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The relationships of solar flares with both sunspot and geomagnetic activity *

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Abstract The relationships between solar flare parameters (total importance, time duration, flare index, and flux) and sunspot activity ($R_{\rm z}$) as well as those between geomagnetic activity (aa index) and the flare parameters can be well described by an integral response model with response time scales of about eight and 13 months, respectively. Compared with linear relationships, the correlation coefficients of the flare parameters with $R_{\rm z}$, of aa with the flare parameters, and of aa with $R_{\rm z}$ based on this model have increased about 6%, 17%, and 47% on average, respectively. The time delays between the flare parameters with respect to $R_{\rm z}$, aa to the flare parameters, and aa to $R_{\rm z}$ at their peaks in a solar cycle can be predicted in part by this model (82%, 47%, and 78%, respectively). These results may be further improved when using a cosine filter with a wider window. It implies that solar flares are related to the accumulation of solar magnetic energy in the past through a time decay factor. The above results may help us to understand the mechanism of solar flares and to improve the prediction of the solar flares.

Key words: Sun: activity—Sun: sunspots—Sun: flares—geomagnetic activity

1 INTRODUCTION

Solar flares are powerful eruptions of solar activity (Ozguc & Atac 1989; Mikic & Linker 1994; Jain et al. 2010; Fang 2011) occurring on time scales of minutes up to a few hours (Chandra et al. 2011) and may produce a series of solar-terrestrial effects, which may be hazardous to both spacecraft and astronauts. Understanding the mechanism of solar flares and forecasting them are important for both solar physics and geophysics. Several mechanisms have been proposed to explain the eruptions of solar flares, such as the photospheric converging and shear motions (Mikic & Linker 1994), flux emergence and cancelation (Gan et al. 1993; Zhang et al. 2001), the catastrophe model (Forbes 1990), and kink instability of coronal flux ropes (Sakurai 1976; Li & Gan 2011). The magnetic reconnection plays an important role in triggering solar flares (Lin et al. 1995; Wheatland & Litvinenko 2001; Forbes et al. 2006; Fang et al. 2010).

To quantify the daily flare activity over 24 hours per day, Kleczek (1952) introduced the 'flare index' defined as

$$Q = i \times t\,, \tag{1}$$

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where 'i' represents the intensity scale of importance and 't' the duration (in minutes) of the flare (Knoska & Petrasek 1984; Atac & Ozguc 1998). This relationship is assumed to give roughly the total energy emitted by a flare (Kleczek 1952). The solar flare activity is found to be closely correlated with sunspots (Ozguc & Atac 1989; Feminella & Storini 1997). Larger flares often appear near larger and more complex active regions (McIntosh 1990; Bachmann & White 1994; Norquist 2011). Sunspot activity is a striking manifestation of magnetic fields on the Sun, associated with the main sites of solar-activity phenomena (Moradi et al. 2010) and related to the energy supplied into the corona (de Toma et al. 2000; Temmer et al. 2003). Studying the relationship between solar flares and sunspot activity is useful to understand and predict the former. The flare frequency of occurrence is often predicted by sunspot groups or numbers (McIntosh 1990; Gallagher et al. 2002; Cui et al. 2006; Yu et al. 2009; Huang et al. 2010), which has increasing applications in space weather.

Solar activity is well known to be at the origin of geomagnetic activity (Snyder et al. 1963; Crooker et al. 1977). Studying the relationship between solar activity, as represented by the International Sunspot Number ($R_{\rm z}$), and geomagnetic activity, as represented by the aa index (Mayaud 1972), is useful for understanding the formation of the latter and the mechanism of the solar cycle (Feynman & Crooker 1978; Legrand & Simon 1989; Du 2011a; Du & Wang 2010, 2011a,b). Conventionally, the relationship between aa and $R_{\rm z}$ is often analyzed by point-point correspondence. However, some questions are hardly understood, such as the significant increase in the aa index over the twentieth century (Feynman & Crooker 1978; Cliver et al. 1998; Lukianova et al. 2009), and the variations in the correlation between aa and $R_{\rm z}$ (Borello-Filisetti et al. 1992; Echer et al. 2004; Du 2011b). It is found that these phenomena can be well explained by an integral response model recently presented by Du (2011c). The value of aa depends not only on the present $R_{\rm z}$ but also on past values.

The geomagnetic activity results from various phenomena which are related to the interplanetary magnetic field (IMF, Stamper et al. 1999), solar wind (Svalgaard 1977; Legrand & Simon 1989; Tsurutani et al. 1995), coronal mass ejection (CME, Legrand & Simon 1989), galactic cosmic rays (Stamper et al. 1999), and others (Legrand & Simon 1989; Stamper et al. 1999). Gosling (1993) pointed out that CMEs, rather than flares, were the critical element for large geomagnetic storms, interplanetary shocks, and major solar energetic particle (SEP) events. This idea was argued as not necessarily true by Richardson et al. (2002).

This study analyzes the relationships between solar flare parameters (Sect. 2) and $R_{\rm z}$ as well as the relationships between the aa index and the flare parameters using an integral response model (Du 2011c) in Sections 3.1–3.4. Conclusions are summarized in Section 4.

2 DATA

The data used are the time series of monthly mean geomagnetic aa index[1] (Mayaud 1972), the International Sunspot Number ($R_{\rm z}$)[2], and solar flare parameters based on the Geostationary Operational Environmental Satellite (*GOES*) soft X-ray flares shown as follows[3]:

(i) I: total importance of flares, $I = 100X + 10M + C + 0.1B$, where X, M, C, and B are the flare classes (Cui et al. 2006).
(ii) T: time duration of flare (in minutes).
(iii) Q: the 'flare index' from Equation (1) by Kleczek (1952).
(iv) F: flux from event start to end (in J m^{-2}).

These parameters are first summed over each day and then averaged over each month to obtain the monthly means of the daily integrated quantities. To filter out high frequency variations in the

[1] *ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/RELATED_INDICES/AA_INDEX/*
[2] *http://www.ngdc.noaa.gov/stp/spaceweather.html*
[3] *ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/*

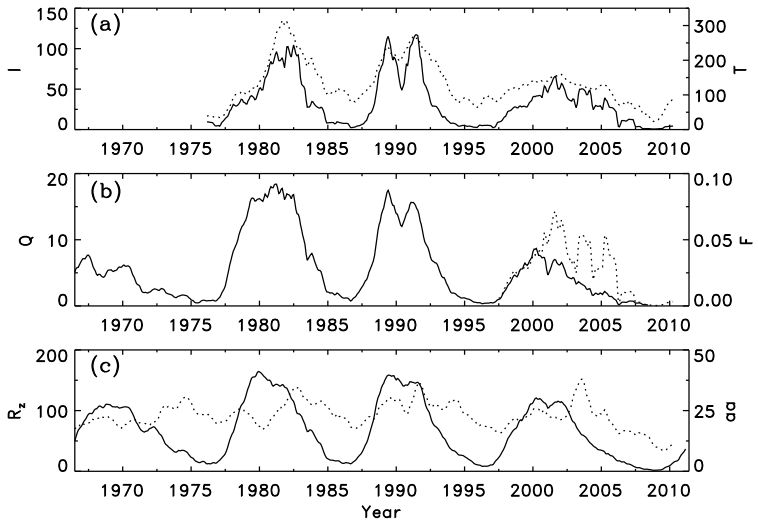

Fig. 1 (a) I (*solid*) and T (*dotted*) since March 1976, with a correlation coefficient of $r = 0.89$. (b) Q (*solid*) since July 1966 and F (*dotted*) since July 1997, with a correlation coefficient of $r = 0.67$. (c) $R_{\rm z}$ (*solid*) and aa (*dotted*) since July 1966, with a correlation coefficient of $r = 0.40$.

data, the parameters are smoothed with the commonly used 13-month running mean technique. The solar flare parameters since July 1966 are shown in Figure 1(a) and (b). It is seen that these parameters are well correlated. For example, I is well correlated with T ($r = 0.89$, Fig. 1(a)), and Q is well correlated with F ($r = 0.67$, Fig. 1(b)), with both being significant at the 99% level of confidence. Figure 1(c) depicts the time series of $R_{\rm z}$ (solid) and aa (dotted) with a correlation coefficient of $r = 0.40$.

3 RESULTS

It is well known that solar flares tend to lag behind sunspot activity by several months (Wheatland & Litvinenko 2001; Temmer et al. 2003) or even a few years (Wagner 1988; Aschwanden 1994). To have a better understanding of the relationships and time delays between solar flares and R_z, we employ the following integral response model (Du 2011c) to study the relationships between the flare parameters ($P = I$, T, Q and F) and $R_{\rm z}$,

$$\begin{aligned} y(t) &= D\int_{-\infty}^{t} x(t')\mathrm{e}^{-(t-t')/\tau}dt' + y_0 \\ &= D\sum_{t'=t_0}^{t} x(t')\mathrm{e}^{-(t-t')/\tau} + y_0\,, \end{aligned} \tag{2}$$

where y_0 is a constant, reflecting the part of $y = P$ that is uncorrelated to $x = R_{\rm z}$ (related to other phenomena); D is the 'dynamic response factor' of y to x, representing the initial generation efficiency of y by x ($\partial y/\partial x|_{t'=t}$); and τ is the 'response time scale' of y to x, indicating the dependence of the current $y(t)$ on the past $x(t')$ through a time decay factor $\mathrm{e}^{-(t-t')/\tau}$ ($\tau = 0$ reflects the point-point correspondence of y to x, i.e. the current $y(t)$ is only related to the current $x(t)$; $\tau = +\infty$ represents y being uncorrelated to x). In application, both y and x are discrete variables. Therefore, we use the second formula in Equation (2) with the summation being taken from the starting time (t_0) of the series (see Fig. 1) to time t. The three parameters (D, τ and y_0) are determined by a nonlinear least-squares fitting algorithm. Besides, as the geomagnetic activity (aa index) often lags

behind solar flares by several months, the relationships between aa and the flare parameters are also analyzed by the same model.

3.1 Relationship between R_z-I-aa

First, we analyze the relationship between $y = I$ and $x = R_z$ since March 1976 (t_0) with Equation (2) in the form of

$$I(t) \;=\; D_1 \sum_{t'=t_0}^{t} R_z(t') \mathrm{e}^{-(t-t')/\tau_1} + I_0 \,. \tag{3}$$

Figure 2(a) plots the reconstructed series I_f (dotted) of I (solid) from R_z (dashed) calculated by Equation (3). Although the correlation coefficient between I and R_z ($r_0 = 0.87$) has not been significantly improved by this model ($r_f = 0.88$), the lag times of I to R_z at their peaks (time differences between the peak timings) for Cycles 21–23 ($L_1 = 31, 24, 17$ with a mean $\overline{L}_1 = 24$ months) can be predicted in part by Equation (3) as shown in Figure 2(a) for the corresponding ones in brackets ($L_{f1} = 4, 8, 5$ with a mean $\overline{L}_{f1} = 6$). It implies that the current flares are related to the accumulation of solar magnetic energy in the past through a time decay factor. Active magnetic structures may evolve from the photosphere to the upper chromosphere with different speeds and times (Lin et al. 1995; Wheatland & Litvinenko 2001).

The relationship between $y = aa$ and $x = I$ can be fitted by

$$aa(t) \;=\; D_2 \sum_{t'=t_0}^{t} I(t') \mathrm{e}^{-(t-t')/\tau_2} + aa'_0 \,, \tag{4}$$

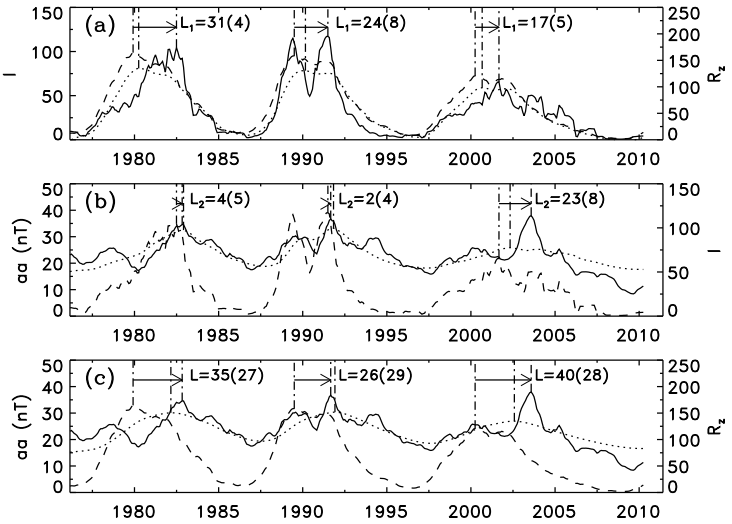

Fig. 2 (a) I (*solid*), R_z (*dashed*), and the reconstructed series (I_f, dotted) calculated by Equation (3). The correlation coefficients of I with R_z and I_f are $r_0 = 0.87$ and $r_f = 0.88$, respectively. The lag times of I (I_f) to R_z at their peaks for Cycles 21–23 are $L_1 = 31, 24, 17$ ($L_{f1} = 4, 8, 5$) months. (b) Similar results for the relationship between aa (*solid*) and I (*dashed*). The correlation coefficients of aa with I and the reconstructed series aa_f calculated by Equation (4) are $r_0 = 0.61$ and $r_f = 0.74$, respectively. The lag times of aa (aa_f) to I at their peaks for Cycles 21–23 are $L_2 = 4, 2, 23$ ($L_{f2} = 5, 4, 8$) months. (c) For the relationship between aa (*solid*) and R_z (*dashed*). The correlation coefficients of aa with R_z and the reconstructed series aa_f calculated by Equation (5) are $r_0 = 0.51$ and $r_f = 0.73$, respectively. The lag times of aa (aa_f) to R_z at their peaks for Cycles 21–23 are $L = 35, 26, 40$ ($L_f = 27, 29, 28$) months.

Table 1 Fitted Results of the Integral Response Model for the Flare Parameter: $P = I, T, Q, F$

y	x	t_0	D	τ	y_0	r_0	r_f	σ	$L_\mathrm{f}/L(21)$	$L_\mathrm{f}/L(22)$	$L_\mathrm{f}/L(23)$	$\overline{L}_\mathrm{f}/\overline{L}$ [a]
I	R_z	Mar. 1976	9.88×10^{-2}	4.9	-2.2	0.87	0.88	14.1	4/31	8/24	5/17	6/24
aa	I	Mar. 1976	1.12×10^{-2}	16.7	16.7	0.61	0.74	3.9	5/4	4/2	8/23	6/10
aa	R_z	Mar. 1976	4.79×10^{-3}	24.5	15.1	0.51	0.73	4.0	27/35	29/26	28/40	28/34
T	R_z	Mar. 1976	9.24×10^{-2}	14.2	41.7	0.79	0.89	29.6	22/21	27/21	25/22	25/21
aa	T	Mar. 1976	7.42×10^{-3}	8.8	14.0	0.66	0.74	4.0	7/14	6/5	2/18	5/12
aa	R_z	Mar. 1976	4.79×10^{-3}	24.5	15.1	0.51	0.73	4.0	27/35	29/26	28/40	28/34
Q	R_z	Jul. 1966	0.094	0.5	-1.3	0.90	0.90	2.3	0/15	0/–1	0/0	0/5
aa	Q	Jul. 1966	2.78×10^{-2}	27.7	19.1	0.37	0.58	3.9	18/20	29/27	24/40	24/29
aa	R_z	Jul. 1966	3.33×10^{-3}	36.9	15.0	0.32	0.64	3.7	31/35	31/26	30/40	31/34
F	R_z	Jul. 1997	3.50×10^{-5}	13.6	0.001	0.79	0.89	0.009	—	—	25/17	25/17
aa	F	Jul. 1997	229.4	0.3	14.0	0.75	0.75	4.4	—	—	0/23	0/23
aa	R_z	Jul. 1997	8.32×10^{-3}	18.6	12.3	0.62	0.79	4.0	—	—	26/40	26/40
Av.(P-R_z) [b]				8.3		0.84	0.89		9/22	12/15	14/14	14/17
Av.(aa-P)				13.4		0.60	0.70		10/13	3/11	9/26	9/19
Av.(aa-R_z)				26.1		0.49	0.72		28/35	30/26	28/40	28/36

[a] Average over Cycles 21–23. [b] Average of the corresponding parameters for the relationships between P ($= I, T, Q, F$) and R_z.

as shown in Figure 2(b): aa (solid), I (dashed), and the reconstructed series aa_f (dotted) calculated by Equation (4). One can see that aa_f well reflects the profile of aa. The correlation coefficient between aa and aa_f ($r_\mathrm{f} = 0.74$) is higher than that between aa and I ($r_0 = 0.61$). About half of the lag times of aa to I at their peaks for Cycles 21–23 ($L_2 = 4, 2, 23$ with a mean $\overline{L}_2 = 10$) can be predicted by Equation (4) as shown in Figure 2(b) for the corresponding ones in brackets ($L_\mathrm{f2} = 5, 4, 8$ with a mean $\overline{L}_\mathrm{f2} = 6$).

The relationship between $y = aa$ and $x = R_\mathrm{z}$ is analyzed by using the following equation,

$$aa(t) \;=\; D\sum_{t'=t_0}^{t} R_\mathrm{z}(t')\mathrm{e}^{-(t-t')/\tau} + aa_0\,. \tag{5}$$

Figure 2(c) illustrates aa (solid), R_z (dashed), and the reconstructed series aa_f (dotted) calculated by this equation. The correlation coefficient between aa and aa_f ($r_\mathrm{f} = 0.73$) is much higher than that between aa and R_z ($r_0 = 0.51$). The lag times of aa to R_z at their peaks for Cycles 21–23 ($L = 35, 26, 40$ with a mean $\overline{L} = 34$) can be well predicted by Equation (5) as shown in Figure 2(c) for the corresponding ones in brackets ($L_\mathrm{f} = 27, 29, 28$ with a mean $\overline{L}_\mathrm{f} = 28$). The above results are listed in Table 1, in which σ refers to the standard deviation, the last column indicates the relevant averages of fitted/observed lag times at the corresponding peaks over Cycles 21–23 ($\overline{L}_\mathrm{f}/\overline{L}$), and the last three rows represent the relevant averages of the parameters for the relationships between P-R_z, aa-P, and aa-R_z, respectively, where $P = I, T, Q, F$.

3.2 Relationship between R_z-T-aa

The relationship between R_z-T-aa since March 1976 (t_0) can also be analyzed by the technique in the previous section, with the results shown in Figure 3 and Table 1. The following can be noted.

(i) The correlation coefficient of T with the reconstructed series T_f ($r_\mathrm{f} = 0.89$) from R_z by Equation (2) is higher than that of T with R_z ($r_0 = 0.79$).
(ii) The correlation coefficient of aa with the reconstructed series aa_f ($r_\mathrm{f} = 0.74$) from T by Equation (2) is higher than that of aa with T ($r_0 = 0.66$).
(iii) The lag times of T to R_z ($\overline{L}_\mathrm{f1}/\overline{L}_1 = 25/21$), aa to T ($\overline{L}_\mathrm{f2}/\overline{L}_2 = 5/12$), and aa to R_z ($\overline{L}_\mathrm{f}/\overline{L} = 28/34$) at their peaks for Cycles 21–23 can be approximately predicted by the model.

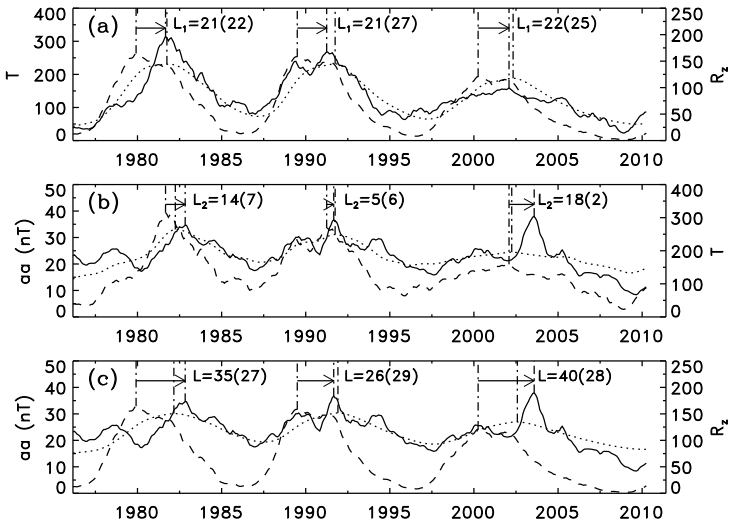

Fig. 3 Similar to Fig. 2 for the relationship between R_z-T-aa.

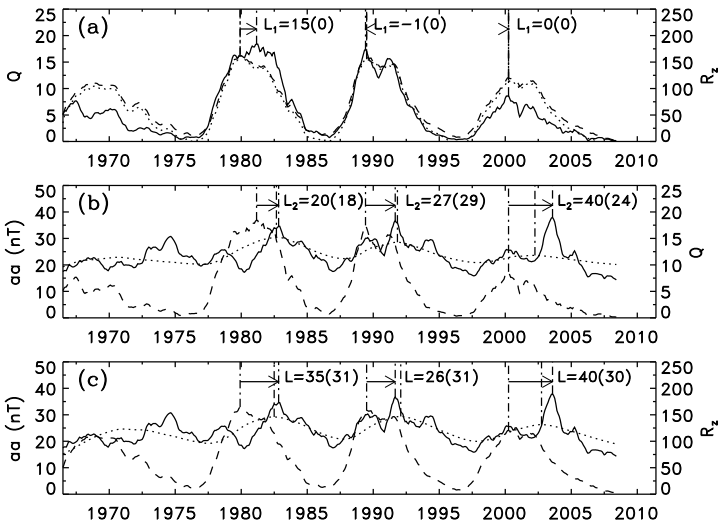

Fig. 4 Similar to Fig. 2 for the relationship between R_z-Q-aa.

3.3 Relationship between R_z-Q-aa

Figure 4 illustrates the relationship between R_z-Q-aa since July 1966 (t_0) by using the technique in Section 3.1. One can note the following.

(i) The correlation coefficient of Q with the reconstructed series Q_f ($r_f = 0.90$) from R_z by using Equation (2) has not improved in comparison to that of Q with R_z ($r_0 = 0.90$), implying that Q and R_z peak nearly at the same time (Kleczek 1952).

(ii) The correlation coefficient of aa with the reconstructed series aa_f ($r_f = 0.58$) from Q by using Equation (2) is much higher than that of aa with Q ($r_0 = 0.37$).

(iii) The correlation coefficient of aa with the reconstructed series $aa_{\rm f}$ ($r_{\rm f} = 0.64$) from $R_{\rm z}$ by using Equation (2) is much higher than that of aa with $R_{\rm z}$ ($r_0 = 0.32$).
(iv) The lag times of Q to $R_{\rm z}$ ($\overline{L}_{\rm f1}/\overline{L}_1 = 0/5$), aa to Q ($\overline{L}_{\rm f2}/\overline{L}_2 = 24/29$), and aa to $R_{\rm z}$ ($\overline{L}_{\rm f}/\overline{L} = 31/34$) at their peaks for Cycles 21–23 can be predicted in part by Equation (2).

3.4 Relationship between $R_{\rm z}$-F-aa

Figure 5 shows the relationship between $R_{\rm z}$-F-aa since July 1997 (t_0) using the technique in Section 3.1. One finds the following.

(i) The correlation coefficient of F with the reconstructed series $F_{\rm f}$ ($r_{\rm f}$ = 0.89) from $R_{\rm z}$ by using Equation (2) is higher than that of F with $R_{\rm z}$ (r_0 = 0.79).
(ii) The correlation coefficient of aa with the reconstructed series $aa_{\rm f}$ ($r_{\rm f}$ = 0.75) from F by using Equation (2) is equal to that of aa with F (r_0 = 0.75).
(iii) The correlation coefficient of aa with the reconstructed series $aa_{\rm f}$ ($r_{\rm f}$ = 0.79) from $R_{\rm z}$ by using Equation (2) is much higher than that of aa with $R_{\rm z}$ (r_0 = 0.62).
(iv) The lag times of F to $R_{\rm z}$ ($L_{\rm f1}/L_1 = 25/17$), and aa to $R_{\rm z}$ ($L_{\rm f}/L = 26/40$) at their peaks for Cycle 23 can be predicted in part by Equation (2). By contrast, the lag time of aa to F ($L_{\rm f2}/L_2 = 0/23$) at their peaks for Cycle 23 has not been predicted by Equation (2) due to the great fluctuations in both aa and F.

These results imply that solar flares depend not only on the present but also on past solar activities ($R_{\rm z}$), reflecting the long-term evolutional characteristics of solar magnetic field structures (energy) evolving from the photosphere to the upper chromosphere (Donnelly 1987; Zhang et al. 2007; Lin 2009). The correlations between solar flare parameters and $R_{\rm z}$ are not simply due to the time shifts (Bachmann & White 1994). Solar flares may play a role for the formation of the geomagnetic activity (aa) from the solar (magnetic field) activity ($R_{\rm z}$), although the processes are not completely clear (Cliver & Hudson 2002).

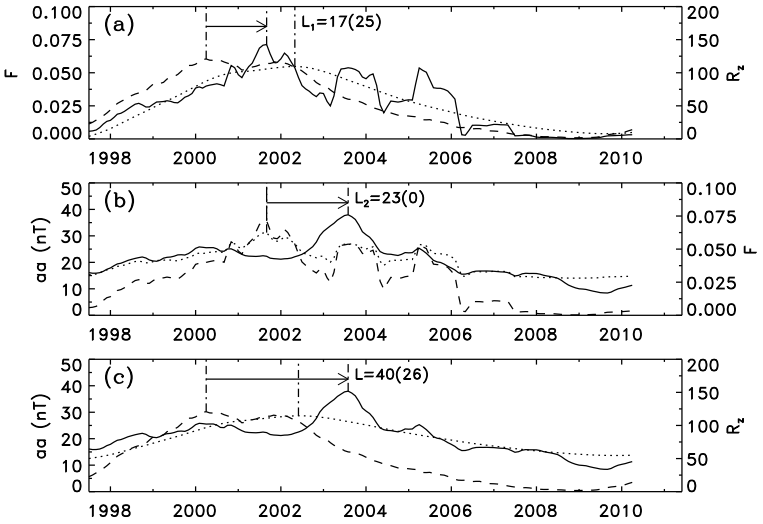

Fig. 5 Similar to Fig. 2 for the relationship between $R_{\rm z}$-F-aa.

4 DISCUSSION AND CONCLUSIONS

In this study, we investigated the relationships between the solar flare parameters ($P = I, T, Q$ and F) and sunspot activity (R_z), and between geomagnetic activity (aa) and the flare parameters via the integral response model (Equation (2)). The results indicate that (i) the correlation coefficients between the flare parameters and R_z have increased about 6% from $\overline{r}_0 = 0.84$ to $\overline{r}_f = 0.89$ on average when using Equation (2) and the time delays at their peaks for Cycles 21–23 can be well predicted by this model, $\overline{L}_{f1}/\overline{L}_1 = 14/17 = 82\%$; (ii) the correlation coefficients between aa and the flare parameters have increased about 17% from $\overline{r}_0 = 0.60$ to $\overline{r}_f = 0.70$ on average when using Equation (2) and half of the time delays at their peaks can be predicted by this model, $\overline{L}_{f2}/\overline{L}_2 = 9/19 = 47\%$; and (iii) the correlation coefficient between aa and R_z has increased about 47% from $\overline{r}_0 = 0.49$ to $\overline{r}_f = 0.72$ on average when using Equation (2) and the time delays at their peaks can be well predicted by this model, $\overline{L}_f/\overline{L} = 28/36 = 78\%$. This model might be used to improve the solar flare prediction, which should be studied in the future.

It is seen in Figure 2(a) and Table 1 that the time delays between I and R_z at their peaks for Cycles 21–23 have not been well predicted by the model (6/24 = 25%). This is due to the large fluctuations in the data. To suppress the fluctuations further, we introduce a cosine filter with the weights given by

$$W_C(\Delta t) = \frac{\pi}{4b}\cos\left(\frac{\pi\Delta t}{2b}\right) \tag{6}$$

for $b = 24$ months. Since $\int_{-b}^{b}\frac{\pi}{4b}\cos(\frac{\pi\Delta t}{2b}) = 1$, the weights $W_C(\Delta t)$ are normalized. Using the series smoothed by this filter, we re-analyze the results in Figure 2, as shown in Figure 6.

The time delays between I and R_z at their peaks are now better predicted, $L_{1f}/L_1 = 5/15$, 5/1 and 4/3 for Cycles 21–23 (Fig. 6(a)), respectively, with a mean of $\overline{L}_{1f}/\overline{L}_1$ = 4.7/6.3 = 74% which is much higher than the original one (6/24 = 25%). The time delays between aa and I at their peaks are also better predicted, $L_{2f}/L_2 = 11/16$, 13/13 and 19/27 for Cycles 21–23 (Fig. 6(b)), respectively, with a mean of $\overline{L}_{2f}/\overline{L}_2$ = 14.3/18.7 = 77% which is higher than the original one (6/10 = 60%). In Figure 6(c), the time delays between aa and R_z at their peaks are predicted as $L_f/L = 19/31$,

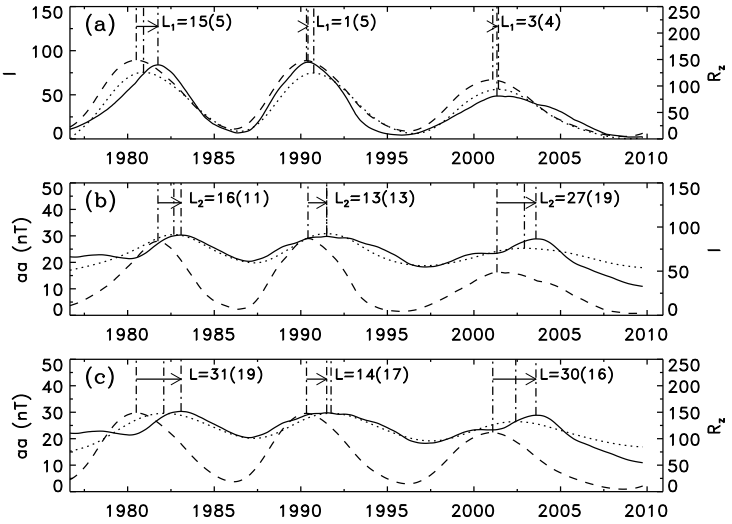

Fig. 6 Same as Fig. 2 but using the cosine filter (Eq. (6)).

17/14 and 16/30 for Cycles 21–23, respectively, with a mean of $\overline{L}_{\rm f}/\overline{L} = 17.3/25 = 69\%$ which is smaller than the original one (28/34 = 82%) due to the great lag time of aa compared to $R_{\rm z}$ (about 30 months) and other sources of aa.

In Equation (2), the output y depends on the past values of input x ($\tau > 0$) rather than only the current value ($\tau = 0$). The stronger the input (x) is, the more it contributes to the output (y), and the longer the lag time of y to x will be (Du 2011c). Therefore, solar flares are related to the accumulation of solar magnetic energy in the past rather than the simple time shifts of occurrences (Bachmann & White 1994). The average response time scale of flare parameters to $R_{\rm z}$ in this model ($\overline{\tau} = 8$) is close to the coronal response time (~ 10 months) derived from a model for dynamical energy balance in the flaring solar corona (Wheatland & Litvinenko 2001; Litvinenko & Wheatland 2004).

There are various types of active regions in a solar cycle. Small active regions with simple magnetic structure are short-lived and produce minor solar flares, while large active regions with complex magnetic structure are long-lived and produce major solar flares (and hard X-ray flares). It is shown in Figure 1(a) that T is well correlated with I ($r = 0.89$), with the regression equation given by

$$T = 67.9 \pm 2.2 + (1.94 \pm 0.05) I \,. \tag{7}$$

According to the above discussions, minor (low-energy) solar flares lag behind the input $R_{\rm z}$ shorter times with shorter durations while major solar flares lag behind $R_{\rm z}$ longer times with longer durations. Therefore, (i) the time delays between flare activities and sunspot activity come mainly from the major flares rather than the weak ones; (ii) major flares tend to have longer durations and may occur until quite late in the decay phase of a solar cycle (Temmer et al. 2003; Tan 2011); and (iii) the upper chromospheric activity indices (Donnelly 1987; Bachmann & White 1994) and the solar flares (Wheatland & Litvinenko 2001; Temmer et al. 2003) tend to lag behind the sunspot number by several months in a hierarchical manner (Bachmann & White 1994).

Although it is unclear how solar flares affect geomagnetic activities (Gosling 1993; Cliver & Hudson 2002), it is apparent that geomagnetic activities are well correlated with the solar flares. For example, aa is well correlated with F ($r_0 = 0.75$). As flares are unable to travel to 1 AU, streams of matter emanating from large flares were considered as the prime cause of geomagnetic storms (Hale 1931; Chapman 1950; Pudovkin et al. 1977). However, Gosling (1993) argued that CMEs, not flares, were the critical element for large geomagnetic storms, interplanetary shocks, and major SEP events. In fact, solar flares may affect geomagnetic activities via different processes related to flare brightening, eruption, particle ejections, and other unknown effects (Cliver & Hudson 2002). Therefore, the relationships between geomagnetic activity (aa) and solar flares can also be well described by Equation (2). Since the geomagnetic activity (aa) can result from various activity phenomena (Legrand & Simon 1989; Tsurutani et al. 1995), it is the integral of the effects of all these phenomena, including solar winds, CMEs, solar flares and others. The lag time of aa to solar flares has not been well predicted by the model (9/19) due to the additional effects of other activities. By contrast, the lag times of both solar flares and aa to $R_{\rm z}$ at their peaks have been well predicted by the model (14/17, 28/36) because the solar magnetic field activity is their main source.

The main conclusions can be drawn as follows:

(i) The relationships between the flare parameters ($P = I, T, Q, F$) and sunspot activity ($R_{\rm z}$) can be well described by an integral response model ($r = 0.89$) with a mean response time scale of about eight months. The time delays between the flare parameters and $R_{\rm z}$ at their peaks can be well predicted by this model (82%).

(ii) The relationships between geomagnetic activity (aa) and the flare parameters can be better described by this model ($r = 0.70$), with a mean response time scale of 13 months, than by a linear dependence ($r = 0.60$). Half of the time delays between aa and the flare parameters at their peaks can be predicted by this model (47%).

(iii) The relationship between aa and R_z can be much better described by this model ($r = 0.72$) with a mean response time scale of about 26 months than by a linear dependence ($r = 0.49$). Part of the time delay between aa and R_z at their peaks can be predicted by this model (78%).

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