, 133
Zimdahl, W., Pavón, D., Chimento, L. P., & Jakubi, A. S. 2005, in The Tenth Marcel Grossmann Meeting, On Recent Developments in Theoretical and Experimental General Relativity, Gravitation and Relativistic Field Theories, eds. M. Novello, S. Perez Bergliaffa, & R. Ruffini, 1794