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Orbit and spin evolution of synchronous binary stars on the main sequence

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Abstract A set of synchronous equations are derived from a set of non-synchronous equations. The analytical solutions are given by solving the set of differential equations. The results of the evolutionary trend of the spin-orbit interaction are that the semi-major axis gradually shrinks with time; the orbital eccentricity gradually decreases with time until orbital circularization occurs; the orbital period gradually shortens with time and the rotational angular velocity of the primary component gradually speeds up with time before the orbit achieves circularization. The theoretical results are applied to evolution of the orbit and spin of synchronous binary stars Algol A and B that are on the main sequence. The circularization time, lifetime and the evolutionary numerical solutions of orbit and spin when circularization time occurs are estimated for Algol A and B.

Key words: binaries: close — rotation — evolution

1 INTRODUCTION

Tidal friction plays an important role in evolution of the orbit and spin of a close binary system. The earliest author who explored this topic was Zahn (1965, 1966a,b,c, 1975). Alexander (1973) firstly studied the dynamical problem of tidal friction in a close binary system using the method employed by Darwin (1879). Later, Hut (1980, 1981) generalized the method given by Alexander (1973). He studied the stability of tidal equilibrium and tidal evolution in a close binary system using the method of energy and angular momentum. However, their research only dealt with a few examples of synchronization. Subsequent research on the synchronization of rotation was given by Zahn (1977, 1978). Rajamohan & Venkatakrishnan (1981) studied synchronization in binary stars. Giuricin et al. (1984a) investigated synchronization in eclipsing binary stars and Giuricin et al. (1984b) also researched synchronization in early-type spectroscopic binary stars. Zahn & Bouchet (1989) mainly studied the orbital circularization of late-type binary stars in the pre-main sequence phase and the theoretical results were given by Zahn (1989). Pan (1996) calculated the timescale for circularization using two mechanisms: one was an equilibrium tidal mechanism described by Zahn (1977), and the other was a purely hydrodynamic mechanism from Tassoul (1987). Keppens et al. (2000) studied the rotational evolution of a binary star system by considering both synchronization and circularization. Huang & Zeng (2000) also examined evolution of non-synchronized binary stars with masses $9M_\odot$ and $6M_\odot$. Meibom et al. (2005) observed tidal synchronization in detached solar-type binary stars and Meibom et al. (2006) also performed an observational study of tidal synchronization in solar–type binary stars in open clusters M35 and M34. Although Li (1998, 2004, 2009) studied some

methods for judging synchronization of rotation in binary stars, he has not studied the evolution of orbital rotation in synchronous binary stars. In this paper, we examine the evolutionary trends of orbit and spin in synchronous binary stars on the main sequence.

2 EVOLUTIONARY EQUATIONS OF SYNCHRONOUS BINARY STARS EXPERIENCING TIDAL FRICTION ON THE MAIN SEQUENCE

Equations describing secular evolution of the semi-major axis a, eccentricity e, and rotational angular velocity Ω due to tidal frication in non-synchronous binary stars are given by Zahn (1989)

$$
\begin{aligned}
\frac{1}{a}\frac{da}{dt} = &-\frac{12}{t_{\rm f}}q(1+q)\Big(\frac{R}{a}\Big)^8\Big\{\lambda^{22}\Big(1-\frac{\Omega}{\omega}\Big) \\
&+e^2\Big[\frac{3}{8}\lambda^{10}+\frac{1}{16}\lambda^{12}\Big(1-2\frac{\Omega}{\omega}\Big)-5\lambda^{22}\Big(1-\frac{\Omega}{\omega}\Big)+\frac{147}{16}\lambda^{32}\Big(3-2\frac{\Omega}{\omega}\Big)\Big]\Big\},
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
\frac{1}{e}\frac{de}{dt} = &-\frac{3}{t_{\rm f}}q(1+q)\Big(\frac{R}{a}\Big)^8\Big[\frac{3}{4}\lambda^{10}-\frac{1}{8}\lambda^{12}\Big(1-2\frac{\Omega}{\omega}\Big) \\
&-\lambda^{22}\Big(1-\frac{\Omega}{\omega}\Big)+\frac{49}{8}\lambda^{32}\Big(3-2\frac{\Omega}{\omega}\Big)\Big],
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
\frac{d}{dt}(I\Omega) = &\frac{6}{t_{\rm f}}q^2MR^2\Big(\frac{R}{a}\Big)^6\Big\{\lambda^{22}(\omega-\Omega)+e^2\Big[\frac{1}{8}\lambda^{12}(\omega-2\Omega) \\
&-5\lambda^{22}(\omega-\Omega)+\frac{49}{8}\lambda^{32}(3\omega-2\Omega)\Big]\Big\},
\end{aligned}
\tag{3}
$$

where M and R respectively denote the mass and radius of the primary star, $q=\frac{M'}{M}$, M' denotes the mass of the secondary star, ω denotes the orbital angular velocity (mean motion), and I denotes the moment of inertia. The convective friction time $t_{\rm f}$ and tidal coefficient λ^{lm} are given by Zahn & Bouchet (1989)

$$
t_{\rm f}=\Big(\frac{MR^2}{L}\Big)^{1/3}, \qquad \lambda^{lm}=\lambda_2(2\pi/|l\omega-m\Omega|).
$$

Here L denotes the luminosity of the primary star.

One can then derive the evolutionary equations of synchronous binary stars. Zahn & Bouchet (1989) pointed out that when the two components rotate and their orbital motion is synchronized so that $|l\omega-m\Omega|=\omega$, then all tidal coefficients are identical ($\lambda^{lm}=\lambda$) except for λ^{22}. Hence in Equations (1) – (3), $\lambda^{11}=\lambda^{10}=\lambda^{12}=\lambda^{32}=\lambda$, $\lambda^{22}\neq\lambda$. When we consider that the two components rotate synchronously, $\Omega=\omega$ or $\frac{\Omega}{\omega}=1$. Substituting these conditions into Equations (1) – (3), the secular Equations (1) – (3) are reduced to the following simplified synchronous secular equations

$$
\frac{1}{a}\frac{da}{dt}=-114q(1+q)\frac{\lambda}{t_{\rm f}}e^2\Big(\frac{R}{a}\Big)^8,
\tag{4}
$$

$$
\frac{1}{e}\frac{de}{dt}=-21q(1+q)\frac{\lambda}{t_{\rm f}}\Big(\frac{R}{a}\Big)^8,
\tag{5}
$$

$$
\frac{1}{\Omega}\frac{d\Omega}{dt}=36q^2\Big(\frac{MR^2}{I}\Big)\frac{\lambda}{t_{\rm f}}e^2\Big(\frac{R}{a}\Big)^6=36q^2\Big(\frac{M}{I}\Big)\frac{\lambda}{t_{\rm f}}e^2a^2\Big(\frac{R}{a}\Big)^8.
\tag{6}
$$

We may also write supplementary secular equations according to Kepler's third law

$$
\frac{1}{\omega}\frac{d\omega}{dt}=-\frac{3}{2}\frac{1}{a}\frac{da}{dt},
\tag{7}
$$

$$\frac{1}{P_{\mathrm{orb}}}\frac{dP_{\mathrm{orb}}}{dt} = \frac{3}{2}\frac{1}{a}\frac{da}{dt}, \tag{8}$$

$$\frac{1}{P_{\mathrm{rot}}}\frac{dP_{\mathrm{rot}}}{dt} = -\frac{1}{\Omega}\frac{d\Omega}{dt}, \tag{9}$$

where P_{orb} denotes the orbital period and P_{rot} denotes the rotational period.

Substituting Equation (5) for de/dt into the following equation, we get the timescale of circularization

$$t_{\mathrm{cir}} = \frac{e}{\frac{de}{dt}} = \frac{t_{\mathrm{f}}}{21q(1+q)\lambda}\left(\frac{a}{R}\right)^8. \tag{10}$$

In the following, we use an analytical method to solve the evolutionary Equations (4)–(9) with eccentricity e as an independent variable.

This paper considers the evolutionary trend of the orbital rotation of synchronous binaries before orbital circularization occurs when the stars are on the main sequence. We assume that the radius of the primary star R may be regarded as not varying, i.e. R is a constant during the main sequence phase of the star, but their separation or semi-major axis is variable due to tidal friction.

Combining Equation (4) with Equation (5), we obtain a differential equation

$$\frac{1}{a}\frac{da}{de} = \frac{38}{7}e. \tag{11}$$

Integrating this equation, we get

$$a = a_0 \exp\left[\frac{19}{7}(e^2 - e_0^2)\right]. \tag{12}$$

Substituting Equation (12) into Equation (5), we obtain

$$\frac{1}{e}\frac{de}{dt} = -21q(1+q)\frac{\lambda}{t_{\mathrm{f}}}\left(\frac{R}{a_0}\right)^8 \exp\left[-\frac{152}{7}(e^2 - e_0^2)\right]. \tag{13}$$

By setting $c = 152/7$, the differential Equation (13) can be written as

$$\frac{1}{e}\exp c(e^2 - e_0^2)de = \exp(-ce_0^2)\frac{\exp(ce^2)}{e}de = -21q(1+q)\frac{\lambda}{t_{\mathrm{f}}}\left(\frac{R}{a_0}\right)^8 dt.$$

Using the expansion of the series

$$\exp(ce^2) = 1 + ce^2 + \frac{1}{2}c^2e^4 + \frac{1}{3}c^3e^6 + ...$$

and integrating the above differential equation yields

$$\exp(-ce_0^2)\left[\ln(e) + \frac{1}{2}ce^2 + \frac{1}{8}c^2e^4 + ...\right]_{e_0}^{e} = -21q(1+q)\frac{\lambda}{t_{\mathrm{f}}}\left(\frac{R}{a_0}\right)^8(t - t_0).$$

We obtain a timescale in terms of e by neglecting the term with e^4

$$t - t_0 = -\frac{\ln(\frac{e}{e_0}) + \frac{1}{2}c(e^2 - e_0^2)}{21q(1+q)Q}. \tag{14}$$

Here

$$Q = \frac{\lambda}{t_{\mathrm{f}}}\left(\frac{R}{a_0}\right)^8 \exp\left(\frac{152}{7}e_0^2\right). \tag{15}$$

Combining Equation (4) with Equation (6), we derive equation

$$ada = -\frac{57}{18}\frac{(1+q)}{q}\left(\frac{I}{M}\right)\frac{d\Omega}{\Omega}.$$

Integrating this equation, we obtain

$$\Omega = \Omega_0 \exp\left[-\frac{9}{57}\left(\frac{q}{1+q}\right)\left(\frac{M}{I}\right)(a^2 - a_0^2)\right]. \tag{16}$$

From Equation (12) $a = a_0 \exp\left[\frac{19}{7}(e^2 - e_0^2)\right]$, $a^2 = a_0^2 \exp\left[2 \times \frac{19}{7}(e^2 - e_0^2)\right]$, hence

$$a^2 - a_0^2 = a_0^2\left\{\exp\left[\frac{38}{7}(e^2 - e_0^2)\right] - 1\right\}. \tag{17}$$

We obtain an expression for the angular velocity of the primary in terms of e

$$\Omega = \Omega_0 \exp\left\{-\frac{9}{57}\left(\frac{q}{1+q}\right)\frac{M}{I}a_0^2\left\{\exp\left[\frac{38}{7}(e^2 - e_0^2)\right] - 1\right\}\right\}. \tag{18}$$

The integrations of Equations (7) – (9) can be obtained as

$$\omega = \omega_0 \exp\left[-\frac{57}{14}(e^2 - e_0^2)\right], \tag{19}$$

$$P_{\text{orb}} = (P_0)_{\text{orb}} \exp\left[\frac{57}{14}(e^2 - e_0^2)\right], \tag{20}$$

$$P_{\text{rot}} = (P_0)_{\text{rot}} \exp\left\{\frac{9}{57}\left(\frac{q}{1+q}\right)\frac{M}{I}a_0^2\left\{\exp\left[\frac{38}{7}(a^2 - a_0^2)\right] - 1\right\}\right\}. \tag{21}$$

Next, we give analytical solutions of the secular evolutionary equations with time t as an independent variable on the main sequence.

For small values of e, as is the case for Algol A and B where $e = 0.015$ and $\frac{1}{2}ce^2 = 0.0024$, the second term on the right-hand side of Equation (14) may be neglected, and we find the eccentricity decreases with time as given in Equation (14)

$$e = e_0 \exp[-21q(1 + q)Q(t - t_0)], \tag{22}$$

$$e^2 - e_0^2 = e_0^2\{\exp[-42q(1 + q)Q(t - t_0)] - 1\}. \tag{23}$$

Substituting Equation (22) or Equation (23) into Equation (12), we get

$$a = a_0 \exp\left\{\frac{19}{7}e_0^2\{\exp[-42q(1 + q)Q(t - t_0)] - 1\}\right\}, \tag{24}$$

$$a^2 - a_0^2 = a_0^2\left\{\exp\left\{\frac{38}{7}e_0^2\{\exp[-42q(1 + q)Q(t - t_0)] - 1\}\right\} - 1\right\}. \tag{25}$$

Substituting Equation (23) into Equation (18) or Equation (25) into Equation (16), we obtain

$$\Omega = \Omega_0 \exp\left\{-\frac{9}{57}\frac{q}{(1+q)}\frac{M}{I}a_0^2\left\{\exp\left\{\frac{38}{7}e_0^2\{\exp[-42q(1 + q)Q(t - t_0)] - 1\}\right\} - 1\right\}\right\}. \tag{26}$$

The integrations of Equations (7) – (9) can be obtained as

$$\omega = \omega_0 \exp\left[-\frac{57}{14}(e^2 - e_0^2)\right], \tag{27}$$

$$P_{\text{orb}} = (P_0)_{\text{orb}} \exp\left\{\frac{57}{14}e_0\{\exp[-42q(1 + q)Q(t - t_0)] - 1\}\right\}, \tag{28}$$

$$P_{\text{rot}} = (P_{\text{rot}})_0 \exp\left\{\frac{9}{57}\frac{q}{(1+q)}\frac{M}{I}a_0^2\left\{\exp\left\{\frac{38}{7}e_0^2\{\exp[-42q(1 + q)Q(t - t_0)] - 1\}\right\} - 1\right\}\right\}. \tag{29}$$

3 EVOLUTION OF THE ORBIT AND SPIN IN SYNCHRONOUS BINARIES (ALGOL A AND B)

The eclipsing binary system Algol (β Per) consists of at least three components: A, B and C. There is actually also a massive but invisible fourth component D (Hopkins 1976). Algol A (primary) is a main sequence star (B_8 V) (Batten et al. 1989). Algol B (secondary) is a subgiant (g K_0) (Brancewicz et al. 1980; Batten et al. 1989). The separation between A and B is small and nearly constant, so the system Algol A and B is regarded as a synchronous binary system due to the tidal friction. Based on the work of Giuricin et al. (1984a), the mean rotational angular velocity of primary A is $v = 56$ km s^{-1}, the mean orbital angular velocity (the synchronized velocity) is $v_k = 55$ km s^{-1}, and based on the work of Tan (1985), $v_{\sin i} = 55$ km s^{-1} and $v_{\rm syn} = 55$ km s^{-1}. So A and B form a nearly synchronous binary system and considering the apparent descriptive method for judging the synchronization of rotation in binary stars, Li (2004, 2009) also concluded that Algol A and B represent a nearly synchronous binary system. Hence this paper selects Algol A and B as an example of synchronous binaries to calculate the orbital circularization and evolution of the orbit and spin before circularization occurs on the main sequence. For the data of Algol A and B, we cite the orbital period $P_{\rm orb} = 2.8672$ day, $a_0 = 14.03$ $R_\odot$, $M = 3.7$ $M_\odot$, $M' = 0.81$ $M_\odot$, $R = 2.74$ $R_\odot$, $R' = 3.60$ $R_\odot$, $q = M'/M = 0.22$, $T_{\rm e} = 12\,010$ K (Brancewicz & Dworak 1980). $e_0 = 0.015$ (Tomkin & Lambert 1978, Harrington 1984), $\Omega_0 = v/R = 2.9365 \times 10^{-5}$ rad s^{-1}, $t_{\rm f} = t_{f\odot}(M/M_\odot)^{1/3}(T_{\rm e}/T_{\rm e\odot})^{-4/3}$, $t_{f\odot} = 0.433$ yr, $T_{\rm e\odot} = 5770$ K, $t_{\rm f} = 0.2519$ yr and $L = 2.2$ $L_\odot$ (Popper 1980), where $T_{\rm e}$ is the effective temperature (Zahn & Bouchet 1989).

Yang et al. (2011) recently presented an XMM-Newton observation of the eclipsing binary Algol. Their results are useful in this field.

Zahn & Bouchet (1989) showed that when the coefficients λ^{lm} are all equal, then

$$\lambda = k_2. \tag{30}$$

Here k_2 is the apsidal motion which is constant. $\lambda = k_2$ is calculated from the formula given by Cowling (1938) and letting $k_1 = k_2 = k$

$$k = \frac{P_{\rm orb}/P'}{(\frac{R}{a})^5(1 + 16\frac{M'}{M}) + (\frac{R'}{a})^5(1 + 16(\frac{M}{M'})}. \tag{31}$$

Here P' denotes the period of the apsidal motion; $P' = 2.476$ year for Algol A and B as given by Hegedüs (1988). Substituting $P_{\rm orb}$, P', a, M, M', R and R' into the above formula, we get

$$\lambda = k = 0.003308. \tag{32}$$

The moment of inertia

$$I = KMR^2,$$

where K is calculated from the formula $\frac{1}{K} = \frac{3}{2}(n + \frac{5}{2})$ (Schatzman 1963), and the polytropic index $n = 3$ for a main sequence star (Algol A). So $K = 4/33 = 0.1212$.

$$\frac{M}{I} = \frac{M}{KMR^2} = \frac{1}{KR^2} = 0.02268 \times 10^{-10} \text{ km}^{-2}, \qquad \exp(152e_0^2/7) = 1.00489.$$

Substituting the values of k, $t_{\rm f}$, R, a and $\exp(152e_0^2/7)$ into Equation (15), we obtain

$$Q = 2.7 \times 10^{-8} \text{ yr}^{-1}. \tag{33}$$

Let us estimate the numerical solutions when the orbit of Algol A and B achieves circularization.

We firstly evaluate the timescale of circularization. Substituting the values of q, R_0, a_0, $t_{\rm f}$ and $\lambda = k = 0.003308$ into Equation (10), we obtain this value as

$$t_{\rm cir} = 6.4589 \times 10^6 \text{ yr}. \tag{34}$$

Next we estimate the numerical solution of the evolutionary trend of the orbit and spin when Algol A and B achieve orbital circularization. By letting the initial time $t_0 = 0$, and substituting $t_{\mathrm{cir}} = 6.4589$ yr into Equations (22), (24) and (26)–(29), we get $a = 14.0226 R_\odot$, $e = 0.0056$, $\omega = 2.1931$ rad d^{-1}, $P_{\mathrm{orb}} = 2.8649$ d, $\Omega = 2.5536$ rad d^{-1}, $P_{\mathrm{rot}} = 2.4605$ d, $\delta a = -0.0074 R_\odot$, $\delta e = -0.0094$, $\delta \omega = 0.0017$ rad d^{-1}, $\delta P_{\mathrm{orb}} = -0.0023$ d, $\delta \Omega = 0.0156$ rad d^{-1} and $\delta P_{\mathrm{rot}} = -0.0160$ d.

The lifetime is based on stellar mass-loss $\dot{M}$, i.e.

$$t_{\mathrm{life}} = \frac{M}{\dot{M}}. \tag{35}$$

Its value may be calculated from the formula given by Bowers & Deeming (1984)

$$\frac{d(M/M_\odot)}{dt} = 3 \times 10^{-8} \frac{(R/R_\odot)(L/L_\odot)}{(M/M_\odot)} (M_\odot/\mathrm{yr}) \tag{36}$$

or calculated from the formula given by Nieuwenhuijzen & de Jager (1990)

$$\log \dot{M} = -14.02 + 1.24 \log(L/L_\odot) + 0.81 \log(R/R_\odot) + 0.16 \log(M/M_\odot). \tag{37}$$

Substituting the values of M, R and L into Equations (36) and (35), we get the lifetime

$$t_{\mathrm{life}} = 7.5703 \times 10^7 \ \mathrm{yr}. \tag{38}$$

The time for the speed up of spin is

$$t_\Omega = \frac{\Omega}{\dot{\Omega}} = \frac{t_{\mathrm{f}}}{36 q^2 \left(\frac{M}{I}\right) k e^2 a^2 \left(\frac{R}{a}\right)^8} = 4.2447 \times 10^8 \ \mathrm{yr}. \tag{39}$$

The orbital decay time (the collapse time of the system) is

$$t_a = \frac{a}{\dot{a}} = \frac{t_{\mathrm{f}}}{114 q (1+q) k e^2 (R/a)^8} = 5.2273 \times 10^9 \ \mathrm{yr}. \tag{40}$$

Times derived in Equations (39) and (40) represent numerical values for Equations (6) and (4) respectively.

4 DISCUSSION AND CONCLUSIONS

(1) The set of Equations (4)–(6) describes synchronous binaries on the pre-main sequence, main sequence and post-main sequence phases according to the radius of a star. The radius of a late type star is variable due to the gravitational contraction on the pre-main sequence phase. The radius of a giant star is variable possibly due to the expansion of the shell in the post-main sequence phase. During the main sequence phase, the radius of a star is stable. Its radius can be regarded as a constant. These cases refer to the radius of the primary star because in Equations (4)–(6), R denotes the radius of the primary star. It is not applicable to the case of a giant star.

(2) The research in this paper differs from that of Zahn & Bouchet (1989) in some aspects. Zahn & Boucher's paper investigates the orbital evolution and circularization of binary stars during the pre-main sequence phase by using an analytical method and for non-synchronous equations by numerical integration. In the analytical method, the radii of binaries are variable due to gravitational contraction, but the semi-major axis is not variable in the main sequence phase. However, the present paper studies the orbit and spin of binary stars on the main sequence by implementing an analytical method in which the star's radius is not variable, but the semi-major axis is variable due to tidal friction. In Zahn & Boucher's paper, they must use numerical integration to solve non-synchronous equations. However, in the present paper, we use the analytical method

to solve the synchronous equations. Zahn & Boucher estimated that the eccentricity decreases from 0.005 to 0.0043 in 10 billion years for binary stars on the main sequence with masses 0.5 $M_\odot$ + 0.5 $M_\odot$. This paper estimates that the eccentricity decreases from 0.015 to 0.0056 in 6.45 million years for binary stars on the main sequence with masses 3.07 $M_\odot$ + 0.81 $M_\odot$. Hence, the different methods give differing results.

(3) The results of the solution for integrating differential equations using the analytical method are a bit different from those using the method of numerical integration. For example, the semi-major axis $a = 14.0226R_\odot$ for Algol A and B when the circularization time (6.4589×10^6 yr) is calculated by the former method and $a = 14.0126R_\odot$ by the latter method. However, this difference is very small.

(4) The circularization time is shorter than the lifetime, and the time required for the speed up of spin and the decay time (the collapse time of the system) are longer than the lifetime. Hence the latter are both meaningless.

(5) In the system of Algol A, B and C, the tidal friction in a triple star system (Kiseleva et al. 1998) and the perturbing effect of the third star (Algol C) (Li 2006) may decircularize the orbit of the secondary star (Algol B). In this paper, we do not consider these effects.

Based on these results, we make the following conclusions:

(1) The eccentricity gradually decreases with time until orbital circularization occurs.
(2) The semi-major axis gradually shrinks with time or with decreases in eccentricity.
(3) The orbital period gradually shortens with time or with circularization.
(4) The rotational angular velocity of the primary component gradually speeds up with time.

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