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The effect of dynamical quark mass on the calculation of a strange quark star's structure

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Abstract We discuss the dynamical behavior of strange quark matter components, in particular the effects of density dependent quark mass on the equation of state of strange quark matter. The dynamical masses of quarks are computed within the Nambu-Jona-Lasinio model, then we perform strange quark matter calculations employing the MIT bag model with these dynamical masses. For the sake of comparing dynamical mass interaction with QCD quark-quark interaction, we consider the one-gluon-exchange term as the effective interaction between quarks for the MIT bag model. Our dynamical approach illustrates an improvement in the obtained equation of state values. We also investigate the structure of the strange quark star using Tolman-Oppenheimer-Volkoff equations for all applied models. Our results show that dynamical mass interaction leads to lower values for gravitational mass.

Key words: Stars: dense matter — equation of state — ISM: structure — supernova remnants

1 INTRODUCTION

Strange quark stars (SQSs) are the most compact solid objects known, with a surface density of $\rho \sim 10^{15}\mathrm{gr\,cm^{-3}}$, which is about 14 orders of magnitude greater than the surface density of neutron stars, and their central density could be up to five times higher than that (Haensel et al. 2007; Glendenning 2000; Weber 1999). Even before the theory of QCD was fully developed, Itoh (1970) first proposed SQSs as being made of strange quark matter (SQM). Later, Bodmer (1971) discussed the fate of an astronomical object collapsing to such a state of matter.

The quark deconfinement hypothesis is one of the most exciting steps in investigating the building blocks of matter. Soon after the predictions of quarks in theories and successful laboratory observations, many hadronic models were developed to describe the probable quark matter proposed in high energy regimes. In the 1970s, after the formulation of QCD, perturbative calculations of the equations of state of SQM took form, but the area of validity for these calculations was restricted to very high densities (Collins & Perry 1975). The existence of SQSs was also discussed by Witten (1984), who conjectured that a first-order QCD phase transition in the early universe could concentrate most of the excess quarks into dense quark nuggets. Witten proposed that SQM composed of light quarks is more stable than nuclei, therefore SQM can be considered as the ground state of matter.

The bulk of an SQS would be composed of the SQM phase consisting of almost equal numbers of up, down and strange quarks, plus a small number of electrons to ensure charge neutrality. A typical electron fraction is less than 10^{-3} and decreases from the surface to the center of an SQS (Haensel et al. 2007; Glendenning 2000; Weber 1999; Camenzind 2007). SQM would have a lower charge-to-baryon ratio compared to nuclear matter and can show itself in the form of an SQS (Witten 1984; Alcock et al. 1986; Haensel et al. 1986; Kettner et al. 1995). The collapse of a massive star could lead to the formation of an SQS. An SQS may also be formed from a neutron star and is denser than a neutron star (Bhattacharyya et al. 2006). If sufficient additional matter is added to an SQS, it will collapse into a black hole. Neutron stars with masses of 1.5–1.8 $M_{\odot}$ with rapid spins are theoretically the best candidates for conversion to an SQS. An extrapolation based on this indicates that up to two quark-novae occur in the observable universe each day. In addition, recent Chandra observations indicate that objects *RX* J$185635-3754$ and 3C 58 may contain SQSs (Prakash et al. 2003). Other investigations also show that the object *SWIFT* J$1749.4-2807$ may be an SQS (Yu & Xu 2010).

The strange quark star, derived from quark matter theory, consists of many unsolved puzzles which are usually involved in the physics of these relativistic objects. The system complexity of these stars prohibits us from considering all the physical and astrophysical properties simultaneously, and it is possible that some parameters entering the equation of state do not represent specific physical properties. For example, in the MIT bag model, one of the models used in this paper, when researchers try to find and fit the bag constant according to information gained from big colliders (Jin & Jennings 1997; Alford et al. 1998; Blaschke et al. 1999; Burgio et al. 2002b; Begun et al. 2011), we should keep this principle as a matter of fact that different parameters like temperature, electromagnetic intensity, density, etc., are important enough on final interpretation for the theoretically calculated bag constant. With this point of view, even constant values of bag pressure can no longer be considered purely as the energy density difference between the perturbative vacuum and the true vacuum. The role of the bag constant for confining quark matter in comparison with gravity confinement for neutron matter may require more attention when we consider it for compact stars. Therefore, it is better to consider the dynamical properties of the parameters for investigating the properties of quarks. Many works have been done to adapt the bag model theory to the physics of ultra-dense matter, such as using a density dependent bag constant (Burgio et al. 2002a), utilizing different values of coupling constants for one gluon exchange (Farhi & Jaffe 1984; Berger & Jaffe 1987), or considering dynamical mass as an effective interaction between particles (Peng et al. 2000; Shao et al. 2011).

From the theory of perturbative QCD, we know that quarks at ultra high densities asymptotically interact. One way of considering the interaction is to assume that quarks exchange one gluon. Therefore we can add a term to the equation of state that is characterized by a coupling constant. However, constant values of this parameter will weaken the power of interaction at lower densities, but higher densities will increase it. One method to solve this problem is to assume a density dependent quark mass to be the effective interaction. This approach was investigated in references (Fowler et al. 1981; Chakrabarty et al. 1989; Chakrabarty 1991, 1994; Benvenuto & Lugones 1995; Lugones & Benvenuto 1995), and was done by adding a term to the rest mass that is characterized by a free parameter determined by stability conditions. The conclusion was that the density dependent mass is flavor independent and that the applied free parameter has the same meaning as the bag constant. Then, by selecting one value of the bag constant for all densities and flavors, these researchers tried to obtain the equation of state of quark matter (Peng et al. 2000). A better approach closer to the current work is to find a solution for the density dependent mass from the Nambu-Jona-Lasinio (NJL) method (Carroll et al. 2009). Carroll et al. calculated the equation of state and the structure of hybrid stars within the MIT bag model, while the numerical values of the density dependent mass entering the energy equation were obtained from dynamical calculations of mass in the NJL model. These numerical values were entered directly into the pressure equation without considering density dependency. The quark masses and NJL constants were also approximate values. The bag constant in that work was density independent; therefore, in addition to the previously known problems of constant values for this parameter (Baldo et al. 2006; Alford & Reddy 2003; Alford et al. 2005), it misinterprets the meaning of the effective interaction in some densities.

In our previous work we considered a hot strange star just after the collapse of a supernova (Bordbar et al. 2011) at finite temperature with a density dependent bag constant. The calculations for the structural properties of the strange star at different temperatures indicate that its maximum mass decreases with increasing temperature. In another work (Bordbar & Peivand 2011), we concentrated on the calculation of a bulk of spin polarized SQM at zero temperature in the presence of a strong magnetic field. We computed the structural properties of this system and found that the presence of a magnetic field leads to a more stable SQS when compared to the structural properties of an unpolarized SQS. In the present paper, we investigate the quark matter equation of state and the strange quark star structure following Carroll et al. (2009). We base our calculations on the MIT bag model, and after following the NJL formalism we extrapolate a density dependent equation from numerical values of dynamical mass obtained using the NJL method. In Section 2, the required equations for the MIT bag model are written, as has been done for the NJL model. In Section 2.3, we describe the formalism applied in this article, and after solving the Tolman-Oppenheimer-Volkoff (TOV) equations in Section 3, we calculate the SQS structure for our method.

2 CALCULATION OF THE EQUATION OF STATE FOR SQM

In this section, we calculate the equation of state of SQM using the MIT and NJL methods, as well as apply the MIT method for the dynamical mass. First we introduce these three models in three separate sections, then we give our results for the energy and the equation of state of SQM in Section 2.4.

2.1 The MIT Bag Model

The total energy of a bulk of deconfined up (u), down (d) and strange (s) quarks within the MIT bag model is as follows (Witten 1984; Farhi & Jaffe 1984; Baym et al. 1985; Berger & Jaffe 1987; Glendenning 1990; Maruyama et al. 2007)

$$\varepsilon = \varepsilon_u + \varepsilon_d + \varepsilon_s + B\,. \tag{1}$$

In Equation (1), B is the bag constant, and

$$\begin{aligned}\varepsilon_f\,(\rho_f) \;=\; & \frac{3{m_f}^4}{8\pi^2}\left[x_f\left(2x_f^2+1\right)\left(\sqrt{1+x_f^2}\right)-\operatorname{arcsin}h\,x_f\right] \\ & -\alpha_c\,\frac{{m_f}^4}{\pi^3}\left[x_f^4-\frac{3}{2}\left[x_f\left(\sqrt{1+x_f^2}\right)-\operatorname{arcsin}h\,x_f\right]^2\right]\,,\end{aligned} \tag{2}$$

where f denotes the flavor of the relevant quark, α_c is the QCD coupling constant and the following term demonstrates the one-gluon-exchange interaction. In the above equation, x_f is defined as follows,

$$x_f = k_{\rm F}{}^{(f)}/m_f\,, \tag{3}$$

where the Fermi momentum $k_{\rm F}{}^{(f)}$ is given by

$$k_{\rm F}{}^{(f)} = \left(\rho_f\,\pi^2\right)^{1/3}\,. \tag{4}$$

For the bag constant (B), we use a density dependent Gaussian parametrization (Burgio et al. 2002a; Baldo et al. 2006)

$$B\,(\rho) = B_\infty + (B_0 - B_\infty)\exp[-\beta\,(\rho/\rho_0)^2]\,, \tag{5}$$

with $B_\infty = B(\rho = \infty) = 8.99$ MeV fm^{-3}, $B_0 = B(\rho = 0) = 400$ MeV fm^{-3} and $\beta = 0.17$. In SQM, the beta-equilibrium and charge neutrality conditions lead to the following relation for the number density of quarks,

$$\rho = \rho_u = \rho_d = \rho_s\,. \tag{6}$$

From the total energy, we can obtain the equation of state of SQM using the following relation,

$$P(\rho) = \rho\frac{\partial\varepsilon}{\partial\rho} - \varepsilon\,. \tag{7}$$

2.2 The Nambu-Jona-Lasinio Model

Here we give a brief introduction regarding the calculations in the NJL method. For the NJL model, we use a common three flavor Lagrangian adopted from (Rehberg et al. 1996) which preserves the chiral symmetry of QCD,

$$\begin{aligned}\mathcal{L} = \bar{q}\left(i\gamma^\mu\partial_\mu - \hat{m_0}\right)q + G\sum_{k=0}^{8}\left[\left(\bar{q}\lambda_k q\right)^2 + \left(\bar{q}i\gamma_5\lambda_k q\right)^2\right] \\ -K\left[\det_f\left(\bar{q}\left(1+\gamma_5\right)q\right) + \det_f\left(\bar{q}\left(1-\gamma_5\right)q\right)\right]\,.\end{aligned} \tag{8}$$

In the adopted Lagrangian, q denotes the quark field with three flavors, u, d and s, and three colors. $\hat{m_0} = \mathrm{diag}(m_0^u,\, m_0^d,\, m_0^s)$ is a 3×3 matrix in flavor space, and λ_k ($0 \le k \le 8$) are the $U(3)$ flavor matrices. We restrict ourselves to the isospin symmetric case, $m_0^u = m_0^d$. We have picked up the parameters from references (Kunihiro 1989; Ruivo et al. 1999; Buballa & Oertel 1999), which are fitted to the pion mass, the pion decay constant, the kaon mass and the quark condensates.

The NJL model is an unrenormalizable method with divergent integrations. To prevent the divergence, we need to introduce some breaking points for the upper limit of integrals which satisfy the physical ranges of our problem. This is usually done by choosing a proper cut-off. In the present paper, the adopted cut-off is called the ultraviolet cut-off that indicates the restoration of chiral symmetry breaking, $\Lambda = 602.3\,\mathrm{MeV}$. G and K are coupling strengths that read $G\Lambda^2 = 1.835,\ K\Lambda^5 = 12.36$. The rest mass of the s quark is $m_0^s = 140.7\,\mathrm{MeV}$, and $m_0^u = m_0^d = 5.5\,\mathrm{MeV}$ for the u and d quarks. The baryon number density is given by

$$\rho_B = \frac{1}{3}n_B = \frac{1}{3}\left(n_u + n_d + n_s\right)\,, \tag{9}$$

where $n_i = \left\langle q_i^\dagger q_i\right\rangle$. Within the mean field approximation, the dynamical mass is calculated by the following gap equation,

$$m_i = m_0^i - 4G\left\langle\bar{q_i}q_i\right\rangle + 2K\left\langle\bar{q_j}q_j\right\rangle\left\langle\bar{q_k}q_k\right\rangle\,. \tag{10}$$

In the above equation, we need to calculate the permutation of all quark flavors. The quark condensate in Equation (10) reads

$$\left\langle\bar{q_i}q_i\right\rangle = -\frac{3}{\pi^2}\int_{P_{\mathrm{F}i}}^{\Lambda}P^2dp\frac{m_i}{\sqrt{m_i^2+p^2}}\,, \tag{11}$$

and $P_{\mathrm{F}i}$, the Fermi momentum of quark i, is obtained from the following relation,

$$P_{\mathrm{F}i} = \left(\pi^2 n_i\right)^{\frac{1}{3}}\,. \tag{12}$$

Equations (10) and (11) have self consistent solutions. This means that for a given number density, n_i, we should calculate the quark condensate and substitute the corresponding value in Equation (10) to reach a consistent dynamical mass result after iterating the process.

In Figure 1, we have plotted the results of density dependent mass for the u, d and s quarks as a function of density. As is clear from Figure 1, quark masses vary from current masses ($5.5\ \mathrm{MeV}$ for the u and d quarks, and $140.7\ \mathrm{MeV}$ for the s quark) at high densities to constituent mass at near zero densities ($368.7\ \mathrm{MeV}$ for the u and d quarks, and $550\ \mathrm{MeV}$ for the s quark).

The solution via the mean field approximation forces us to stabilize the equations by diminishing energy density and pressure in a vacuum. This is satisfied by defining a parameter which has the same meaning of bag constant in the MIT bag model (Buballa & Oertel 1999)

$$B = \sum_{i=u,d,s} \left(\frac{3}{\pi^2} \int_0^{\Lambda} p^2 dp \left(\sqrt{p^2 + m_i^2} - \sqrt{p^2 + {m^i}_0^2} \right) - 2G \left\langle \bar{q}_i q_i \right\rangle^2 \right) + 4K \left\langle \bar{u}u \right\rangle \left\langle \bar{d}d \right\rangle \left\langle \bar{s}s \right\rangle \ . \quad (13)$$

Now we can calculate the equation of state of SQM in the NJL model,

$$p = -\varepsilon + \sum_{i=u,d,s} n_i \sqrt{P_{\mathrm{F}i}^2 + m_i^2} \,, \quad (14)$$

where

$$\varepsilon = \sum_{i=u,d,s} \frac{3}{\pi^2} \int_0^{P_{\mathrm{F}i}} p^2 dp \sqrt{p^2 + m_i} - (B - B_0) \,. \quad (15)$$

Parameter B is the bag pressure, which is explained by Buballa (2005), and is a dynamical consequence of the mean field solution, not a parameter inserted by hand, as was done in the MIT bag model. It is shown in Figure 1 that the matter in the NJL method acquires dynamical mass in nonzero baryon densities, but in the MIT bag model the given mass remains constant for all densities. Consequently, this will lead to dissimilar chiral symmetry behavior as the density changes. In the NJL model, since quarks acquire dynamical mass, the chiral symmetry spontaneously breaks at lower densities, but in the MIT bag model it will happen physically when quarks change their directions by hitting the bag (what is not considered theoretically in the ordinary MIT bag model). The bag constant versus density is presented in Figure 2 for our models. It is apparent from Figure 2 that the chiral symmetry in our calculations is fully restored in densities greater than $\rho \simeq 2.5\ \mathrm{fm}^{-3}$. It is also important to mention that the vacuum in the MIT bag model is totally free of particles (the flow of the particles' wave function is restricted by the confinement), while in the NJL model no confinement is produced. In other words, the vacuum in the NJL model is made of paired quasi-quarks that lower the energy density of particles in comparison to the MIT bag model. From the above discussions, it seems reasonable to add an effective bag constant to the energy equation (Buballa 2005),

$$\begin{aligned} B_0 &= B \,|_{n_u=n_d=n_s=0} \,, \\ B_{\mathrm{eff}} &= B - B_0 \,. \end{aligned} \quad (16)$$

From Figure 2, it seems that the effective bag constant diminishes at zero density. Then the correct interpretation for the effective bag constant is the energy per volume needed to fully break the quark-antiquark pairs in order to completely restore chiral symmetry at ultra high densities. Even the maximum value of the dynamical NJL bag constant is smaller than that of the MIT's, because it reduces the energy per particle due to quark-antiquark pairing at lower densities (Buballa 2005).

Figure 2 shows that the rate of decrease of the MIT bag constant is higher than that of the NJL. This indicates that the MIT bag model represents a gross approximation over the physics of matter in the middle and higher densities ($\rho > 0.8\ \mathrm{fm}^{-3}$). Therefore, the density dependent bag constant

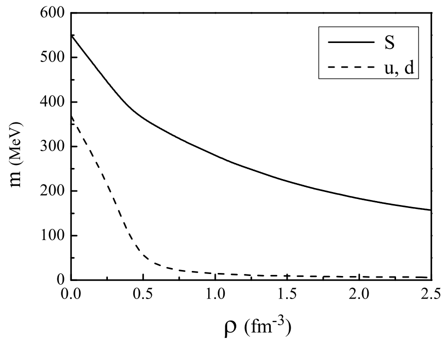

Fig. 1 Density dependent mass (m) versus density (ρ) obtained from the dynamical NJL model.

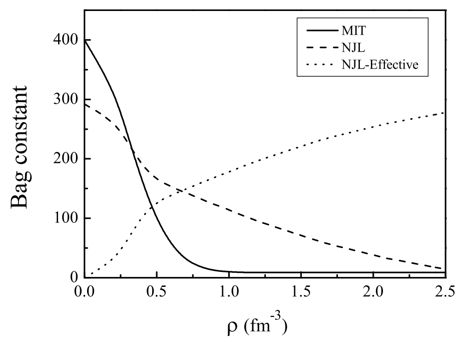

Fig. 2 Bag constant as a function of density for the NJL and MIT models.

should be corrected by another parameter sensitive to a higher density. This could not be achieved by a one gluon exchange term that considers the interaction with a constant strength in all energy regimes. Figure 2 indicates that at the density $\rho \simeq 0.45\ \mathrm{fm}^{-3}$, there is a crossing point for the effective bag constant of the NJL model and the bag constant of the MIT model. As is mentioned in the above discussions, the bag pressure is the energy needed to confine particles and the effective bag constant is the energy needed to destabilize the quark-antiquark pairs. Now, we can suggest that the hadron-quark phase transition can take place at the density $\rho \simeq 0.45\ \mathrm{fm}^{-3}$. This is in good agreement with the results of others (Heinz 2001; Heinz & Jacob 2000).

2.3 The MIT Bag Model with Dynamical Mass

In the MIT bag model with dynamical mass, we consider the effect of the dynamical behavior of the quark mass in calculating the equation of state of SQM within the MIT bag model using NJL numerical mass results. In fact, we use the dynamical masses (Fig. 1) for the u, d and s quarks in Equation (2) instead of their fixed values.

2.4 Our Results for the Energy and Equation of State of SQM

To distinguish numerous outcomes, we present the results of our calculations in the three following models:

- Model 1: The MIT model derived by a density dependent bag constant and one gluon exchange $(\alpha_c = 0,\ 0.16,\ 0.5)$ as the effective interaction.
- Model 2: The NJL model.
- Model 3: The MIT bag model derived by a density dependent bag constant, dynamical mass and one gluon exchange $(\alpha_c = 0,\ 0.16,\ 0.5)$ as the effective interaction.

Our results for the energy of SQM versus density calculated with the above models have been plotted in Figure 3. We see that for both MIT based calculations (models 1 and 3), at lower densities $(\rho < 0.5\ \mathrm{fm}^{-3})$, the energy of SQM suddenly increases as the density decreases. This shows the concept of confinement (Buballa 2005). For these two models, we also see that the energy of SQM achieves a minimum, then increases at a small rate. Figure 3 shows that for model 1 and model 3, the energies of the different coupling constants are nearly identical for densities $\rho < 0.5\ \mathrm{fm}^{-3}$. However, they have a substantial difference as the density increases. We can see that at lower densities $(\rho < 0.7\ \mathrm{fm}^{-3})$, the results of model 3 are considerably different from those of model 1. This difference

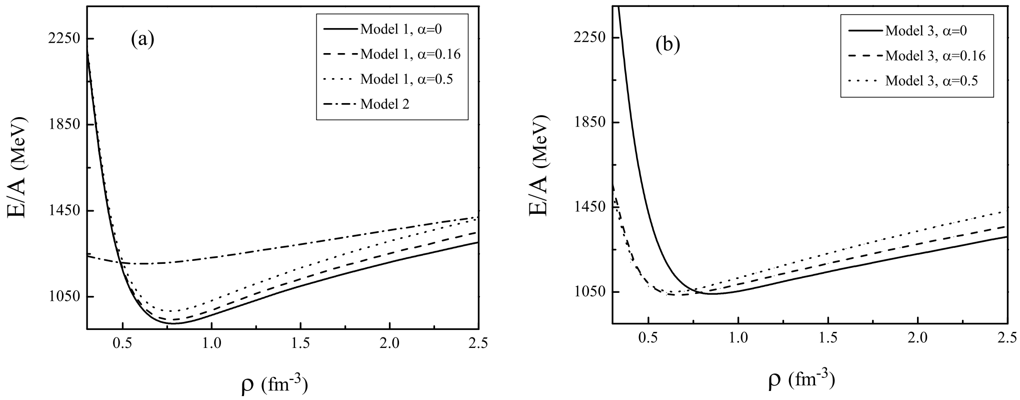

Fig. 3 Energy per baryon versus density for models 1 and 2 (a), and model 3 (b).

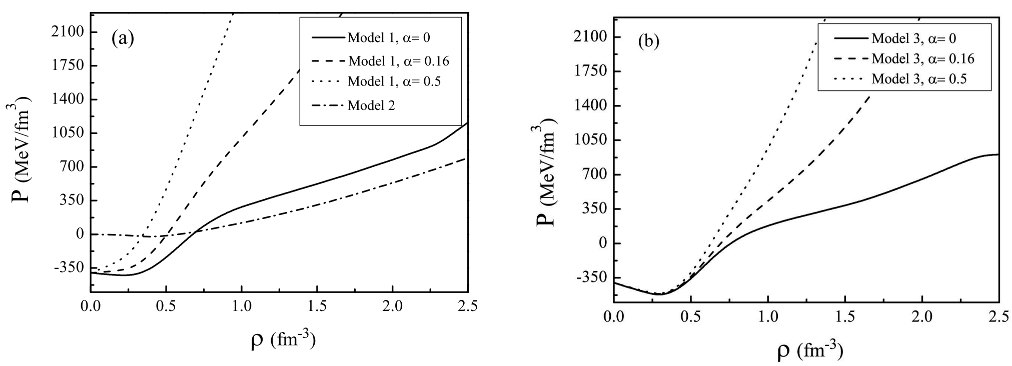

Fig. 4 Pressure as a function of density for models 1 and 2 (a), and model 3 (b).

becomes small as density increases, especially for lower values of the coupling constant, due to asymptotic freedom from the simple MIT bag model without interaction. From Figure 3, it is seen that the energy of SQM in model 2 (the NJL model) has finite values even at low densities showing no confinement. We also see that the energy of SQM from model 3 with smaller values of the coupling constant is lower than that of model 2 for $\rho > 0.7\ \mathrm{fm}^{-3}$, indicating a more stable state of quark matter at these densities. However, at very high densities, the difference between the results of these two models becomes negligible.

In Figure 4, our results for the pressure of SQM have been plotted versus density. It can be found that for the MIT bag model, the higher values of the coupling constant lead to a stiffer equation of state for SQM. Figure 4 shows that by considering a dynamical mass for the quarks (density dependent mass) in the MIT model, we get lower values for the pressure of SQM. For $\alpha_c = 0.0$, we see that the result of model 3 for the equation of state of SQM is nearly identical with that of model 1. It can be seen that for $\rho > 0.6\ \mathrm{fm}^{-3}$, our results for the pressure of SQM calculated by the NJL model are nearly identical to those of model 3 and model 1 for $\alpha_c = 0.0$, but at lower densities, there is a considerable difference between them.

In order to investigate quark matter stability, the energy of SQM versus pressure has been plotted in Figure 5. It is clearly seen that at zero pressure, the MIT bag model with $\alpha_c = 0$ leads to the lowest value for the energy of SQM ($950\ \mathrm{MeV\ fm}^{-3}$) compared to the other models. This value

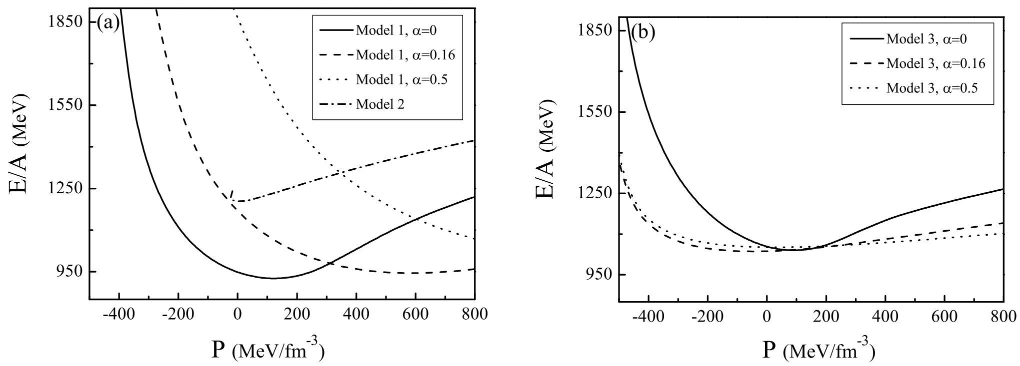

Fig. 5 Energy per particle versus pressure for models 1 and 2 (a), and model 3 (b).

is comparable with the result for the binding energy per particle of ^{56}Fe (930 MeV fm^{-3}) (Witten 1984), which indicates that among the different models used in this work, the MIT model with $\alpha_c = 0$ shows the most stable state of SQM.

3 CALCULATION OF THE STRANGE QUARK STAR STRUCTURE

The gravitational mass (M) and radius (R) of compact stars are of special interest in astrophysics. In this section, we calculate the structural properties of an SQS for our three models. Using the equation of state of SQM for the models applied in this work, we can obtain M and R by numerically integrating the general relativistic equations of hydrostatic equilibrium, the TOV equations, which are as follows (Shapiro & Teukolsky 1983),

$$\frac{dm}{dr} = 4\pi r^2 \varepsilon\left(r\right) \ , \tag{17}$$

$$\frac{dp}{dr} = -\frac{Gm\left(r\right)\varepsilon\left(r\right)}{r^2}\left(1+\frac{p\left(r\right)}{\varepsilon\left(r\right)c^2}\right)\left(1+\frac{4\pi r^3 p\left(r\right)}{m\left(r\right)c^2}\right)\left(1-\frac{2Gm\left(r\right)}{c^2 r}\right)^{-1} \ , \tag{18}$$

where $\varepsilon\left(r\right)$ is the energy density, G is the gravitational constant, and

$$m(r) = \int_0^r 4\pi \acute{r}^2 \, \varepsilon(\acute{r}) d\acute{r} \tag{19}$$

has the interpretation of the mass inside radius r. By selecting a central energy density ε_c, under the boundary conditions $P(0) = P_c$ and $m(0) = 0$, we integrate the TOV equation outwards to a radius $r = R$, at which P vanishes.

In Figure 6, we have presented our results for the gravitational mass of an SQS versus the central energy density. Figure 6 shows that at low energy densities, the gravitational mass increases rapidly by increasing the energy density, and it finally reaches a limiting value (maximum gravitational mass) at higher energy densities. It is seen that the increasing rate of mass for model 3 with higher values of the coupling constant is substantially higher than those of the other models.

Table 1 summarizes the maximum gravitational masses of the different applied models and the corresponding radii. As seen from Table 1, we can conclude that using dynamical mass in the energy equation and equation of state of SQM reduces the calculated maximum mass. This is in good agreement with many of the observational data obtained from low mass compact stars (Zhang et al. 2007).

Table 1 Maximum gravitational mass $(M_{\max})$ and the corresponding radius (R) for different applied models.

	$M_{\max}\,(M_{\odot})$	R (km)
Model 1; $\alpha_c = 0$	1.43	7.61
Model 1; $\alpha_c = 0.16$	1.73	8.17
Model 1; $\alpha_c = 0.5$	2.6	10.6
Model 2	0.98	5.59
Model 3; $\alpha_c = 0$	1.05	6.03
Model 3; $\alpha_c = 0.16$	1.65	6.98
Model 3; $\alpha_c = 0.5$	2.3	8.69

It is interesting that despite considering dynamical mass as the effective interaction in the MIT bag model (model 3 with $\alpha_c = 0$), we find the smaller maximum SQS mass to be comparable to the MIT bag model (model 1) even without interaction ($\alpha_c = 0$). As is obvious from Table 1, for

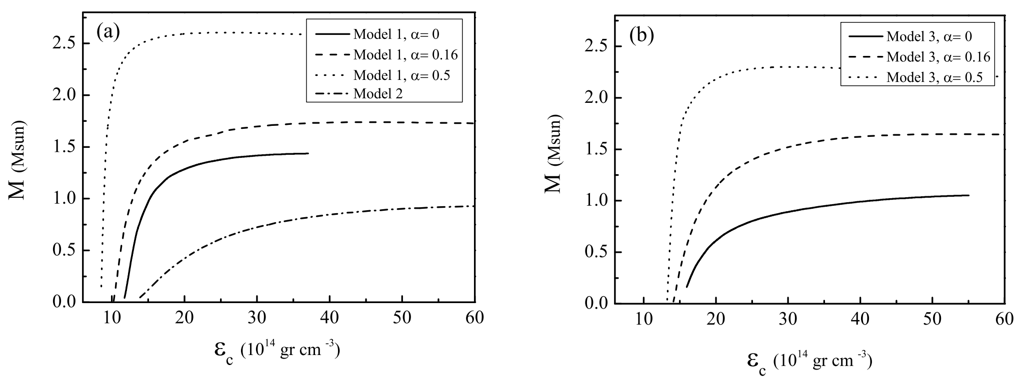

Fig. 6 Gravitational mass (M) in units of solar mass $(M_{\odot})$ versus central energy density (ε_c) for models 1 and 2 (a), and model 3 (b).

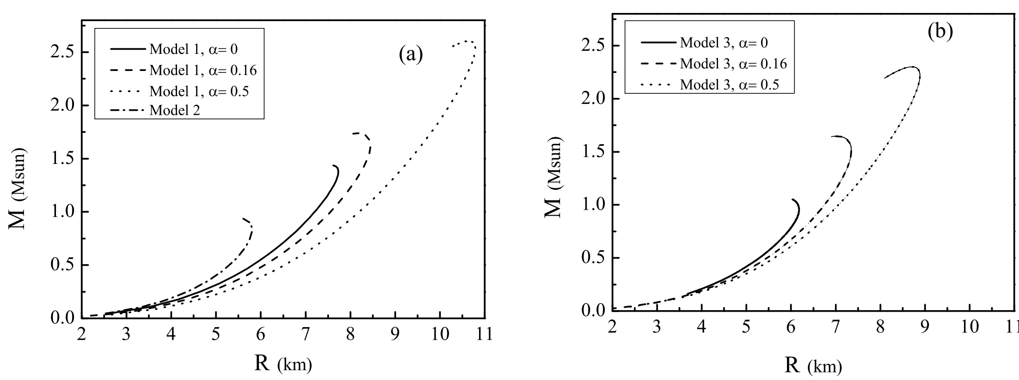

Fig. 7 Gravitational mass (M) in units of solar mass $(M_{\odot})$ versus radius (R) for models 1 and 2 (a), and model 3 (b).

models 1 and 3, the calculated maximum mass increases as the strong coupling constant increases. This behavior demonstrates that ultra massive SQSs with masses greater than $M = 1.05\ M_{\odot}$ are stars that are composed of strongly interacting SQM.

We note that some studies indicate that there is a large uncertainty about the mass and radius of ultra massive stars with $M > 1.9\ M_{\odot}$ (Lattimer & Prakash 2010). These studies showed that the observed data of the mass and radius for these stars, which commonly belong to X-ray stars, were wrongly calculated and the calculations were revised to the smaller values for mass and radius. The best example is pulsar PSR J 0751+1807 which was initially believed to have a mass of $M = 2.2 \pm 0.2\ M_{\odot}$, but this was recently revised to $M = 1.26\ M_{\odot}$ (Lattimer & Prakash 2010).

We have also plotted the gravitational mass of SQS versus radius for our three models in Figure 7. It is seen that for all models, the mass increases by increasing the radius, but with different increasing rates for the different models. Figure 7 shows that for a given value of radius, the dynamical model (model 3) gives the smaller mass with respect to that of the MIT bag model (model 1); however, for $\alpha_c = 0$, it is close to the result of the NJL model (model 2).

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