# A new photometric study of the triple star system EF Draconis * 

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#### Abstract

We present new charge-coupled device (CCD) photometry for the triple star EF Draconis, obtained in 2009 and 2011. Using the updated Wilson-Devinney program, the photometric solutions were deduced from two sets of light curves. The results indicate that EF Dra is an A-type W UMa binary with a contact degree of $f=46.7 \%( \pm 0.6 \%)$ and a third light of $l_{3} \simeq 1.5 \%$. Through analyzing the $O-$ $C$ curve, it is found that the orbital period shows a long-time increase with a lighttime orbit. The period, semi-amplitude and eccentricity of the third body are $P_{\bmod }=$ $17.20( \pm 0.18) \mathrm{yr}, A=0.0039^{\mathrm{d}}\left( \pm 0.0002^{\mathrm{d}}\right)$ and $e=0.49( \pm 0.02)$ respectively. This kind of tertiary companion may extract angular momentum from the central system. The orbital period of EF Dra secularly increases at a rate of $d P / d t=+3.72( \pm 0.07) \times$ $10^{-7} \mathrm{~d} \mathrm{yr}^{-1}$, which may be interpreted by mass transfer from the less massive to the more massive component. As period increases, the separation between components may increase, which will cause the contact degree to decrease. With mass transferring, the spin angular momentum will increase, while the orbital angular momentum will decrease. Only if the contact configuration would merge at $J_{\text {spin }}>\frac{1}{3} J_{\text {orb }}$ could this kind of deep-contact binary with period increasing, such as EF Dra, evolve into a rapidly-rotating single star.


Key words: stars (including multiple): binaries: close - stars: individual (EF Draconis)

## 1 INTRODUCTION

Formation of binaries is a fascinating subject (Zinnecker \& Mathieu 2001). The difficulties of this complex process have been recently reviewed by many authors (i.e. Kondo et al. 2002; Tohline 2002; Bate 2004). Studies of close binary stars can provide some key information for the thermal relaxation oscillation and angular momentum loss (e.g., Robertson \& Eggleton 1977). Most close binary stars exist in multiple systems, although this hypothesis may be further confirmed by observations and statistical investigations.

Table 1 tabulates 35 contact triple or multiple binaries, including their spectroscopic parameters, which are observed at the David Dunlap Observatory (DDO). This kind of additional companion may remove angular momentum from the central system via Kozai oscillation (Kozai 1962) or a

[^0]Table 1 Spectroscopic Orbital Elements for Some Tertiary or Quadruple components of W UMatype Binaries

| No. | Star Name | Spectral | $q_{\text {sp }}$ | Period (d) | $\left(M_{1}+M_{2}\right) \sin ^{3} i$ | Ref. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\mathrm{BD}+7^{\circ} 3142$ | K2V | 0.662(8) | 0.275277 | 1.050(14) | [1] |
| 2 | V410 Aur | G0/2V | 0.144(13) | 0.366340 | $1.42(10)$ | [2] |
| 3 | 44i Boo | K2V | 0.487(6) | 0.267818 | $1.132(11)$ | [3] |
| 4 | EL Boo | K1/2V | 0.248(7) | 0.413772 | $1.448(28)$ | [4] |
| 5 | TZ Boo | F/G5 | 0.207(5) | 0.297160 | 1.118(18) | [4] |
| 6 | EE Cet | F8V | 0.315(5) | 0.379917 | $1.706(41)$ | [5] |
| 7 | KR Com | G0IV | 0.091(2) | 0.407968 | 0.517 (8) | [5] |
| 8 | V401 Cyg | F0V | 0.290(11) | 0.582714 | 2.008(130) | [5] |
| 9 | EF Dra | F8/9V | 0.160(14) | 0.424000 | 1.970(124) | [6] |
| 10 | CT Eri | F2/3V | 0.30(9) | 0.634200 | - | [7] |
| 11 | V345 Gem | F7V | 0.142(3) | 0.274769 | 1.054(13) | [8] |
| 12 | V899 Her | F5 | 0.566(18) | 0.421173 | 2.331(77) | [3] |
| 13 | SZ Hor | F3V | 0.47(4) | 0.625118 | - | [7] |
| 14 | CE Hyi | F5V | - | 0.4408 | - | [9] |
| 15 | VZ Lib | G0 | 0.237(68) | 0.358263 | 1.704(120) | [3] |
| 16 | AM Leo | F5V | 0.459(4) | 0.365799 | 1.882(12) | [8] |
| 17 | XY Leo | K0V | 0.729(7) | 0.284098 | $1.188(12)$ | [8] |
| 18 | VW LMi | F5V | 0.416(4) | 0.477550 | 2.282(18) | [10] |
| 19 | DE Oct | A9IV | - | 0.555592 | - | [9] |
| 20 | V2388 Oph | F3V | 0.186(2) | 0.802298 | 1.926(20) | [5] |
| 21 | V2610 Oph | F8/G2V | 0.291(9) | 0.426512 | 1.441 (31) | [4] |
| 22 | ER Ori | F7/8 | 0.656(12) | 0.423403 | 2.285(45) | [11] |
| 23 | V1387 Ori | A4V | 0.165(5) | 0.730166 | 0.969(16) | [4] |
| 24 | V592 Per | F2IV | 0.408(7) | 0.715722 | 2.521(72) | [12] |
| 25 | HI Pup | F6V | 0.19(6) | 0.432616 | - | [7] |
| 26 | Y Sex | F5/6 | 0.195(8) | 0.419820 | 1.663(45) | [11] |
| 27 | II UMa | F5III | 0.172(4) | 0.825220 | 2.180(80) | [5] |
| 28 | FT UMa | K0V | 0.984(19) | 0.654704 | $2.077(10)$ | [4] |
| 29 | HX UMa | F4V | 0.291(9) | 0.379156 | 0.775 (30) | [2] |
| 30 | TU UMi | F2 | 0.16(7) | 0.377088 | $0.65(27)$ | [11] |
| 31 | TV UMi | F8V | 0.739(21) | 0.415549 | 0.879(12) | [10] |
| 32 | AG Vir | A5V | 0.382(21) | 0.642651 | 2.563(41) | [10] |
| 33 | HT Vir | F8V | 0.812(8) | 0.407670 | 2.285(38) | [3] |
| 34 | PY Vir | K1/2V | $0.773(5)$ | 0.311251 | 1.387(10) | [1] |
| 35 | GSC 1387-0475 | K3/5V | 0.474(8) | 0.217811 | 0.288(8) | [13] |

Notes: The values of $\left(M_{1}+M_{2}\right) \sin ^{3} i$ are in solar mass units, while the period is in days. The standard errors in the table are expressed in units of the last decimal places quoted, which are given in parentheses after each value.
Reference: [1] Rucinski et al. (2008); [2] Rucinski et al. (2003); [3] Rucinski et al. (2001); [4] Pribulla et al. (2009b); [5] Rucinski et al. (2002); [6] Lu \& Rucinski (1999); [7] Duerbeck \& Rucinski (2007); [8] Pribulla et al. (2007); [9] Rucinski \& Duerbeck (2006); [10] Pribulla et al. (2006); [11] Pribulla et al. (2009a); [12] Rucinski et al. (2005); [13] Rucinski \& Pribulla (2008).
combination of the Kozai cycle and tidal friction (e.g., Fabrycky \& Tremaine 2007). Therefore, this will result in a low angular momentum remnant in the close binary.

EF Dra ( $=1 E 1806.1+6944$ ) was discovered as an X-ray source in the Einstein Observatory Extended Medium Sensitivity Survey (Gioia et al. 1987). Based on X-ray observations and three radial velocity measurements, Fleming et al. (1989) suggested that it is a W UMa-type binary, with a distance of $d=153.5 \mathrm{pc}$ and a soft-X-ray flux of $f_{\mathrm{X}}=1.38$. They also gave its spectral type as F9, which was recently revised to be F8/9V (Pribulla et al. 2009a). Robb \& Scarfe (1989) published $V R I$ observations with large scatter and improved the period value to $0.42400^{\text {d }}$. Kazarovets \& Samus (1990) designated it as EF Dra and cataloged it as an EW/KW-type binary. Plewa et al. (1991) obtained incomplete light curves and derived a mass ratio of $q_{\mathrm{ph}}=0.125$. Wunder \& Agerer
(1992) also photoelectrically observed this star and revised its period to be $0.42402394^{\text {d }}$. Pribulla et al. (2001a) published $B V$ light curves of EF Dra and determined its absolute parameters. The orbital inclination and fill-out factor are $i=78.13^{\circ}\left( \pm 0.33^{\circ}\right)$ and $f=45.5 \%( \pm 2.3 \%)$, respectively. Based on 35 light minimum times only spanning 11 yr , they derived a parabolic ephemeris superimposed by a light time orbit, whose period and amplitude are $P_{3}=1360( \pm 150) \mathrm{d}$ and $A=0.002^{\mathrm{d}}$, respectively. Alternatively, they pointed out that the orbital period increase may be a part of a light time orbit due to another companion with a period of at least 12 yr . Therefore, one cannot decide which case is true due to the small number of observations with large scatters. Based on 43 spectra observed in 1991 and 1992, Lu (1993) concluded that EF Dra is a triple star system. Then Lu \& Rucinski (1999) published 16 high dispersion spectra. The radial velocity of $-38 \mathrm{~km} \mathrm{~s}^{-1}$ for the third companion is very close to the systemic velocity of $-42 \mathrm{~km} \mathrm{~s}^{-1}$ for the eclipsing pair. Therefore, EF Dra with a physical companion is an A-type W UMa star with an extremely low mass ratio of $q_{\mathrm{sp}}=0.160( \pm 0.014)$. Due to the extremely low mass ratio and high fill-out factor, the triple star system EF Dra was included in our observing program in order to study the nature of period changes and its evolutionary status.

## 2 OBSERVATIONS AND REDUCTIONS

The multicolor photometry of the binary EF Dra was carried out using the $60-\mathrm{cm}$ telescope in 2009 and the $85-\mathrm{cm}$ telescope (Zhou et al. 2009) in 2011 at the Xinglong Station of National Astronomical Observatories of China (NAOC). Both telescopes were equipped with the standard Johnson-CousinBessel $U B V R_{c} I_{c}$ systems. Image reductions were done using the Image Reduction (IMRED) and Aperture Photometry (APPHOT) packages in the Image Reduction and Analysis Facility (IRAF) in a standard fashion.

On 2009 September 12 and 28, and October 13, EF Dra were observed using the $60-\mathrm{cm}$ telescope. In the field of EF Dra, the nearest star GSC 4433-0827 ( $10.6^{\mathrm{m}}$ in the $V$ band) has been identified to be a multi-periodic $\delta$ Scuti star (Yang et al. 2011). Fainter comparison and check stars (i.e. C 1 and Ch 1 ) were chosen, and are listed in Table 2. Typical exposure times for $B V R$ bands were $45 \mathrm{~s}, 30 \mathrm{~s}$ and 20 s . We obtained a total of 403,444 and 407 images in $B V R$ bands. The light curves (i.e. $\mathrm{LC}_{1}$ ) are displayed in the left panel of Figure 1, where there exists a short gap near the secondary eclipse. The phases were computed with a period of $0.42403^{\text {d }}$ (Kreiner 2004). The mean differences between the magnitude of the check star and that of the comparison one are very large,


Fig. $1 B V R$ light curves of the eclipsing binary EF Dra, observed in 2009 (left panel) and 2011 (right panel). The solid lines were constructed by new photometric solutions.

Table 2 Coordinates of the Variable, Comparison Stars and Check Stars

| Star | $\alpha_{\text {J2000.0 }}$ | $\delta_{\text {J2000.0 }}$ | $\operatorname{mag}(\mathrm{V})$ | Telescope |
| :--- | :---: | :---: | :---: | :---: |
| EF Dra | $18: 05: 30.35$ | $+69: 45: 15.4$ | 10.6 |  |
| C1 | $18: 06: 08.52$ | $+69: 43: 42.6$ | 12.5 | $60-\mathrm{cm}$ |
| C2 | $18: 04: 22.68$ | $+69: 52: 47.7$ | 11.2 | $85-\mathrm{cm}$ |
| Ch1 | $18: 06: 53.70$ | $+69: 45: 35.7$ | 12.5 | $60-\mathrm{cm}$ |
| Ch2 | $18: 05: 07.36$ | $+69: 55: 22.2$ | 12.0 | $85-\mathrm{cm}$ |

up to $0.01^{\mathrm{m}}$, which may result from the faint comparison star because the nearest star GSC 44330827 is identified to be variable. Using the $85-\mathrm{cm}$ telescope at the Xinglong station of NAOC, we again observed this binary on three consecutive nights of 2011 May 30 and 31, and June 1. Another two stars were used as the comparison and check stars (i.e. C2 and Ch2), whose coordinates are also listed in Table 2. The exposure times were adopted to be $20 \mathrm{~s}, 15 \mathrm{~s}$ and 10 s for $B V R$ bands, respectively. A set of complete light curves (i.e. $\mathrm{LC}_{2}$ ), including 565,564 and 548 observations in $B V R$ bands, is shown in the right panel of Figure 1. The precisions of individual points do not exceed $0.005^{\mathrm{m}}$. All observations of two sets of light curves, i.e. the magnitude differences in the sense of the variable minus the comparison star together with their heliocentric Julian dates, are available upon request. From Figure 1, EF Dra is an A-type total eclipsing binary, with a duration of totality being about 50 minutes. The primary eclipse is deeper than the secondary eclipse by up to $0.02^{\mathrm{m}}$. The amplitudes of variable light are $0.33^{\mathrm{m}}, 0.31^{\mathrm{m}}$ and $0.30^{\mathrm{m}}$ for $B V R$ bands respectively, which agree with the value of $\Delta V \approx 0.32^{\mathrm{m}}$ (Robb \& Scarfe 1989). Moreover, EF Dra was observed for other nights in 2009 and 2011 in order to measure its eclipse times. From the new observations, ten light minimum timings were determined by the K-W method (Kwee \& van Woerden 1956). Those individual light minimum times, together with their errors, are listed in Table 3.

## 3 ANALYSIS OF PERIOD CHANGES

Based on a 12-year eclipsing cycle of EF Dra, Pribulla et al. (2001a) suggested that there exists a light-time orbit with a period of $P_{3} \approx 3.5 \mathrm{yr}$ or $P_{3}>12 \mathrm{yr}$. In order to study orbital period changes, we collected a total of 93 light minimum times ( 5 visual, 50 photoelectric, 38 CCD measurements).

Table 4 tabulates those compiled eclipsing times. In Col. (3) of this table, "vi," "pe" and "CCD" refer to visual observations, photoelectric measurements, and charge-coupled device, respectively. According to various measurement precisions, weights of 1 and 10 are assigned to visual, and photoelectric or CCD measurements, respectively. Using the linear ephemeris (Kreiner 2004),

$$
\begin{equation*}
\text { Min.I }=\text { HJD } 2452500.3076+0.42403 \times E, \tag{1}
\end{equation*}
$$

we can calculate the residuals of $(O-C)_{1}$ for all light minimum times, which are listed in Col. (6) of Table 4, and are displayed in Figure 2.

The general trend of the $(O-C)_{1}$ curve can be obviously described by an upward parabolic curve, indicating that the orbital period may be increasing. In the calculation process, weights 1 and 10 are assigned to visual, and photoelectric or CCD measurements, respectively. A linear fitting method yielded the following quadratic ephemeris,

$$
\begin{equation*}
\text { Min. } \mathrm{I}=\text { HJD } 2452500.3086(2)+0.42402945(3) \times E+3.16(4) \times 10^{-10} \times E^{2}, \tag{2}
\end{equation*}
$$

where the parentheses contain the standard errors in units of the last decimal place. Equation (2) is plotted in Figure 2 as a solid line. With the coefficient of the quadratic term, a continuous period increase rate of $d P / d t=+3.72( \pm 0.07) \times 10^{-7} \mathrm{~d} \mathrm{yr}^{-1}\left(+3.22( \pm 0.06) \mathrm{s}\right.$ century $\left.{ }^{-1}\right)$ can be obtained. The computed residuals of $(O-C)_{2}$ are listed in Col. (7) of Table 4. The $(O-C)_{2}$ curve is shown in the upper section of top panel of Figure 3, whose shape appears to have a quasi-sinusoidal

Table 3 New Obtained Light Minimum Times of EF Dra

| JD (Hel.) | Error | Min. | Filter | Telescope |
| :--- | :--- | :--- | :--- | :--- |
| 2455087.32057 | $\pm 0.00050$ | I | $B$ | $60-\mathrm{cm}$ |
| 2455087.32033 | $\pm 0.00057$ | I | $V$ | $60-\mathrm{cm}$ |
| 2455087.32032 | $\pm 0.00064$ | I | $R$ | $60-\mathrm{cm}$ |
| 2455103.00713 | $\pm 0.00052$ | I | $B$ | $60-\mathrm{cm}$ |
| 2455103.01080 | $\pm 0.00059$ | I | $V$ | $60-\mathrm{cm}$ |
| 2455103.01116 | $\pm 0.00045$ | I | $R$ | $60-\mathrm{cm}$ |
| 2455118.06499 | $\pm 0.00099$ | II | $B$ | $60-\mathrm{cm}$ |
| 2455118.06542 | $\pm 0.00123$ | II | $V$ | $60-\mathrm{cm}$ |
| 2455118.06587 | $\pm 0.00129$ | II | $R$ | $60-\mathrm{cm}$ |
| 2455165.98809 | $\pm 0.00103$ | II | $B$ | $60-\mathrm{cm}$ |
| 2455165.98378 | $\pm 0.00070$ | II | $V$ | $60-\mathrm{cm}$ |
| 2455165.98665 | $\pm 0.00080$ | II | $R$ | $60-\mathrm{cm}$ |
| 2455675.24953 | $\pm 0.00033$ | II | $B$ | $85-\mathrm{cm}$ |
| 2455675.25148 | $\pm 0.00034$ | II | $V$ | $85-\mathrm{cm}$ |
| 2455675.25180 | $\pm 0.00036$ | II | $R$ | $85-\mathrm{cm}$ |
| 2455678.21810 | $\pm 0.00030$ | II | $B$ | $85-\mathrm{cm}$ |
| 2455678.21799 | $\pm 0.00025$ | II | $V$ | $85-\mathrm{cm}$ |
| 2455678.21812 | $\pm 0.00029$ | II | $R$ | $85-\mathrm{cm}$ |
| 2455709.17140 | $\pm 0.00005$ | II | $B$ | $85-\mathrm{cm}$ |
| 2455709.17384 | $\pm 0.00062$ | II | $V$ | $85-\mathrm{cm}$ |
| 2455709.17334 | $\pm 0.00058$ | II | $R$ | $85-\mathrm{cm}$ |
| 2455712.14253 | $\pm 0.00026$ | II | $B$ | $85-\mathrm{cm}$ |
| 2455712.14241 | $\pm 0.00023$ | II | $V$ | $85-\mathrm{cm}$ |
| 2455712.14211 | $\pm 0.00029$ | II | $R$ | $85-\mathrm{cm}$ |
| 2455713.20105 | $\pm 0.00013$ | I | $B$ | $85-\mathrm{cm}$ |
| 2455713.20110 | $\pm 0.00012$ | I | $V$ | $85-\mathrm{cm}$ |
| 2455713.20118 | $\pm 0.00016$ | I | $R$ | $85-\mathrm{cm}$ |
| 2455714.26222 | $\pm 0.00049$ | II | $B$ | $85-\mathrm{cm}$ |
| 2455714.26186 | $\pm 0.00061$ | II | $V$ | $85-\mathrm{cm}$ |
| 2455714.26180 | $\pm 0.00058$ | II | $R$ | $85-\mathrm{cm}$ |
|  |  |  |  |  |
|  |  |  |  |  |



Fig. $2(O-C)_{1}$ diagram of EF Dra. The open and filled circles refer to visual observations, and photoelectric and CCD measurements, respectively. The solid line was constructed by Eq. (2).

Table 4 All Available Times of Light Minimum for EF Dra

| JD (Hel.) | Method | Epoch | Min | $(O-C)_{1}$ | $(O-C)_{2}$ | $(O-C)_{3}$ | Ref. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2447701.8201 | pe | -11316.5 | II | $+0.0480$ | +0.0003 | +0.0019 | [1] |
| 2447701.8201 | pe | -11316.5 | II | +0.0480 | +0.0003 | +0.0019 | [1] |
| 2447715.8125 | pe | -11283.5 | II | +0.0474 | +0.0000 | +0.0016 | [1] |
| 2447716.8710 | pe | -11281.0 | I | +0.0458 | -0.0016 | +0.0000 | [1] |
| 2447727.8971 | pe | -11255.0 | I | +0.0472 | +0.0000 | +0.0016 | [1] |
| 2447730.8650 | pe | -11248.0 | I | +0.0468 | -0.0003 | +0.0013 | [1] |
| 2447734.8902 | pe | -11238.5 | II | +0.0438 | -0.0033 | -0.0017 | [1] |
| 2447752.4872 | pe | -11197.0 | 1 | $+0.0435$ | -0.0032 | -0.0016 | [2] |
| 2447759.4876 | pe | -11180.5 | II | $+0.0474$ | $+0.0008$ | $+0.0024$ | [2] |
| 2447760.3305 | pe | -11178.5 | II | $+0.0423$ | -0.0043 | -0.0027 | [2] |
| 2447763.5069 | pe | -11171.0 | I | $+0.0384$ | -0.0081 | -0.0065 | [2] |
| 2447792.3481 | pe | -11103.0 | I | $+0.0456$ | -0.0004 | $+0.0012$ | [2] |
| 2447820.3334 | pe | -11037.0 | I | +0.0449 | -0.0006 | +0.0009 | [2] |
| 2448467.3956 | pe | -9511.0 | I | +0.0373 | +0.0025 | +0.0029 | [3] |
| 2448475.4498 | pe | -9492.0 | I | +0.0350 | +0.0003 | $+0.0006$ | [4] |
| 2448488.3835 | pe | -9461.5 | II | +0.0357 | +0.0012 | +0.0015 | [4] |
| 2448491.5633 | pe | -9454.0 | I | +0.0353 | +0.0009 | $+0.0012$ | [4] |
| 2448502.3729 | pe | -9428.5 | II | +0.0322 | $-0.0021$ | -0.0018 | [3] |
| 2448624.2819 | pe | -9141.0 | 1 | +0.0325 | $+0.0001$ | $+0.0001$ | [3] |
| 2448775.4460 | pe | -8784.5 | II | +0.0299 | $-0.0003$ | -0.0008 | [3] |
| 2448909.4420 | pe | -8468.5 | II | $+0.0325$ | $+0.0042$ | $+0.0033$ | [3] |
| 2449132.4789 | pe | -7942.5 | II | $+0.0296$ | $+0.0043$ | $+0.0026$ | [5] |
| 2449132.4793 | pe | -7942.5 | II | $+0.0300$ | $+0.0047$ | $+0.0030$ | [6] |
| 2449416.5707 | pe | -7272.5 | II | $+0.0213$ | -0.0004 | $-0.0033$ | [7] |
| 2449465.5495 | pe | -7157.0 | I | $+0.0246$ | $+0.0035$ | $+0.0004$ | [7] |
| 2449580.4600 | pe | -6886.0 | I | $+0.0230$ | +0.0032 | -0.0004 | [8] |
| 2449763.6375 | pe | -6454.0 | I | +0.0195 | +0.0018 | $-0.0027$ | [9] |
| 2449995.3760 | vi | -5907.5 | II | +0.0256 | +0.0103 | $+0.0048$ | [9] |
| 2449999.4050 | vi | -5898.0 | I | +0.0263 | +0.0111 | +0.0055 | [9] |
| 2450180.4570 | vi | -5471.0 | I | +0.0175 | $+0.0040$ | -0.0020 | [9] |
| 2450189.5730 | vi | -5449.5 | II | +0.0169 | $+0.0035$ | -0.0025 | [9] |
| 2450195.5050 | vi | -5435.5 | II | +0.0125 | $-0.0008$ | -0.0068 | [9] |
| 2450301.5187 | pe | -5185.5 | II | $+0.0187$ | $+0.0063$ | $+0.0003$ | [10] |
| 2450571.4095 | pe | -4549.0 | 1 | $+0.0144$ | $+0.0043$ | -0.0010 | [11] |
| 2451331.4810 | pe | -2756.5 | II | $+0.0121$ | $+0.0072$ | $+0.0049$ | [12] |
| 2451680.4526 | pe | -1933.5 | II | $+0.0070$ | +0.0037 | $+0.0026$ | [13] |
| 2451705.4590 | CCD | -1874.5 | II | -0.0044 | $-0.0076$ | $-0.0087$ | [14] |
| 2451789.4263 | pe | -1676.5 | II | $+0.0050$ | $+0.0022$ | $+0.0014$ | [15] |
| 2451796.4220 | pe | -1660.0 | I | $+0.0042$ | $+0.0014$ | $+0.0006$ | [15] |
| 2451798.3308 | pe | -1655.5 | II | $+0.0049$ | $+0.0021$ | $+0.0013$ | [15] |
| 2451968.5769 | pe | -1254.0 | I | $+0.0029$ | +0.0007 | $+0.0004$ | [16] |
| 2452031.3346 | pe | -1106.0 | I | $+0.0042$ | +0.0022 | $+0.0021$ | [17] |
| 2452041.5040 | CCD | -1082.0 | I | -0.0031 | -0.0051 | -0.0052 | [18] |
| 2452139.4607 | pe | -851.0 | 1 | $+0.0026$ | +0.0009 | $+0.0011$ | [17] |
| 2452189.2824 | pe | -733.5 | II | $+0.0008$ | -0.0008 | -0.0005 | [17] |
| 2452203.2757 | pe | -700.5 | II | $+0.0011$ | $-0.0005$ | -0.0002 | [17] |
| 2452252.2516 | pe | -585.0 | I | +0.0016 | +0.0001 | $+0.0006$ | [17] |
| 2452277.6938 | CCD | -525.0 | I | $+0.0020$ | $+0.0006$ | $+0.0011$ | [19] |
| 2452321.5799 | pe | -421.5 | II | $+0.0009$ | -0.0004 | $+0.0002$ | [17] |
| 2452348.5043 | CCD | -358.0 | I | -0.0006 | -0.0019 | -0.0012 | [17] |
| 2452352.3203 | CCD | -349.0 | I | -0.0008 | -0.0021 | -0.0014 | [17] |
| 2452352.5360 | CCD | -348.5 | II | +0.0029 | +0.0016 | $+0.0023$ | [17] |
| 2452369.4948 | CCD | -308.5 | II | +0.0005 | -0.0007 | $+0.0000$ | [20] |
| 2452401.5073 | pe | -233.0 | I | -0.0013 | -0.0025 | -0.0017 | [17] |
| 2452415.5014 | CCD | -200.0 | I | -0.0002 | -0.0014 | -0.0006 | [21] |
| 2452509.4210 | pe | +21.5 | II | -0.0032 | -0.0042 | -0.0032 | [22] |
| 2452510.4852 | pe | +24.0 | I | +0.0009 | -0.0001 | +0.0009 | [22] |
| 2452511.3349 | pe | +26.0 | I | +0.0025 | $+0.0015$ | $+0.0025$ | [22] |
| 2452511.5439 | pe | +26.5 | II | -0.0005 | -0.0015 | -0.0005 | [22] |

Table 4 - Continued.

| JD (Hel.) | Method | Epoch | Min | $(O-C)_{1}$ | $(O-C)_{2}$ | $(O-C)_{3}$ | Ref. |
| :--- | :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 2452550.3423 | pe | +118.0 | I | -0.0008 | -0.0018 | -0.0007 | $[17]$ |
| 2452568.3659 | CCD | +160.5 | II | +0.0015 | +0.0005 | +0.0016 | $[23]$ |
| 245271.4122 | CCD | +512.0 | I | +0.0012 | +0.0004 | +0.0017 | $[23]$ |
| 2452717.6180 | CCD | +512.5 | II | -0.0050 | -0.0058 | -0.0045 | $[23]$ |
| 2452718.4702 | CCD | +514.5 | II | -0.0008 | -0.0016 | -0.0003 | $[20]$ |
| 2452835.5067 | CCD | +790.5 | II | +0.0034 | +0.0026 | +0.0040 | $[24]$ |
| 2452908.4374 | CCD | +962.5 | II | +0.0009 | +0.0001 | +0.0016 | $[25]$ |
| 2452909.4968 | CCD | +965.0 | I | +0.0002 | -0.0006 | +0.0009 | $[25]$ |
| 2453082.5013 | pe | +1373.0 | I | +0.0005 | -0.0004 | +0.0012 | $[25]$ |
| 2453084.4092 | CCD | +1377.5 | II | +0.0003 | -0.0006 | +0.0010 | $[25]$ |
| 2453098.4049 | CCD | +1410.5 | II | +0.0030 | +0.0021 | +0.0037 | $[25]$ |
| 2453285.3991 | pe | +1851.5 | II | +0.0000 | -0.0011 | +0.0006 | $[25]$ |
| 2453410.2776 | CCD | +2146.0 | I | +0.0016 | +0.0003 | +0.0021 | $[26]$ |
| 2453452.4636 | pe | +2245.5 | II | -0.0034 | -0.0048 | -0.0030 | $[25]$ |
| 2453458.6159 | CCD | +2260.0 | I | +0.0005 | -0.0009 | +0.0009 | $[14]$ |
| 2453466.4614 | pe | +2278.5 | II | +0.0014 | +0.0000 | +0.0018 | $[25]$ |
| 2453507.8055 | CCD | +2376.0 | I | +0.0026 | +0.0011 | +0.0029 | $[27]$ |
| 2453848.5136 | CCD | +3179.5 | II | +0.0026 | +0.0001 | +0.0018 | $[28]$ |
| 2453911.4791 | CCD | +3328.0 | I | -0.0003 | -0.0030 | -0.0013 | $[28]$ |
| 2454091.6936 | CCD | +3753.0 | I | +0.0014 | -0.0020 | -0.0005 | $[14]$ |
| 2454270.4216 | CCD | +4174.5 | II | +0.0008 | -0.0035 | -0.0022 | $[14]$ |
| 2454279.5377 | CCD | +4196.0 | I | +0.0002 | -0.0041 | -0.0028 | $[14]$ |
| 2454570.4248 | CCD | +4882.0 | I | +0.0027 | -0.0032 | -0.0024 | $[29]$ |
| 2454943.5757 | CCD | +5762.0 | I | +0.0072 | -0.0012 | -0.0013 | $[30]$ |
| 2455087.3204 | CCD | +6101.0 | I | +0.0058 | -0.0037 | -0.0043 | $[31]$ |
| 2455103.0097 | CCD | +6138.0 | I | +0.0060 | -0.0036 | -0.0042 | $[31]$ |
| 2455118.0654 | CCD | +6173.5 | II | +0.0086 | -0.0011 | -0.0018 | $[31]$ |
| 2455165.9862 | CCD | +6286.5 | II | +0.0140 | +0.0039 | +0.0031 | $[31]$ |
| 2455327.7494 | CCD | +6668.0 | I | +0.0098 | -0.0017 | -0.0030 | $[32]$ |
| 2455675.2509 | CCD | +7487.5 | II | +0.0187 | +0.0040 | +0.0013 | $[31]$ |
| 2455678.2181 | CCD | +7494.5 | II | +0.0177 | +0.0030 | +0.0002 | $[31]$ |
| 2455709.1729 | CCD | +7567.5 | II | +0.0183 | +0.0033 | +0.0004 | $[31]$ |
| 2455712.1424 | CCD | +7574.5 | II | +0.0196 | +0.0046 | +0.0017 | $[31]$ |
| 2455713.2011 | CCD | +7577.0 | I | +0.0182 | +0.0032 | +0.0003 | $[31]$ |
| 2455714.2620 | CCD | +7579.5 | II | +0.0190 | +0.0039 | +0.0010 | $[31]$ |

Notes: [1] Robb \& Scarfe (1989); (2) Plewa et al. (1991); [3] Wunder \& Agerer (1992); [4] Hübscher et al. (1992); [5] Hübscher et al. (1993); [6] Hübscher et al. (1994); [7] Agerer \& Hübscher (1995); [8] Agerer \& Hübscher (1996); [9] Molik (2007); [10] Agerer \& Hüebscher (1997); [11] Agerer \& Hübscher (1999); [12] Agerer et al. (2001); [13] Agerer \& Hübscher (2002); [14] Brát et al. (2007); [15] Pribulla et al. (2001a); [16] Pribulla et al. (2001b); [17] Pribulla et al. (2002); [18] Diethelm (2001); [19] Sarounova \& Wolf (2005); [20] Agerer \& Hübscher (2003); [21] Diethelm (2002); [22] Drozdz \& Ogloza (2005); [23] Diethelm (2003); [24] Hübscher (2005); [25] Pribulla et al. (2005); [26] Zejda et al. (2006); [27] Nelson (2006); [28] Parimucha et al. (2007); [29] Borkovits et al. (2008); [30] Hübscher et al. (2010); [31] Present work; [32] Diethelm (2010).
variation, i.e. a light-time orbit (Irwin 1952), which is more flexible than a sinusoidal one. In order to search for the period of oscillation, Fourier analysis was carried out for the residuals of $(O-C)_{2}$, whose result is displayed in the bottom panel of Figure 3. We found a significant peak in the power spectrum mainly located around the frequency of $f_{1}=1.6225 \times 10^{-4} \mathrm{~d}^{-1}$, which corresponds to a period of 6160 d ( $\sim 17 \mathrm{yr}$ ). The Levenberg-Marquardt technique (Press et al. 1992) was applied in the calculation process. A nonlinear least-squares fitting method led to the following equation,

$$
\begin{equation*}
O-C=-0.0020( \pm 0.0002)+A \times\left(\frac{1-e^{2}}{1+e \cos \nu}+e \sin \omega\right) \tag{3}
\end{equation*}
$$



Fig. 3 Top panel: the $(O-C)_{2}$ curve (upper) and corresponding curve of residuals $(O-C)_{3}$ (lower). The solid line represents the light-time orbit in Eq. (3). Other symbols are the same as in Fig. 2. Bottom panel: the power spectrum of the residuals $(\mathrm{O}-\mathrm{C})_{2}$.
where $A=a_{12} \sin i / c$ is the semi-amplitude, and other parameters are taken from Irwin (1952). The fitted parameters are listed as follows: $A=0.0039^{\mathrm{d}}\left( \pm 0.0002^{\mathrm{d}}\right), P_{\mathrm{mod}}=17.20( \pm 0.18) \mathrm{yr}$, $e=0.49( \pm 0.02), \omega=1.4064( \pm 0.0209)$ and $T_{p}=\operatorname{HJD} 24501665( \pm 65)$. The derived period is consistent with one of the predicted values (i.e. $P_{3}>15 \mathrm{yr}$; Pribulla et al. 2001a). The solid line is constructed by Equation (3) in the upper section of top panel of Figure 3. The corresponding residuals of $(O-C)_{3}$, listed in Col. (8) of Table 4, are also shown in its lower section of top panel. From this figure, no regularity is apparent although there exists a bit of scatter.

## 4 LIGHT-CURVE SOLUTION

Two sets of multi-color light curves for EF Dra ( $\mathrm{LC}_{1}$ in 2009 and $\mathrm{LC}_{2}$ in 2011) were applied to derive the photometric solution using the W-D code (Wilson \& Devinney 1971), including the Kurucz (1993) stellar atmosphere model and a detailed reflection treatment (Wilson 1990). During the calcu-
lation, Mode 3 (i.e. the contact configuration) was always used. We adopted a set of fixed parameters $\left(q, T_{1}, A_{1,2}, g_{1,2}, X_{1,2}\right.$ and $Y_{1,2}, x_{1,2}$ and $y_{1,2}$ ), while other parameters $\left(i, T_{2}, \Omega_{1}, L_{1}\right.$, and $L_{3}$ ) were adjustable in the range of commonly used values.

The spectroscopic elements for the triple star EF Dra were published by Lu \& Rucinski (1999). The mass ratio was fixed at $q_{\mathrm{sp}}=0.16$ and the third light $L_{3}$ was adjusted. The mean effective temperature of Star 1 was adopted to be $T_{1}=6250 \mathrm{~K}$ based on the spectral type of F8V (Pribulla et al. 2009a). Following Lucy (1967) and Ruciński (1973), gravity darkening exponents of $g_{1,2}=0.32$ and bolometric albedo coefficients of $A_{1,2}=0.5$ were set, which are appropriate for stars with convective envelopes. According to the effective temperatures of both components, the logarithmic bolometric ( $X$ and $Y$ ) and monochromatic ( $x$ and $y$ ) limb-darkening coefficients were interpolated from the tables in van Hamme (1993). After some iterations, the photometric solutions can be deduced from $\mathrm{LC}_{1}$ and $\mathrm{LC}_{2}$, whose results are listed in Table 5 where two sets of parameters are in agreement. The contributions of the third light to the total light $\left(l_{3} \simeq 1.7 \%\right.$ in 2009 and $l_{3} \simeq 1.45 \%$ in 2011) are much smaller than the value of $l_{3} \simeq 0.14$ (Pribulla et al. 2001a). The corresponding theoretical light curves were shown as solid lines in both panels of Figure 1. The value of $\Sigma(O-C)_{i}^{2}$ is 0.2817 for $\mathrm{LC}_{1}$, which is larger than that of 0.2624 for $\mathrm{LC}_{2}$. Therefore, we accepted Sol. 2 (i.e. $\mathrm{LC}_{2}$ in 2011) as the final solution. The contact degree of $f=46.7 \%( \pm 0.6 \%)$ approximately agrees with that of $f=45.5 \%( \pm 2.3 \%)$ (Pribulla et al. 2001a), indicating that EF Dra is a deep-contact triple star system.

Table 5 Photometric Solutions of the Triple Star System EF Dra

| Parameter | Sol.1 $\left(\mathrm{LC}_{1}\right.$ in 2009) | Sol.2 $\left(\mathrm{LC}_{2}\right.$ in 2011) |
| :--- | :---: | :---: |
| $i\left({ }^{\circ}\right)$ | $78.0( \pm 0.5)$ | $77.8( \pm 0.2)$ |
| $q=M_{2} / M_{1}$ |  | 0.16 |
| $T_{1}(\mathrm{~K})$ | $6299( \pm 20)$ | 6250 |
| $T_{2}(\mathrm{~K})$ | $6186( \pm 7)$ |  |
| $\Omega_{1}=\Omega_{2}$ | $2.0790( \pm 0.0047)$ | $2.0818( \pm 0.0014)$ |
| $f$ | $49.5 \%( \pm 2.3 \%)$ | $46.7 \%( \pm 0.6 \%)$ |
| $L_{1 B} /\left(L_{1 B}+L_{2 B}\right)$ | $0.8207( \pm 0.0292)$ | $0.8368( \pm 0.0101)$ |
| $L_{1 V} /\left(L_{1 V}+L_{2 V}\right)$ | $0.8235( \pm 0.0269)$ | $0.8356( \pm 0.0091)$ |
| $L_{1 R} /\left(L_{1 R}+L_{2 R}\right)$ | $0.8248( \pm 0.0261)$ | $0.8347( \pm 0.0084)$ |
| $L_{3 B}$ | $1.73 \%( \pm 0.15 \%)$ | $1.45 \%( \pm 0.05 \%)$ |
| $L_{3 V}$ | $1.69 \%( \pm 0.14 \%)$ | $1.48 \%( \pm 0.05 \%)$ |
| $L_{3 R}$ | $1.69 \%( \pm 0.14 \%)$ | $1.43 \%( \pm 0.04 \%)$ |
| $r_{1}$ (pole) | $0.5163( \pm 0.0012)$ | $0.5156( \pm 0.0004)$ |
| $r_{1}$ side) | $0.5711( \pm 0.0019)$ | $0.5700( \pm 0.0006)$ |
| $r_{1}$ (back $)$ | $0.5962( \pm 0.0023)$ | $0.5947( \pm 0.0007)$ |
| $r_{2}$ (pole $)$ | $0.2335( \pm 0.0015)$ | $0.2326( \pm 0.0004)$ |
| $r_{2}$ (side) | $0.2506( \pm 0.0018)$ | $0.2440( \pm 0.0005)$ |
| $r_{2}$ (back) | $0.2960( \pm 0.0043)$ | $0.2934( \pm 0.0013)$ |
| $\Sigma(O-C)_{i}^{2}$ | 0.2817 | 0.2624 |

## 5 DISCUSSION AND CONCLUSIONS

The photometric solutions were deduced from two sets of $B V R$ light curves, indicating that EF Dra is a triple star system with a high fill-out factor. Combining the photometric solution (Sol.2) and spectroscopic elements (Lu \& Rucinski 1999), the absolute physical parameters can be updated as follows: $a=3.045( \pm 0.028) R_{\odot}, M_{1}=1.815( \pm 0.032) M_{\odot}, M_{2}=0.290( \pm 0.026) M_{\odot}$, $R_{1}=1.702( \pm 0.002) R_{\odot}, R_{2}=0.777( \pm 0.002) R_{\odot}, L_{1}=3.961( \pm 0.008) L_{\odot}$ and $L_{2}=$ $0.793( \pm 0.004) L_{\odot}$. The $O-C$ curve shows its orbital period may change in a complicated mode, i.e. a secular period increase superimposed with a light-time orbit. This kind of period variation may appear in other deep-contact binaries, whose spectroscopic parameters are listed in Table 6. From

Table 6 Parameters of Some Deep-contact Binaries with a Period Increase and a Cyclic Variation

| Star | Period <br> $(\mathrm{d})$ | $q$ | $f$ <br> $(\%)$ | $d P / d t$ <br> $\left(\times 10^{-7} \mathrm{~d} \mathrm{yr}^{-1}\right)$ | A <br> $(\mathrm{d})$ | $P_{\bmod }$ <br> $(\mathrm{yr})$ | Ref. |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| XY Boo | 0.3706 | 0.186 | 55.9 | +6.54 | 0.0045 | 30.4 | $[1]$ |
| RR Cen | 0.6057 | 0.205 | 35.1 | +1.21 | 0.0124 | 60.1 | $[2]$ |
| AH Cnc | 0.3604 | 0.168 | 58.5 | +3.99 | 0.0237 | 36.5 | $[3]$ |
| V1191 Cyg | 0.3134 | 0.107 | 68.6 | +4.5 | 0.023 | 26.7 | $[4]$ |
| EF Dra | 0.4240 | 0.160 | 46.7 | +3.72 | 0.0038 | 17.2 | $[5]$ |
| V345 Gem | 0.2748 | 0.142 | 72.9 | +0.12 | 0.0019 | 1.77 | $[6]$ |
| BB Peg | 0.3440 | 0.149 | 95.3 | +0.30 | 0.0055 | 27.9 | $[7]$ |
| EM Psc | 0.3440 | 0.149 | 95.3 | +39.7 | 0.0110 | 3.3 | $[8]$ |

References: [1] Yang (2006); [2] Yang et al. (2005); [3] Qian et al. (2006); [4] Zhu et al. (2011); [5] Present work; [6] Yang et al. (2010); [7] Kalomeni et al. (2007); [8] Qian et al. (2008).
this table, the period increase rate is a typical value. The semi-amplitude and period of the light-time orbit are smaller values.

### 5.1 Light-time Effect due to the Third Body

From Equation (3), the period and semi-amplitude of the light-time orbit are $P_{\bmod }=$ $17.20( \pm 0.18)$ yr and $A=0.0039^{\mathrm{d}}\left( \pm 0.0002^{\mathrm{d}}\right)$, respectively. The existence of the tertiary component of EF Dra has been identified by the photometry and spectroscopy. From the fitted parameters of Equation (3), we can easily calculate the value of $a_{12} \sin i=0.6752( \pm 0.0346) \mathrm{AU}$, where $a_{12}$ and $j$ are the radius and inclination of the orbit of the third body around the mass center of the binary system. Using the known equation,

$$
\begin{equation*}
f(m)=\frac{4 \pi^{2}}{G P_{\bmod }^{2}} \times\left(a_{12} \sin i\right)^{3}=\frac{\left(M_{3} \sin i\right)^{3}}{\left(M_{1}+M_{2}+M_{3}\right)^{2}}, \tag{4}
\end{equation*}
$$

the mass function of $f(m)=1.04( \pm 0.16) \times 10^{-3} M_{\odot}$ for the third body was computed. Therefore, for some orbital inclination $i$, the unknown mass $M_{3}$ can be derived by the iteration method. With the absolute parameters of EF Dra, the relations between the orbital inclination $i$, radii $a_{12}$ and mass $M_{3}$ for the third body are displayed in Figure 4. According to the photometric solution, the value of $L_{3} / L_{1}$ is about 0.018 . The visual magnitude of EF Dra is $4.0^{\mathrm{m}}$, which is based on its spectral type of F8V (Cox 2000). Therefore, the value of the tertiary companion approximates $M_{\mathrm{v}}=$ $8.56^{\mathrm{m}}$, corresponding to a spectral type of K9 and a mass of $0.54 M_{\odot}$. From Equation (4), we can deduce the orbital radius and inclination for the third component, i.e. $i=21.0^{\circ}\left( \pm 1.1^{\circ}\right)$ and $a_{12}=$ $7.33( \pm 0.38) \mathrm{AU}$, which are shown as solid circles in both panels of Figure 4. Using the Harrington (1977) sufficient condition for the stability of a triple system, EF Dra should be dynamically stable.

### 5.2 Mass Transfer and Its Evolutionary Status

According to the binary-star evolution code (i.e. BSE; Hurley et al. 2002), we constructed the zeroage main sequence (ZAMS) and the terminal-age main sequence (TAMS) in a $\log L-\log M$ diagram in Figure 5, where the open circles and triangles represent the primary and the secondary of the Atype low-temperature contact binary systems (Yakut \& Eggleton 2005). From this figure, it is found that the more massive component is close to the TAMS line, indicating that the primary component may be a normal main-sequence star. Moreover, the less massive component lies upon the TAMS line. This may be due to the energy transferring from the more massive component to the less massive one during its evolutionary process. Therefore, the secondary component is overluminous.


Fig. 4 Relations between the inclination $i$ and radius $a_{12}$ and mass $M_{3}$ for the third body from Eq. (4). The filled circles in both panels represent the positions of the third body, whose spectral type is $K 9$, estimated from the third light of the photometric solution.


Fig. 5 Mass-luminosity diagram of the triple star system EF Dra. The open and filled circles refer to the primary and secondary components for the A-type low-temperature contact binaries (Yakut \& Eggleton 2005). The solid and dotted lines show the ZAMS and TAMS lines, constructed by the BSE Code (Hurley et al. 2002).

From Equation (2), the orbital period of EF Dra increases at a rate of $d P / d t=+3.72( \pm 0.07) \times$ $10^{-7} \mathrm{~d} \mathrm{yr}^{-1}$, indicating that the mass transfers from the less massive component to the more massive one. Assuming there is conserved mass transfer, its rate can be estimated using the following equation (Singh \& Chaubey 1986),

$$
\begin{equation*}
\frac{\dot{P}}{P}=3 \frac{1-q}{q} \frac{\dot{M}_{1}}{M_{1}}, \tag{5}
\end{equation*}
$$

where $M_{1}$ and $\dot{M}_{1}$ are the mass of the primary component and its mass transfer rate, respectively. Inserting the values of $\dot{P}, P, M_{1}$ and $q$ of EF Dra into Equation (5), the mass transfer rate can be estimated to be $\dot{M}_{1}=+1.01( \pm 0.02) \times 10^{-7} M_{\odot} \mathrm{yr}^{-1}$. This situation occurs in other deep-contact binaries, listed in Table 6. With the period continuously increasing, the orbital angular momentum decreases while the spin angular momentum increases. Meanwhile, the separation between both components may increase, which may cause the contact degree to decrease. Moreover, a tertiary companion may remove angular momentum from the central system via a Kozai (1962) oscillation or a combination of a Kozai cycle and tidal friction (e.g., Fabrycky \& Tremaine 2007), which may play an important role for the formation and evolution of the contact binary. For this kind of deepcontact binary, instability occurs when it meets the more familiar criterion that the orbital angular momentum is less than three times the total spin angular momentum (Hut 1980). Only if the contact configuration does not break down at $J_{\text {orb }}>3 J_{\text {spin }}$ could EF Dra merge into a rapidly rotating single star. In our future work, it is necessary for us to obtain high-precision eclipsing times in order to check the nature of its orbital period changes.
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