A numerical study of self-gravitating protoplanetary disks

Kazem Faghei

School of Physics, Damghan University, Damghan 36715-364, Iran; *kfaghei@du.ac.ir*

Abstract The effect of self-gravity on protoplanetary disks is investigated. The mechanisms of angular momentum transport and energy dissipation are assumed to be the viscosity due to turbulence in the accretion disk. The energy equation is considered in a situation where the released energy by viscosity dissipation is balanced with cooling processes. The viscosity is obtained by equality of dissipation and cooling functions, and is used to derive the angular momentum equation. The cooling rate of the flow is calculated by a prescription, $du/dt = -u/\tau_{\rm cool}$, where u and $\tau_{\rm cool}$ are the internal energy and cooling timescale, respectively. The ratio of local cooling to dynamical timescales $\Omega\tau_{\rm cool}$ is assumed to be a constant and also a function of the local temperature. The solutions for protoplanetary disks show that in the case of $\Omega\tau_{\rm cool} = {\rm constant}$, the disk does not exhibit any gravitational instability over small radii for a typical mass accretion rate, $\dot{M} = 10^{-6} M_\odot \, {\rm yr}^{-1}$, but when choosing $\Omega\tau_{\rm cool}$ to be a function of temperature, gravitational instability can occur for this value of mass accretion rate or even less in small radii. Also, by studying the viscosity parameter α, we find that the strength of turbulence in the inner part of self-gravitating protoplanetary disks is very low. These results are qualitatively consistent with direct numerical simulations of protoplanetary disks. Also, in the case of cooling with temperature dependence, the effect of physical parameters on the structure of the disk is investigated. These solutions demonstrate that disk thickness and the Toomre parameter decrease by adding the ratio of disk mass to central object mass. However, the disk thickness and the Toomre parameter increase by adding mass accretion rate. Furthermore, for typical input parameters such as mass accretion rate $10^{-6} M_\odot \, {\rm yr}^{-1}$, the ratio of the specific heat $\gamma = 5/3$ and the ratio of disk mass to central object mass $q = 0.1$, gravitational instability can occur over the whole radius of the disk excluding the region very near the central object.

Key words: accretion, accretion disks — planetary systems: protoplanetary disks — planetary systems: formation

1 INTRODUCTION

Accretion disks are important for many astrophysical phenomena, including protoplanetary systems, different types of binary stars, binary X-ray sources, quasars, and Active Galactic Nuclei (AGNs). Historically, theories of accretion disks have concentrated on the non self-gravitating cases and occasionally the effect of self-gravity had been studied (Paczynski 1978; Kolykhalov & Syunyaev 1979; Lin & Pringle 1987, 1990). On the other hand, in recent years, the importance of study of disk

self-gravity has increased, especially in protostellar disks and AGN disks. Evidence confirming the existence of self-gravitating disks has accumulated, possibly due to an increase of computational resources in simulation of self-gravitating accretion disks and their observational results, including cases from AGNs to protostars (Lodato 2007 and references therein). Also, it appears the development of gravitational instability is important for cool regions of accreting gas where angular momentum transport by magneto-rotational instability (MRI) becomes weak (Fleming et al. 2000; Masada & Sano 2008; Faghei 2011) and angular momentum can be transported by gravitational instability.

The structure of self-gravitating disks has been studied both through self-similar solutions assuming steady and unsteady states (Mineshige & Umemura 1996, 1997; Tsuribe 1999; Bertin & Lodato 1999, 2001; Shadmehri & Khajenabi 2006; Abbassi et al. 2006; Shadmehri 2009) and through direct numerical simulations (Gammie 2001; Rice et al. 2003, 2005, 2010; Rice & Armitage 2009; Cossins et al. 2010; Meru & Bate 2011a).

Mineshige & Umemura (1996) investigated the role of self-gravity on the classical self-similar solution of advection dominated accretion flows (ADAF, Narayan & Yi 1994) and found global one-dimensional solutions influenced by self-gravity both in the radial and in the perpendicular directions of the disk. They extended the previous steady state solutions to the time-dependent case while the effect of self-gravity of the disk was taken into account. They used an isothermal equation, and so their solutions describe viscous accretion disks in the slow accretion limit. Tsuribe (1999) studied unsteady viscous accretion in self-gravitating disks. Taking into account the growth of the central point mass, Tsuribe (1999) derived a series of self-similar solutions for rotating isothermal disks. The solutions showed, as a core mass increases, the rotation law changes from flat rotation to Keplerian rotation in the inner disk and in addition to the central point mass, the inner disk grows by mass accumulation due to the differing mass accretion rates in the inner and outer radii. Bertin & Lodato (1999) considered a class of steady-state self-gravitating accretion disks for which efficient cooling mechanisms are assumed to operate so that the disk is self-regulated at a condition of the approximate marginal Jeans stability. They investigated the entire parameter space available for such self-regulated accretion disks. In another study, Bertin & Lodato (2001) followed the model such that, when the disk is sufficiently cold, the stirring due to Jeans-related instabilities acts as a source of effective heating. With the corresponding reformulation of the energy equations, they demonstrated how self-regulation can be established, so that the stability parameter Q is maintained close to a threshold value, with a weak dependence on radius. Abbassi et al. (2006) studied the effect of viscosity on the time evolution of axisymmetric, polytropic self-gravitating disks around a new born central object. Thus, they ignored the gravitational effect of the central object and only self-gravity of the disk played an important role. They compared effects of the α-viscosity prescription (Shakura & Sunyaev 1973) and β-viscosity prescription (Duschl et al. 2000) on disk structure. They found that accretion rate onto the central object for β-disks is more than that for α-disks, at least in the outer regions where β-disks are more efficient. Also, their results showed gravitational instability can occur everywhere on the β-disks and thus they suggested that β-disks can be a good candidate for the origin of planetary systems. Shadmehri & Khajenabi (2006) examined steady self-similar solutions of isothermal self-gravitating disks in the presence of a global magnetic field. Similar to Abbassi et al. (2006), they neglected the range of values from the mass of the central object to the disk mass. By studying the Toomre parameter, they showed that the magnetic field can be important in gravitational stability of the disk.

An accretion disk can become gravitationally unstable if the Toomre parameter becomes smaller than its critical value, $Q < Q_{crit}$ (Toomre 1964). For axisymmetric instabilities $Q_{crit} \sim 1$, while for non-axisymmetric instabilities Q_{crit} values are as high as 1.5–1.7 (Durisen et al. 2007). One possible outcome is that unstable disks fragment to produce bound objects and this has been suggested as a possible mechanism for forming giant planets (Boss 1998, 2002). However, recently it has been realized that the above condition is not sufficient to guarantee fragmentation. Gammie (2001) showed that in addition to the above instability criterion, the disk must cool at a fast enough rate. Let the

cooling timescale τ_{cool} be defined as the gas internal energy divided by the volumetric cooling rate. For a power-law equation of state with τ_{cool} prescribed to be some value over an annulus of the disk, the thin shearing box simulations of Gammie (2001) show that fragmentation occurs if and only if $\Omega\tau_{\mathrm{cool}} \lesssim \beta_{\mathrm{crit}}$, where $\beta_{\mathrm{crit}} \sim 3$ and Ω is the angular velocity of the disk or inverse of the dynamical timescale $\Omega = \tau_{\mathrm{dyn}}^{-1}$. The critical value of $\Omega\tau_{\mathrm{cool}}$ can be somewhat larger than three for more massive and physically thicker disks (Rice et al. 2003), a larger adiabatic index (Rice et al. 2005), and higher resolution of simulations (Meru & Bate 2011b). Using a smoothed-particle hydrodynamics simulation, Cossins et al. (2010) studied the effects of opacity regimes on the stability of self-gravitating protoplanetary disks fragmenting into bound objects. They showed that $\Omega\tau_{\mathrm{cool}}$ has a strong dependence on the local temperature. Thus, they found that without temperature dependence, for radii $\lesssim 10$ AU, a very large accretion rate $10^{-3}M_{\odot}\,\mathrm{yr}^{-1}$ is required for fragmentation, but this is reduced to 10^{-4} with cooling, which is dependent on temperature.

As mentioned, typically semi-analytical studies of self-gravitating disks are modeling polytropic disks (Abbassi et al. 2006), isothermal disks (Mineshige & Umemura 1996, 1997; Tsuribe 1999; Shadmehri & Khajenabi 2006), ADAFs in the extreme limit of no radiative cooling (Shadmehri 2004), and disks without a central object (Mineshige & Umemura 1996, 1997; Tsuribe 1999; Shadmehri & Khajenabi 2006; Abbassi et al. 2006). In this paper, it will be interesting to understand under which conditions gravitational instability can occur in accretion disks by a suitable energy equation and assuming a Newtonian potential of a mass point that is located at the disk's center. Thus, to obtain these conditions, we will use a prescription for cooling rate that is introduced by Gammie (2001), $du/dt = -u/\tau_{\mathrm{cool}}$, where u and τ_{cool} are internal energy and cooling timescale, respectively. The ratio of local cooling to dynamical timescales $\Omega\tau_{\mathrm{cool}}$ is assumed to be a power-law function of temperature in adapting the result of Cossins et al. (2010), $\Omega\tau_{\mathrm{cool}} = \beta_0(T/T_0)^{\delta}$, where T_0 and δ are free parameters, and β_0 is a free parameter in Gammie (2001). When $\delta = 0$, $\Omega\tau_{\mathrm{cool}}$ reduces to the Gammie (2001) model where $\Omega\tau_{\mathrm{cool}}$ is a constant, while non-zero δ is qualitatively consistent with the results of Cossins et al. (2010). We will examine the effects of the δ parameter on gravitational stability of the disk. We will show that the present model is qualitatively consistent with direct numerical simulations (Rice & Armitage 2009; Cossins et al. 2010; Rice et al. 2010) and can provide conditions such that gravitational instability can occur over the whole radius, excluding the region very near the central object.

In Section 2, the basic equations of constructing a model for a steady self-gravitating disk will be defined. In Section 3, we will find asymptotic solutions for the outer edge of the disk. In Section 4, by exploiting asymptotic solutions as boundary conditions for system equations, we will numerically investigate the effects of physical parameters on the structure and stability of the disk. The summary and discussion of the model will appear in Section 5.

2 BASIC EQUATIONS

We use cylindrical coordinates (r, φ, z) centered on the accreting object and make the following standard assumptions:

 (i) The flow is assumed to be steady and axisymmetric $\partial_t = \partial_\varphi = 0$, so all flow variables are a function of r and z;
 (ii) The gravitational force of the central object on a fluid element is characterized by the Newtonian potential of a point mass, $\Psi = -GM_*/r$, with G representing the gravitational constant and M_* standing for the mass of the central star;
(iii) The equations written in cylindrical coordinates are integrated in the vertical direction, hence all quantities of the flow variables will be expressed in terms of cylindrical radius r;

The governing equations of the self-gravitating accretion disk for such assumptions are as follows. The continuity equation is

$$\frac{1}{r}\frac{d}{dr}(r\Sigma v_r) = 0\,, \tag{1}$$

where v_r is the radial infall velocity and Σ is the surface density, which is defined as $\Sigma = 2\rho h$, and ρ and h are density and the disk half-thickness, respectively. The half-thickness of the disk with the assumption of hydrostatic equilibrium in the vertical direction is $h = c_{\rm s}/\Omega$, where $c_{\rm s}$ is the sound speed, which is defined as $c_{\rm s}^2 = p/\rho$, p is the gas pressure and Ω represents the angular velocity of the flow. Equation (1) implies that

$$\dot{M} = -2\pi r\Sigma v_r = {\rm constant}\,,$$

where $\dot{M}$ is the mass accretion rate and is a constant in the present model. The simulation results of protoplanetary disks show that the disk reaches a quasi-steady state in $20\,000$ years or less and might imply that these systems are rarely out of equilibrium. Also, the simulations show that the mass of the disk redistributes itself to produce a state in which the accretion rate, $\dot{M}$, is largely independent of r (Rice & Armitage 2009; Rice et al. 2010). Thus, we can use the mass accretion as a constant and it cannot be a limitation for the present model. The momentum equations are

$$v_r\frac{dv_r}{dr} = -\frac{1}{\Sigma}\frac{d}{dr}(\Sigma c_{\rm s}^2) - G\left[\frac{M_* + M(r)}{r^2}\right] + r\Omega^2\,, \tag{2}$$

$$\Sigma v_r\frac{d}{dr}(r^2\Omega) = \frac{1}{r}\frac{d}{dr}\left[\nu\Sigma r^3\frac{d\Omega}{\partial r}\right]\,, \tag{3}$$

where ν is the kinematic viscosity coefficient, γ is the adiabatic index, and $M(r)$ is the mass of a disk within a radius r. As mentioned in Mineshige & Umemura (1997), we adopt the monopole approximation for the radial gravitational force due to the self-gravity of the disk, which considerably simplifies the calculations and is not expected to introduce any significant error as long as the surface density profile is steeper than $1/r$ (e.g. Li & Shu 1997; Saigo & Hanawa 1998; Tsuribe 1999; Krasnopolsky & Königl 2002; Shadmehri 2009). Now, we can write

$$\frac{dM(r)}{dr} = 2\pi r\Sigma\,. \tag{4}$$

The energy equation is

$$\frac{\Sigma v_r}{\gamma - 1}\frac{dc_{\rm s}^2}{dr} + \frac{\Sigma c_{\rm s}^2}{r}\frac{d}{dr}(rv_r) = \Gamma - \Lambda\,, \tag{5}$$

where Γ is the heating rate of the gas by dissipation processes such as turbulent viscosity and Λ represents the energy loss through radiative cooling processes. The forms of the dissipation and cooling functions can be written as

$$\Gamma = r^2\Sigma\nu\left|\frac{d\Omega}{dr}\right|^2\,, \tag{6}$$

$$\Lambda = \frac{1}{\gamma(\gamma - 1)}\frac{\Sigma c_{\rm s}^2}{\tau_{\rm cool}}\,, \tag{7}$$

where $\tau_{\rm cool}$ is the cooling timescale. As noted in the introduction, we are interested in considering the effect of the cooling function on the structure of self-gravitating disks. Thus, similar to Rice & Armitage (2009) we will study the effects of it in the case where the heating rate in the disk is equal to the cooling rate, $\Gamma = \Lambda$.

Since fragmentation requires fast cooling, Gammie (2001) suggested the cooling timescale can be parameterized as $\beta = \Omega\tau_{\rm cool}$, where β is a free parameter. Gammie (2001) showed fragmentation requires $\beta \lesssim \beta_{\rm crit}$, where $\beta_{\rm crit} \approx 3$ for the adiabatic index of $\gamma = 2$. Rice et al. (2005) performed

3D simulations to show the dependence of $\beta_{\rm crit}$ on γ: for disks with $\gamma = 5/3$ and $7/5$, $\beta_{\rm crit} \approx 6 - 7$ and $\approx 12 - 13$, respectively. Recently, Cossins et al. (2010) studied β as a function of temperature. They showed that β has a strong dependence on the local temperature. They found that without temperature dependence, for radii $\lesssim 10$ AU a very large accretion rate $10^{-3}\,M_\odot\,{\rm yr}^{-1}$ is required for fragmentation, but this is reduced to $10^{-4}\,M_\odot\,{\rm yr}^{-1}$ with cooling, which is dependent on temperature. So, for simplicity in this paper we will use a cooling timescale with a power-law dependence on temperature for study of Equations (1)–(5)

$$\tau_{\rm cool} = \frac{\beta_0}{\Omega}\left(\frac{T}{T_0}\right)^\delta = \frac{\beta_0}{\Omega}\left(\frac{c_{\rm s}}{c_{\rm s_0}}\right)^{2\delta}, \tag{8}$$

where δ and β_0 are free parameters. If we select T_0 as a temperature of the outer part of the disk, then $c_{\rm s_0}$ will be the sound speed there. From Equation (8) and $\delta = 0$, we expect that $\Omega\tau_{\rm cool}$ becomes a constant that is the same as that in the Gammie (2001) model. So non-zero δ is qualitatively consistent with the Cossins et al. (2010) model. It is important to stress that the above description for cooling rate does not mean to reproduce any specific cooling law, but is just a convenient way of exploring the role of the cooling timescale in the outcome of the gravitational instability.

Here, the kinematic coefficient of viscosity can be obtained by equating the heating and cooling rates

$$\nu = \frac{1}{\gamma(\gamma - 1)}\frac{\left|\frac{d\Omega}{dr}\right|^{-2}}{r^2}\frac{c_{\rm s}^2}{\tau_{\rm cool}}. \tag{9}$$

Thus, by exploiting Equation (9) we do not need to use viscosity descriptions, such as α and β prescriptions that were introduced by Shakura & Sunyaev (1973) and Duschl et al. (2000), respectively. Equation (9) implies that the kinematic coefficient of viscosity in the present model depends on physical quantities of the system, especially the cooling timescale. The kinematic coefficient of viscosity in the α-prescription is $\nu = \alpha c_{\rm s}h$, where α is a free parameter and is less than unity (Shakura & Sunyaev 1973). By using Equation (9) for the α parameter we can write

$$\alpha = \frac{\nu}{c_{\rm s}h} = \frac{1}{\gamma(\gamma - 1)}\frac{\left|\frac{d\Omega}{dr}\right|^{-2}}{r^2 h}\frac{c_{\rm s}}{\tau_{\rm cool}}. \tag{10}$$

The above equation implies that the α parameter is not a constant and varies by position and strongly depends on the cooling timescale. We will study the α parameter in Section 4 and will show that in the present model it increases with radius.

As mentioned in the introduction, the gravitational stability of the disk can be investigated by the Toomre parameter (Toomre 1964). The Toomre parameter for epicyclic motion can be written as

$$Q = \frac{c_{\rm s}k}{\pi G\Sigma}, \tag{11}$$

where

$$k = \Omega\sqrt{4 + 2\frac{d\log\Omega}{d\log r}} \tag{12}$$

is the epicyclic frequency which can be replaced by the angular frequency, Ω.

Equations (1)–(5) and (9) provide a set of ordinary differential equations that describe physical properties of the self-gravitating disk. Since these equations are nonlinear, we will need suitable boundary conditions to solve them numerically. Thus, in the next section we will try to obtain an asymptotic solution in the outer edge of the disk and then by exploiting this asymptotic solution as a boundary condition, we can integrate the system of equations inward from a point very near the outer edge of the disk.

Before covering the next sections and examining the numerical study of the model, we shall express all quantities in units with values typical for a protostellar disk. We will choose astronomical unit (AU) and the Sun's mass ($M_\odot$) as the units of length and mass, respectively. Thus, the time unit is given by $\sqrt{\mathrm{AU}^3/GM_\odot}$, which is equal to a year divided by 2π.

3 OUTER LIMIT

Here, the asymptotic behavior of the system of equations expressed as $r \to R$ is investigated, where R is the outer radius of the disk. The asymptotic solutions are given by

$$\Sigma(r) \sim \frac{\Sigma_0}{R^{1/2}} \left(1 + a_1 \frac{s}{R} + \cdots\right),\tag{13}$$

$$v_r(r) \sim -c_1 \sqrt{\frac{M_* + M_{\mathrm{disk}}}{R}} \left(1 + a_2 \frac{s}{R} + \cdots\right),\tag{14}$$

$$\Omega(r) \sim c_2 \sqrt{\frac{M_* + M_{\mathrm{disk}}}{R^3}} \left(1 + a_3 \frac{s}{R} + \cdots\right),\tag{15}$$

$$c_s^2(r) \sim c_3 \frac{M_* + M_{\mathrm{disk}}}{R} \left(1 + a_4 \frac{s}{R} + \cdots\right),\tag{16}$$

$$M(r) \sim M_{\mathrm{disk}} - \int_r^R 2\pi r' \Sigma(r') dr',\tag{17}$$

where $s = R - r$, M_{disk} is the disk mass, and the coefficients of c_i, a_i and Σ_0 must be determined. Using these solutions, from the continuity, momentum, angular momentum, energy and viscosity, by using Equations (1)–(5) and (9), we can obtain the coefficients of c_i that have the following forms:

$$c_1 = \frac{\dot{M}}{2\pi\Sigma_0\sqrt{M_* + M_{\mathrm{disk}}}},\tag{18}$$

$$c_2^2 + \left[\frac{a_3\gamma(\gamma-1)\beta_0\dot{M}(a_3-2)(a_1+a_4)}{2\pi\Sigma_0\sqrt{M_* + M_{\mathrm{disk}}}(a_1+a_3+a_4-1)}\right] c_2$$

$$+ \left[\frac{a_2\dot{M}^2}{4\pi^2\Sigma_0^2(M_* + M_{\mathrm{disk}})} - 1\right] = 0,\tag{19}$$

$$c_3 = \left(\frac{a_3\gamma\beta_0(a_3-2)(\gamma-1)\dot{M}}{2\pi\Sigma_0(a_1+a_3+a_4-1)\sqrt{M_* + M_{\mathrm{disk}}}}\right) c_2,\tag{20}$$

where

$$a_4 = (1+a_2)(1-\gamma).\tag{21}$$

The value of mass accretion rate can be determined by observational data of the protoplanetary disks. Also, Σ_0 can approximately be determined by disk mass, $M_{\mathrm{disk}} \sim \pi R^2 \Sigma$. Thus, after determining the values of Σ_0 and $\dot{M}$ from the observations, the value of the c_3 coefficient is only dependent on the value of c_2. On the other hand, the value of c_2 can be obtained by Equation (19). Since we only have one equation for coefficients of a_i (Eq. (21)), we will select the below values for them in the process of numerical integration of the system of equations to obtain physical results

$$a_1 < -2 + \frac{3}{2}\gamma, \qquad 3\,a_2 = a_3 = \frac{3}{2}, \qquad a_4 = (1+a_2)(1-\gamma).\tag{22}$$
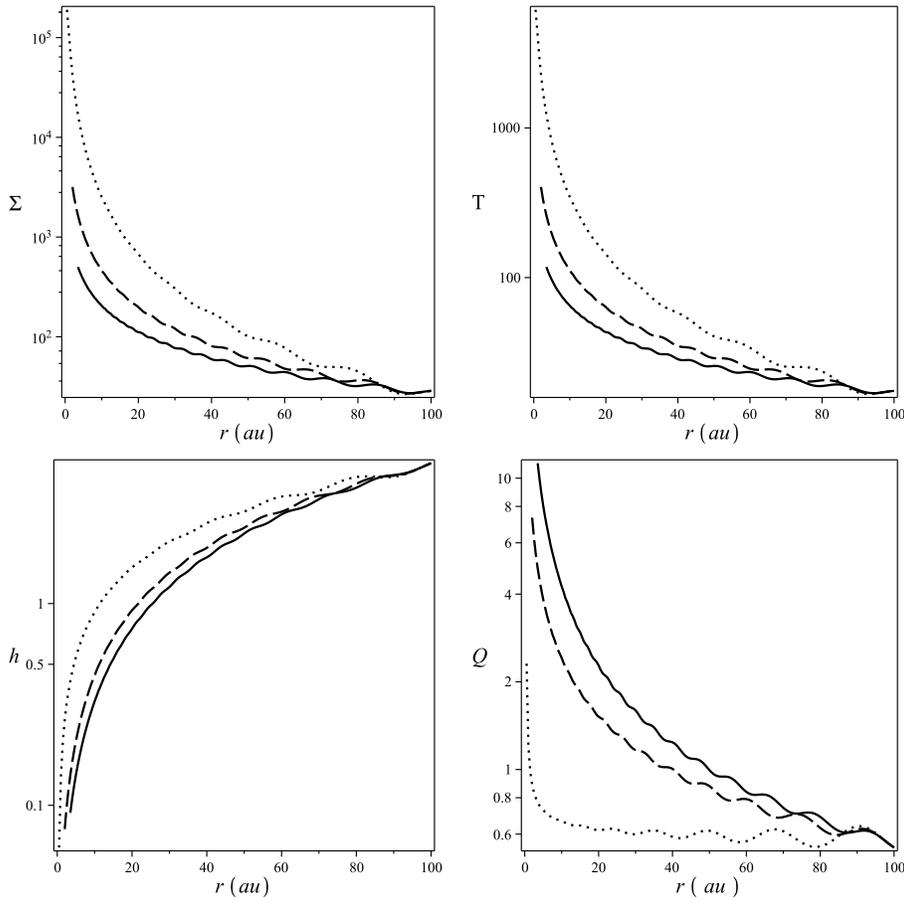
Fig. 1 Surface density, thickness, temperature and the Toomre parameter of the disk as a function of radius, for several values of δ. The surface density and the temperature are expressed in the cgs system, and the thickness and the distance are in AU. The solid lines represent $\delta = 0$, the dashed lines represent $\delta = 0.75$ and the dotted lines represent $\delta = 1.5$. The input parameters are set to the disk mass $M_{\mathrm{disk}} = 0.1 M_\odot$, the star mass $M_* = M_\odot$, the mass accretion rate $\dot{M} = 10^{-6} M_\odot \,\mathrm{yr}^{-1}$, the ratio of the specific heats is set to be $\gamma = 5/3$ and $\beta_0 = 2$.

4 NUMERICAL RESULTS

If the value of R is initialized, the equations describing the Fehlberg-Runge-Kutta fourth-fifth order method can be integrated inwards from a point very near the outer edge of the disk, using the above expansions. Examples of such solutions for surface density, half-thickness of the disk, temperature, the Toomre parameter and the viscosity parameter of α as a function of radius are presented in Figures 1–5. The delineated quantity of T in Figures 1–4 is the mid-plane temperature which can then be determined using

$$T = \left(\frac{\mu m_{\mathrm{p}}}{k_{\mathrm{B}}} \right) c_{\mathrm{s}}^2 ,$$

where $\mu = 2$ is the mean molecular weight, m_{p} is the proton mass and k_{B} is Boltzmann's constant.
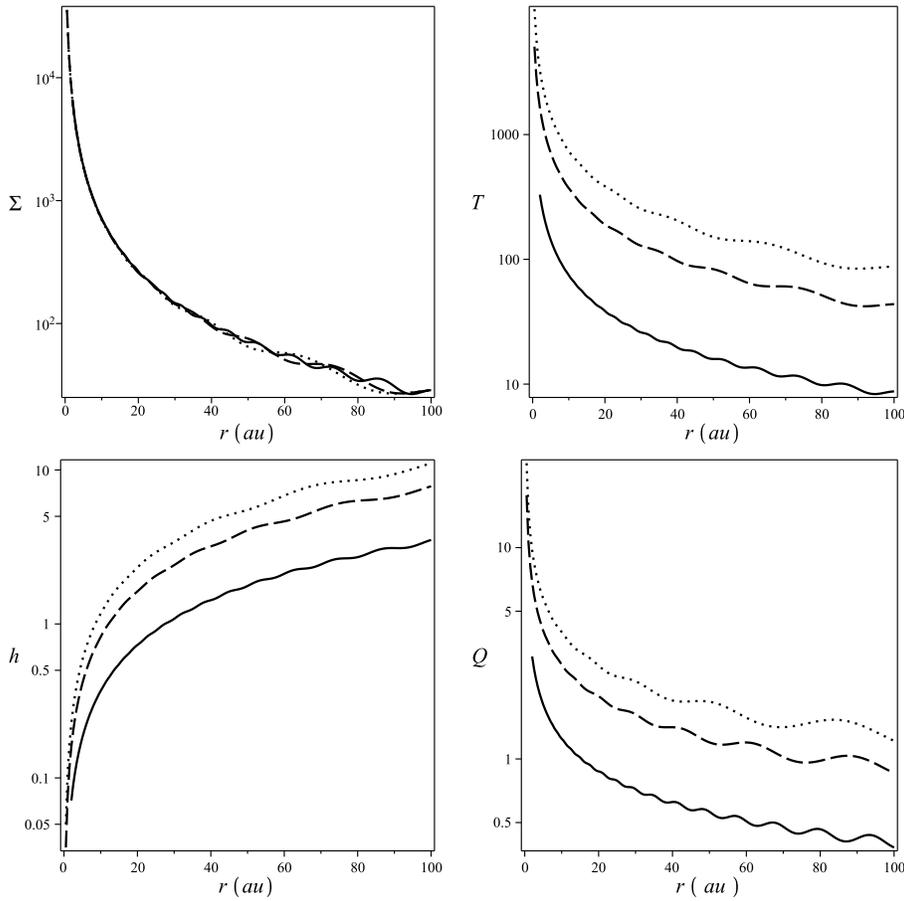
Fig. 2 Surface density, thickness, temperature and the Toomre parameter of the disk as a function of radius, for several values of β_0. The surface density and the temperature are expressed in the cgs system, and the thickness and the distance are in AU. The solid lines represent $\beta_0 = 1$, the dashed lines represent $\beta_0 = 5.0$ and the dotted lines represent $\beta_0 = 10$. The input parameters are set to the disk mass $M_{\mathrm{disk}} = 0.1 M_\odot$, the star mass $M_* = M_\odot$, the mass accretion rate $\dot{M} = 10^{-6} M_\odot \, \mathrm{yr}^{-1}$, the ratio of the specific heats is set to be $\gamma = 5/3$ and $\delta = 1.0$.

4.1 The Influences of Physical Parameters on the Results

The free parameters in the present model are the degree of influence of temperature on the cooling timescale, δ, the mass accretion rate, $\dot{M}$, the parameter β_0 and the ratio of disk mass to star mass, $q = M_{\mathrm{disk}}/M_*$.

4.1.1 δ parameter

The effects of the δ parameter on the physical quantities are presented in Figure 1. The profiles of surface density and temperature show that they increase by adding δ. However, the increase of surface density is more than temperature. Thus, the Toomre parameter ($Q \propto c_{\mathrm{s}}/\Sigma \propto \sqrt{T}/\Sigma$) decreases by adding the δ parameter. The profiles of the Toomre parameter represent those for small
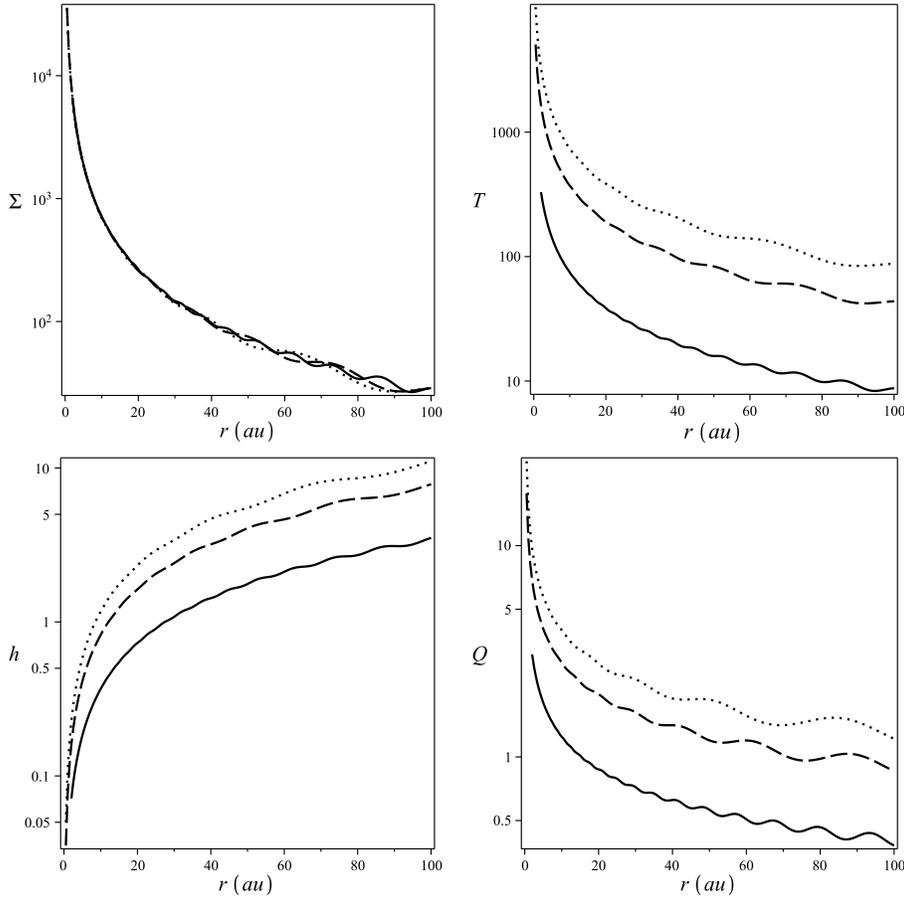
Fig. 3 Surface density, thickness, temperature and the Toomre parameter of the disk as a function of radius, for several values of $\dot{M}$. The surface density and the temperature are expressed in the cgs system, and the thickness and the distance are in AU. The solid lines represent $\dot{M} = 10^{-7} M_\odot \, \mathrm{yr}^{-1}$, the dashed lines represent $\dot{M} = 5 \times 10^{-7} M_\odot \, \mathrm{yr}^{-1}$ and the dotted lines represent $\dot{M} = 10^{-6} M_\odot \, \mathrm{yr}^{-1}$. The input parameters are set to the disk mass $M_{\mathrm{disk}} = 0.1 M_\odot$, the star mass $M_* = M_\odot$, the ratio of the specific heats is set to be $\gamma = 5/3$, $\beta_0 = 10$ and $\delta = 1.0$.

δ, only the outer part of the disk is gravitationally unstable, and the gravitational instability can extend to the inner radii by adding the δ parameter. For $\delta_{\mathrm{crit}} \sim 1.5$, the Toomre parameter in terms of radii $\gtrsim 5 \, \mathrm{AU}$ becomes smaller than the critical Toomre parameter ($Q_{\mathrm{crit}} \sim 1$) and the disk becomes gravitationally unstable. In other words, the profiles of the Toomre parameter represent the gravitational instability of the flow, which strongly depends on the cooling timescale, with a temperature dependence. This result is qualitatively consistent with direct numerical simulations (e.g. Cossins et al. 2010). The disk thickness increases by adding the δ parameter. It can be due to the increase of the temperature ($h \propto c_{\mathrm{s}} \propto \sqrt{T}$).

Equations (8) and (9) imply that

$$\frac{\nu_{(\delta \neq 0)}}{\nu_{(\delta = 0)}} = \left(\frac{c_{\mathrm{s}}}{c_{\mathrm{s}_0}} \right)^{-2\delta} . \tag{23}$$
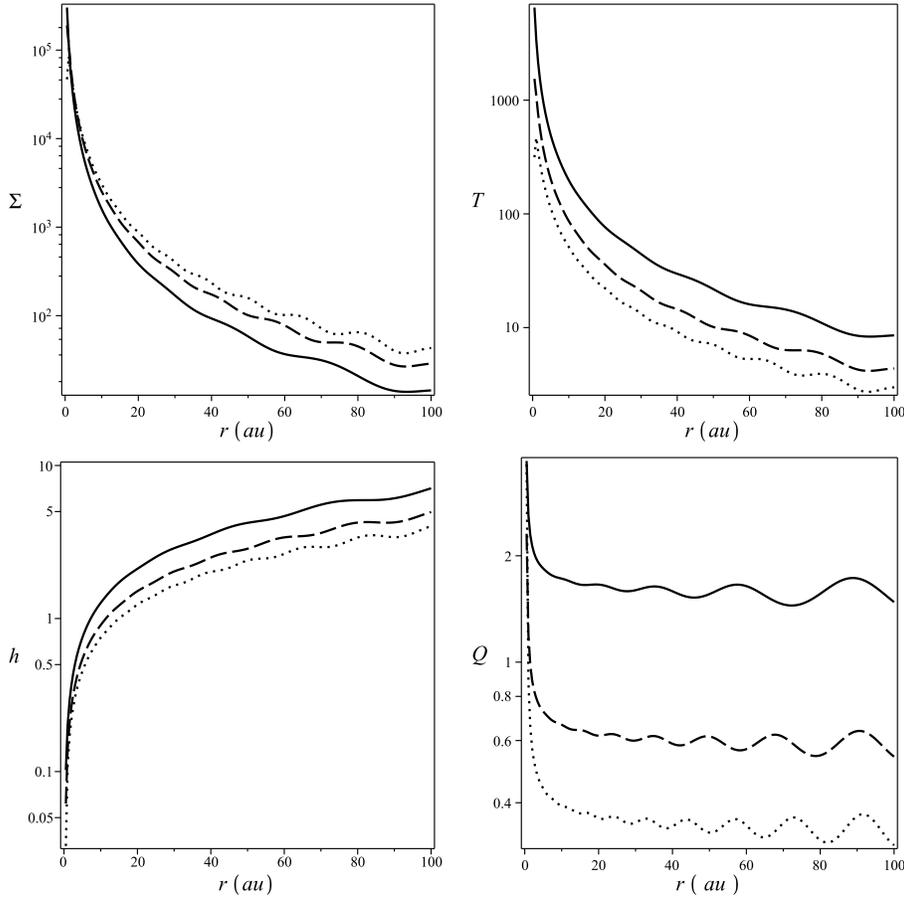
Fig. 4 Surface density, thickness, temperature and the Toomre parameter of the disk as a function of radius, for several values of $q = M_{disk}/M_*$. The surface density and the temperature are expressed in the cgs system, and the thickness and the distance are in AU. The solid lines represent $q = 0.05$, the dashed lines represent $q = 0.1$ and the dotted lines represent $q = 0.15$. The input parameters are set to the star mass $M_* = 1 M_\odot$, the mass accretion rate $\dot{M} = 10^{-6} M_\odot \, \text{yr}^{-1}$, the ratio of the specific heats is set to be $\gamma = 5/3$, $\beta_0 = 2$ and $\delta = 1.5$.

Since $c_s \geq c_{s_0}$ the right-hand side of the above equation is less than or equal to one. On the other hand, non-zero δ constrains the viscosity to lower values for hotter regions of the disk. The study of gravitational instability shows that it is enhanced with lower viscosity (Abbassi et al. 2006; Shadmehri & Khajenabi 2006; Khajenabi & Shadmehri 2007). Thus, the gravitational instability can be enhanced by adding the δ parameter for hotter regions, but there is a limitation for the value of the δ parameter that we discuss in the next section.

4.1.2 β_0 parameter

The influences of parameter β_0 are shown in Figure 2. Thus, as we know from simulations of a self-gravitating disk (Gammie 2001; Rice et al. 2003), the reduction of this parameter leads to grav-
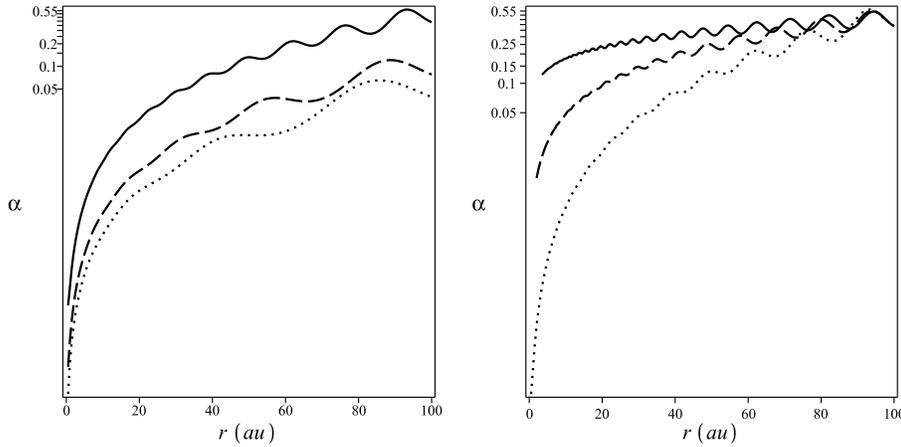
Fig. 5 Viscosity parameter of α as a function of radius (AU). The input parameters are set to the star mass $M_* = 1 M_\odot$, the mass accretion rate $\dot{M} = 10^{-6} M_\odot \text{ yr}^{-1}$ and the ratio of the specific heats is set to be $\gamma = 5/3$. *Left panel* is for several values of Gammie's parameter β_0, the solid line represents $\beta_0 = 1$, the dashed line represents $\beta_0 = 5$, the dotted line represents $\beta_0 = 10$ and $\delta = 1.5$. *Right panel* is for several values of parameter of δ, the solid line represents $\delta = 0.5$, the dashed line represents $\delta = 1.0$, the dotted line represents $\delta = 1.5$ and $\beta_0 = 1.0$.

itational instability and consequently fragmentation of the disk. The profiles of surface density show that it does not change by adding the β_0 parameter and the addition only results in small deviations over large radii. The disk temperature increases by adding the β_0 parameter. Therefore, the increase of this parameter reduces the rate of cooling. For a large value of β_0 (~ 10), the disk is gravitationally stable, but when reducing its value to 5, the gravitational instability can occur over large radii, and for a small value of it ($\beta_0 \sim 1$), we can expect gravitational instability over the whole the disk excluding the region near the star. These results are qualitatively consistent with direct numerical simulations of a protoplanetary disk (Gammie 2001; Rice et al. 2003; Cossins et al. 2010). Also, the solutions show that the disk thickness increases by adding the β_0 parameter.

4.1.3 The mass accretion rate

Rice & Armitage (2009) showed that beyond 1 AU the disk reaches a quasi-steady state in 20 000 years and the mass itself is redistributed to produce a state in which the accretion rate is largely independent of r. The mass accretion rate in their simulations finally reached $10^{-6} - 10^{-7} M_\odot \text{ yr}^{-1}$ (see fig. 4 in their paper). We will study the behavior of the present model in Figure 3 for several values of the mass accretion rate (10^{-7}, 5×10^{-7} and $10^{-6} M_\odot \text{ yr}^{-1}$). The solutions imply that the disk temperature is sensitive to the value of the mass accretion rate and increases by adding the mass accretion rate. However, the surface density is not sensitive to the mass accretion rate and only shows small variations over large radii. Thus, the behavior of the temperature only specifies the behavior of the Toomre parameter ($Q \propto \sqrt{T}/\Sigma$). The profiles of the Toomre parameter indicate that it increases by adding the mass accretion rate. Also, the solutions show the disk thickness increases by adding mass accretion rate, which is due to the increase of the disk temperature. The solutions show that for a low mass accretion rate ($\sim 10^{-7} M_\odot \text{ yr}^{-1}$), but cooling timescale with temperature dependence ($\delta \sim 1$), the gravitational instability can occur for radii $\gtrsim 10$ AU.

4.1.4 Mass ratio

As noted in the introduction, semi-analytical studies of self-gravitating disks are considering disks without central objects. This simplification is relevant to protostellar disks at the beginning of the accretion phase, during which the mass of the central object is small and only self-gravity of the disk plays an important role. Also, this simplification can correspond to disks at large radii because the effects of the central mass become unimportant in the outer regions of the disk. Moreover, the central object is important in the present model and its effects are not ignored. Thus, the present model does not have the limitations of previous studies of semi-analytical self-gravitating disks and can be applied for all regions of the disk. Figure 4 presents the effects of the ratio of the disk mass to the star mass $q = M_*/M_{\mathrm{disk}}$ in the present model. The solutions show the surface density increases and the temperature decreases. Each of the individual surface density increases and the temperature decreases can reduce the Toomre parameter. Thus, we expect that the Toomre parameter decreases by adding the q parameter and the profiles of the Toomre parameter confirm this behavior. The disk thickness profiles represent the disk thickness decreases by adding the disk mass. This property is qualitatively consistent with the two-dimensional study of a self-gravitating disk (e.g. Ghanbari & Abbassi 2004).

4.2 The Viscosity Parameter α

In the present model, the viscosity parameter α depends on the physical quantities of the disk (Equation 10), especially the local cooling rate which depends on the local temperature. The profiles of the viscosity parameter α show that its increases in radii agree with simulation results of Rice & Armitage (2009) and Rice et al. (2010). As mentioned in the introduction, the minimum cooling timescale depends on the equation of state (Rice et al. 2005) with fragmentation occurring for $\tau_{\mathrm{cool}} \leq 3\Omega^{-1}$ when the specific heat ratio $\gamma = 5/3$ (Gammie 2001). Rice et al. (2005) showed that fragmentation occurs for $\alpha > 0.06$ and this boundary is independent of the specific heat ratio γ. The left panel of Figure 5 presents the viscosity parameter α as a function of radius for several values of the β_0 parameter. The solutions show the viscosity α strongly depends on the β_0 parameter. In addition, the α parameter decreases by a factor of β_0. The solutions for small values of β_0 show the viscosity α can reach its critical value for fragmentation. The right panel of Figure 5 represents the viscosity parameter of α as a function of radius for several values of the δ parameter. The solutions which present the α parameter but exclude the outer region of the disk strongly depend on the δ parameter. For $\delta = 0.5$, the value of the viscosity α over the entire disk is in the region for fragmentation. However, Rafikov (2005) suggested that it is extremely difficult to see how fragmentation can occur within 10 AU even for relatively massive disks. In $\delta = 1.0$ and $\delta = 1.5$, the viscosity α in the inner disk ($r \lesssim 10$ and 40 AU, respectively) is well below that required for fragmentation.

The requirements for fragmentation are $Q \lesssim 1$ and $\alpha > 0.06$ (Rice et al. 2005, 2010; Rice & Armitage 2009). In the present model, apparently the increase of the δ parameter reduces the possibility of fragmentation (right panel of Fig. 5). On the other hand, the increase of the δ parameter can lead the disk into a situation of gravitational instability (Fig. 1). Thus, by having a suitable value for the δ parameter, the disk can obtain two requirements for fragmentation. Figures 1 and 5 imply that this value for small β_0 can be between 0.5 and 1.0.

5 SUMMARY AND DISCUSSION

In this paper, we have studied self-gravitating accretion disks in the presence of a Newtonian potential of a point mass. We have used a prescription for cooling that is introduced by Gammie (2001). However, due to recent results of Cossins et al. (2010), we have assumed that the cooling timescale in units of the dynamical timescale is a power-law function of temperature. As a result, the system of equations is non-linear and there is no self-similar solution for it. First, we have obtained asymptotic

solutions for the system of equations and then using them as boundary conditions, we integrated the system of equations numerically.

The solutions showed that the structure of the disk strongly depends on the present cooling function. Thus, by adding the importance degree of temperature in the cooling timescale, gravitational instability extends from outer to inner radii. The solutions showed that in the case of cooling with temperature dependence, the disk thickness increases. However, this change of thickness is important in the region with a smaller Toomre parameter. In the present model, the effect of physical parameters is studied, such as mass accretion rate, β_0 parameter and the ratio of the disk mass to central object mass. The results showed the structure of the disk is sensitive to these parameters. For example, the disk becomes gravitationally stable in a larger mass accretion rate. The gravitational instability can occur over a larger disk mass. Also, the disk thickness increases by adding the mass accretion rate and decreases by adding the ratio of the disk mass to the star mass. The study of the viscosity parameter α in the present model shows that it increases with radius and this result is consistent with direct numerical simulations (e.g. Rice & Armitage 2009; Rice et al. 2010). Also, the solution implies that the viscosity α in the outer part of the disk becomes larger than its critical value (~ 0.06), which might lead to the condition for fragmentation.

Here, the solutions imply that the disk thickness is very sensitive to input parameters. Thus, the present study in a two dimensional approach may be an interesting subject for future works. Also, it will be interesting to obtain a suitable δ value for fragmentation by direct numerical simulations.

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