Bianchi type V universe with bulk viscous matter and time varying gravitational and cosmological constants

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Abstract We consider spatially homogeneous and anisotropic Bianchi type V space-time with a bulk viscous fluid source, and time varying gravitational constant G and cosmological term Λ. The coefficient of bulk viscosity ζ is assumed to be a simple linear function of the Hubble parameter H (i.e. $\zeta = \zeta_0 + \zeta_1 H$, where ζ_0 and ζ_1 are constants). The Einstein field equations are solved explicitly by using a law of variation for the Hubble parameter, which yields a constant value of the deceleration parameter. Physical and kinematical parameters of the models are discussed. The models are found to be compatible with the results of astronomical observations.

Key words: bulk viscosity — Bianchi V — relativity: equation of state — cosmology: cosmological parameters — miscellaneous

1 INTRODUCTION

In Einstein's theory of gravity, the Newtonian gravitational constant G and the cosmological term Λ are considered to be fundamental constants. The Newtonian constant of gravitation G plays the role of a coupling constant between geometry of space and matter in Einstein's field equations. In an evolving universe, it is natural to take this constant as a function of time. Ever since Dirac (1937) first considered the possibility of a variable G, there have been numerous modifications of general relativity to allow a variable G (Wesson 1980). Nevertheless, these theories have not gained wide acceptance. However, recently a modification (Berman 1991; Beesham 1986a; Lau 1985; Abdel-Rahman 1992) was proposed in Einstein's field equations that treated G and Λ as coupling variables within the framework of general relativity. Canuto & Narlikar (1980) showed that the G varying cosmology is consistent with whatever cosmological observations are available. Beesham (1986a); Levitt (1980); Abdel-Rahman (1990) discussed the possibility of an increasing G.

Cosmological models with a cosmological constant are currently serious candidates for describing the expansion history of the universe. The huge difference between the small cosmological constant inferred from observations and vacuum energy density resulting from quantum field theories has long been a difficult and perplexing problem (Weinberg 1989; Sahni & Starobinsky 2000; Carneiro 2003; Krauss & Turner 1995) for cosmologists and field theory researchers. A wide range of observations (Perlmutter et al. 1999; Garnavich et al. 1998; Riess et al. 2004; Schmidt et al. 1998) suggest that the universe possesses a non-zero cosmological constant. At the same time, theories are increasingly exploring the possibility that this parameter is dynamical, with the effective value in the

early universe perhaps being quite different from the one we measure today. One possible explanation for a small Λ term is to assume that it is dynamically evolving and is not constant, that is, as the universe evolves from an earlier hotter and denser epoch, the effective cosmological term also evolves and decreases to its present value (Özer & Taha 1987; Freese et al. 1987; Frieman et al. 1995; Coble et al. 1997). Bertolami (1986) obtained the time-dependent G and Λ solutions. Cosmological models with variable G and Λ terms have been studied by a number of authors for homogeneous isotropic (Abdel-Rahman 1990; Kalligas et al. 1992; Abdussattar & Vishwakarma 1997; Beesham 1986b; Arbab 1997) and anisotropic (Singh & Beesham 2010; Saha 2006; Bali & Tinker 2009; Singh et al. 2008; Yadav 2010; Pradhan et al. 2005; Yadav et al. 2012) space-times.

The investigations of relativistic cosmological models usually assume the cosmic fluid to be a perfect fluid. However, these models do not incorporate dissipative mechanisms responsible for smoothing out initial anisotropies. It is believed that during neutrino decoupling, the matter behaved like a viscous fluid (Klimek 1976) in the early stages of evolution. Coley (1990) studied Bianchi V viscous fluid cosmological models for a barotropic fluid distribution. Misner (1967); Murphy (1973); Heller & Klimek (1975) studied the role of viscosity in avoiding the initial big bang singularity. Padmanabhan & Chitre (1987) investigated the effect of bulk viscosity on the evolution of the universe at large. They showed that bulk viscosity leads to inflationary like solutions. Saha (2005) discussed a Bianchi type I universe with a viscous fluid. Pradhan et al. (2004) scrutinized cosmological models with viscous fluid in an LRS Bianchi type V universe with varying Λ. Bulk viscous cosmological models with time-dependent G and Λ terms have been studied by Bali & Tinker (2009); Arbab (1998); Beesham et al. (2000), where bulk viscosity is taken as a power function of energy density. Recently Singh & Baghel (2010) examined Bianchi type V cosmological models with bulk viscosity, where the coefficient of bulk viscosity is assumed to be a power function of energy density ρ or volume expansion θ.

Among the physical quantities of interest in cosmology, the deceleration parameter q is currently a serious candidate to describe the dynamics of the universe. The prediction of standard cosmology, that the present universe is decelerating, is contradictory to the recent observational evidence of high redshift type Ia supernovae (Riess et al. 1998; Knop et al. 2003; Tonry et al. 2003). Observations reveal that instead of slowing down, the expanding universe is speeding up. Models with a constant deceleration parameter have recently received considerable attention. The law of variation for the Hubble parameter was initially proposed by Berman (1983) for FRW models that yield a constant value of the deceleration parameter. Cosmological models with such a law of variation for the Hubble parameter have been studied by a number of authors (Maharaj & Naidoo 1993; Singh & Kumar 2009; Reddy et al. 2007). Recently Singh & Baghel (2009b,a) proposed a similar law for the Hubble parameter and generated the solution for Bianchi type V space-time in general relativity. According to the proposed law, the relation between the Hubble parameter H and average scale factor R is given by

$$H = \beta R^{-m}, \tag{1}$$

where $\beta > 0$ and $m \geq 0$ are constants. For this variation law the deceleration parameter q comes out to be constant, i.e.

$$q = m - 1. \tag{2}$$

For $m > 1$, the model represents a decelerating universe and $m < 1$ corresponds to the accelerating phase of the universe. When $m = 1$, we obtain $H \sim \frac{1}{t}$ and $q = 0$. Therefore galaxies move with a constant speed and the model represents an anisotropic Milne universe (Landsberg & Evans 1977) for $m = 1$. For $m = 0$, we get $H = \beta$ and $q = -1$. Thus the observed Hubble parameter is a true constant equal to its present value H_0 and the model represents an accelerating phase of the universe.

The relevance of the study of Bianchi type V cosmological models has already been discussed in our earlier papers (Singh & Baghel 2009b,a), where we studied the viscous fluid models in some detail. As a natural sequel to that study, here we incorporate time varying G and Λ terms in the bulk

viscous Bianchi type V models and exactly solve the coupled field equations. In this paper we take the coefficient of bulk viscosity as a simple linear function of the Hubble parameter H (Meng et al. 2007).

2 METRIC AND FIELD EQUATIONS

We consider Bianchi type V space-time in an orthogonal form represented by the line-element

$$ds^2 = -dt^2 + A^2(t)dx^2 + e^{2\alpha x}\left\{B^2(t)dy^2 + C^2(t)dz^2\right\}, \tag{3}$$

where α is a constant. We assume that the cosmic matter is represented by the energy-momentum tensor of an imperfect bulk viscous fluid

$$T_{ij} = (\rho + \bar{p})v_i v_j + \bar{p}g_{ij}, \tag{4}$$

where $\bar{p}$ is the effective pressure given by

$$\bar{p} = p - \zeta v^i_{;i}, \tag{5}$$

satisfying a linear equation of state

$$p = \omega\rho. \tag{6}$$

Here p is the equilibrium pressure, ρ is the energy density of matter, ζ is the coefficient of bulk viscosity and v^i is the flow vector of the fluid satisfying $v_i v^i = -1$. The semicolon in Equation (5) stands for covariant differentiation. On thermodynamical grounds, bulk viscosity coefficient ζ is positive, assuring that the viscosity pushes the dissipative pressure $\bar{p}$ towards negative values. However, correction to the thermodynamical pressure p due to bulk viscosity pressure is very small. Therefore, the dynamics of cosmic evolution do not fundamentally change by the inclusion of a viscosity term in the energy-momentum tensor.

The Einstein field equations with time-dependent cosmological term Λ and gravitational constant G are

$$R_{ij} - \frac{1}{2}R^k_k g_{ij} = -8\pi G(t)T_{ij} + \Lambda(t)g_{ij}. \tag{7}$$

For the metric (3) and matter distribution (4) in a comoving system of coordinates ($v_i = -\delta^4_i$), Equation (7) yields

$$8\pi G\bar{p} - \Lambda = \frac{\alpha^2}{A^2} - \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC}, \tag{8}$$

$$8\pi G\bar{p} - \Lambda = \frac{\alpha^2}{A^2} - \frac{\ddot{C}}{C} - \frac{\ddot{A}}{A} - \frac{\dot{C}\dot{A}}{CA}, \tag{9}$$

$$8\pi G\bar{p} - \Lambda = \frac{\alpha^2}{A^2} - \frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB}, \tag{10}$$

$$8\pi G\rho + \Lambda = -\frac{3\alpha^2}{A^2} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC}, \tag{11}$$

$$0 = \frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}, \tag{12}$$

where an overhead dot (.) denotes ordinary differentiation with respect to cosmic time t. Due to the contracted Bianchi identity, the divergence of Einstein tensor $G_{ij} = R_{ij} - \frac{1}{2}R^k_k g_{ij}$ is zero and we get

$$8\pi G\left\{\dot{\rho} + (\rho + \bar{p})\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\right\} + 8\pi\dot{G} + \dot{\Lambda} = 0. \tag{13}$$

The usual conservation equation $T^{j}_{i\,;j} = 0$ splits the above equation into

$$\dot{\rho} + (\rho + \bar{p})\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0 \tag{14}$$

and

$$8\pi\rho\dot{G} + \dot{\Lambda} = 0. \tag{15}$$

From (15), one concludes that, when Λ is a constant or zero, G turns out to be a constant for non-zero energy density.

We define the average scale factor R for Bianchi V space-time as $R^3 = ABC$. From Equations (8)–(10) and (12), we obtain

$$\frac{\dot{A}}{A} = \frac{\dot{R}}{R}, \tag{16}$$

$$\frac{\dot{B}}{B} = \frac{\dot{R}}{R} - \frac{k}{R^3}, \tag{17}$$

$$\frac{\dot{C}}{C} = \frac{\dot{R}}{R} + \frac{k}{R^3}, \tag{18}$$

with k being a constant of integration. On integration, Equations (16)–(18) give

$$A = m_1 R, \tag{19}$$

$$B = m_2 R \exp\left(-k \int \frac{dt}{R^3}\right), \tag{20}$$

$$C = m_3 R \exp\left(k \int \frac{dt}{R^3}\right), \tag{21}$$

where m_1, m_2 and m_3 are constants of integration satisfying $m_1 m_2 m_3 = 1$.

In analogy with an FRW universe, we define a generalized Hubble parameter H and generalized deceleration parameter q as

$$H = \frac{\dot{R}}{R} = \frac{1}{3}(H_1 + H_2 + H_3) \tag{22}$$

and

$$q = -1 - \frac{\dot{H}}{H^2}, \tag{23}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble's factors along x, y and z directions respectively.

We introduce volume expansion θ and shear σ as usual

$$\theta = v^i_{;i} \quad \text{and} \quad \sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij},$$

with σ^{ij} being the shear tensor. For the Bianchi V metric, expressions for θ and σ come out to be

$$\theta = \frac{3\dot{R}}{R} \tag{24}$$

and

$$\sigma = \frac{k}{R^3}. \tag{25}$$

We can express Equations (8)–(11) and (14) in terms of R, H, q and σ as

$$8\pi G\bar{p} - \Lambda = (2q-1)H^2 - \sigma^2 + \frac{\alpha^2}{R^2}, \tag{26}$$

$$8\pi G\rho + \Lambda = 3H^2 - \sigma^2 - \frac{3\alpha^2}{R^2}, \tag{27}$$

$$\dot{\rho} + 3(\rho + \bar{p})H = 0. \tag{28}$$

It can be noted that energy density of the universe is a positive quantity. It is believed that at the early stages of evolution, when the average scale factor R was close to zero, the energy density of the universe was infinitely large. On the other hand, with expansion of the universe, i.e. with an increase of R, the energy density decreases and an infinitely large R corresponds to ρ close to zero. In that case from (27), we obtain $\frac{\rho_v}{\rho_c} \to 1$, where $\rho_v = \frac{\Lambda}{8\pi G}$ and $\rho_c = \frac{3H^2}{8\pi G}$. For $\Lambda \geq 0$, $\rho < \rho_c$. Also from (27), we observe that $0 < \frac{\sigma^2}{\theta^2} < \frac{1}{3}$ and $0 < \frac{8\pi G\rho}{\theta^2} < \frac{1}{3}$ for $\Lambda \geq 0$. Thus a positive Λ restricts the upper limit of anisotropy whereas a negative Λ will increase the anisotropy.
From (26) and (27), we get

$$\frac{d\theta}{dt} = \Lambda + 12\pi G\zeta\theta - 4\pi G(\rho + 3p) - 2\sigma^2 - \frac{\theta^2}{3}, \tag{29}$$

which is the Raychaudhuri equation for the given distribution. We observe that for negative Λ and in the absence of viscosity, the universe will always be in a decelerating phase provided the strong energy conditions (Hawking & Ellis 1975) hold, whereas in the presence of viscosity, positive Λ will slow down the rate of decrease of volume expansion. Also, $\dot{\sigma} = -\sigma\theta$ implies that σ decreases in an evolving universe and for an infinitely large value of R, σ becomes negligible.

3 SOLUTION OF THE FIELD EQUATIONS

Integrating (1), we obtain
$$R = (m\beta t + t_1)^{\frac{1}{m}} \qquad \text{for } m \neq 0 \tag{30}$$

and
$$R = \exp\{\beta(t - t_0)\} \qquad \text{for } m = 0, \tag{31}$$

where t_1 and t_0 are constants of integration. From (19)–(21) with the use of (30), we obtain

$$A = m_1(m\beta t + t_1)^{\frac{1}{m}},$$

$$B = m_2(m\beta t + t_1)^{\frac{1}{m}} \exp\left\{\frac{-k(m\beta t + t_1)^{\frac{m-3}{m}}}{\beta(m-3)}\right\},$$

$$C = m_3(m\beta t + t_1)^{\frac{1}{m}} \exp\left\{\frac{k(m\beta t + t_1)^{\frac{m-3}{m}}}{\beta(m-3)}\right\}.$$

For this solution, the metric (3) assumes the following form after a suitable transformation of coordinates

$$ds^2 = -dT^2 + (m\beta T)^{\frac{2}{m}} dX^2 + (m\beta T)^{\frac{2}{m}} \exp\left\{2\alpha X - \frac{2k(m\beta T)^{\frac{m-3}{m}}}{\beta(m-3)}\right\} dY^2$$

$$+ (m\beta T)^{\frac{2}{m}} \exp\left\{2\alpha X + \frac{2k(m\beta T)^{\frac{m-3}{m}}}{\beta(m-3)}\right\} dZ^2. \tag{32}$$

Equations (19)–(21) together with (31) give

$$A = m_1 \exp\{\beta(t - t_0)\},$$

$$B = m_2 \exp\left\{\beta(t - t_0) + \frac{k}{3\beta} e^{-3\beta(t-t_0)}\right\},$$

$$C = m_3 \exp\left\{\beta(t - t_0) - \frac{k}{3\beta} e^{-3\beta(t-t_0)}\right\}.$$

The line-element (3) for this solution can be written as

$$ds^2 = -dT^2 + e^{2\beta T} dX^2 + \exp\left(2\alpha X + 2\beta T + \frac{2k}{3\beta} e^{-3\beta T}\right) dY^2$$

$$+ \exp\left(2\alpha X + 2\beta T - \frac{2k}{3\beta} e^{-3\beta T}\right) dZ^2 . \tag{33}$$

4 DISCUSSION

To determine the coefficient of bulk viscosity, ζ is assumed to be a simple linear function of the Hubble parameter H (Meng et al. 2007), i.e.

$$\zeta = \zeta_0 + \zeta_1 H, \tag{34}$$

where ζ_0 (≥ 0) and ζ_1 are constants. For this choice, Equation (28) reduces to

$$\dot{\rho} + 3(1 + \omega)H\rho = 9(\zeta_0 + \zeta_1 H)H^2. \tag{35}$$

We discuss the models for $m \neq 0$ and $m = 0$.

4.1 Cosmology for $m \neq 0$

For the model (32), the average scale factor R is given by

$$R = (m\beta T)^{\frac{1}{m}}. \tag{36}$$

Volume expansion θ, the Hubble parameter H and shear scalar σ for this model are

$$\theta = 3H = \frac{3}{mT}, \tag{37}$$

$$\sigma = k(m\beta T)^{\frac{-3}{m}}. \tag{38}$$

Using (37) in (34) and (35), we obtain

$$\zeta = \zeta_0 + \frac{\zeta_1}{mT}, \tag{39}$$

$$\rho = \frac{9\zeta_0}{m(3 + 3\omega - m)T} + \frac{9\zeta_1}{(3 + 3\omega - 2m)m^2 T^2} + \frac{a}{T^{3(1+\omega)/m}}, \tag{40}$$

where a is an integration constant. The gravitational constant G and the cosmological term Λ are obtained as

$$G = \frac{1}{4\pi}\left[\frac{mT^2\left\{\alpha^2(m\beta T)^{\frac{4}{m}} + k^2\right\} - (m\beta T)^{\frac{6}{m}}}{\left(\frac{3m\zeta_0}{m-3\omega-3}\right)T^{\frac{m+6}{m}} + \left(\frac{6\zeta_1}{2m-3\omega-3}\right)T^{\frac{6}{m}} - ma(1+\omega)T^{\frac{2m+3(1-\omega)}{m}}}\right], \tag{41}$$

$$\Lambda = 18 \left\{ \frac{a}{9T^{\frac{3(1+\omega)}{m}}} + \frac{\zeta_0}{(3+3\omega-m)mT} + \frac{\zeta_1}{(3+3\omega-2m)m^2T^2} \right\}$$

$$\cdot \left[\frac{(m\beta T)^{\frac{6}{m}} - mT^2 \left\{ \alpha^2 (m\beta T)^{\frac{4}{m}} + k^2 \right\}}{\left(\frac{3m\zeta_0}{m-3\omega-3} \right) T^{\frac{m+6}{m}} + \left(\frac{6\zeta_1}{2m-3\omega-3} \right) T^{\frac{6}{m}} - ma(1+\omega)T^{\frac{2m+3(1-\omega)}{m}}} \right]$$

$$+ \frac{3}{m^2T^2} - \frac{k^2}{(m\beta T)^{\frac{6}{m}}} - \frac{3\alpha^2}{(m\beta T)^{\frac{2}{m}}}. \tag{42}$$

We observe that the model is not tenable for $m = 3$. The model has a singularity at $T = 0$. At $T = 0$, ρ, Λ, ζ, θ and σ all diverge whereas G becomes a constant provided $m > 3$. In the limit of large times, ρ, θ and σ become zero, $\zeta = \zeta_0$ and $G \to \infty$, $\Lambda \to$ constant for $m > 3$. Also for $T \to \infty$, $\sigma/\theta \to 0$ when $m < 3$. Therefore the model asymptotically approaches isotropy. For large values of T, the model becomes conformally flat (Singh & Baghel 2009b). Behaviors of the matter energy density ρ and vacuum energy density Λ with respect to cosmic time t are plotted in Figures 1 and 2. The integral

$$\int_{T_0}^{T} \frac{dt}{R(t)} = \frac{1}{\beta(m-1)} \left[(m\beta T)^{\frac{m-1}{m}} \right]_{T_0}^{T} \tag{43}$$

is finite provided $m \neq 1$. Therefore, a particle horizon exists in the model. It should be noted that for $m = 1$, the model does not have a horizon.

4.2 Cosmology for $m = 0$

We now discuss the model (33). Average scale factor R, expansion scalar θ, the Hubble parameter H, shear σ and deceleration parameter q are given by

$$R = e^{\beta T}, \tag{44}$$

$$\theta = 3H = 3\beta, \tag{45}$$

$$\sigma = ke^{-3\beta T}, \tag{46}$$

$$q = -1. \tag{47}$$

We obtain the bulk viscosity coefficient ζ and matter density ρ as

$$\zeta = \zeta_0 + \zeta_1 \beta, \tag{48}$$

$$\rho = 3\beta(\zeta_0 + \zeta_1 \beta) + be^{-3\beta(1+\omega)T}, \tag{49}$$

where b is a constant of integration. Expressions for gravitational constant G and cosmological term Λ are

$$G = -\frac{(k^2 + \alpha^2 e^{4\beta T})}{4\pi b(1+\omega)e^{3(1-\omega)\beta T}}, \tag{50}$$

$$\Lambda = 3\beta^2 + \left(\frac{1-\omega}{1+\omega} \right) \frac{k^2}{e^{6\beta T}} - \left(\frac{1+3\omega}{1+\omega} \right) \frac{\alpha^2}{e^{2\beta T}} + \frac{6\beta(\zeta_0 + \zeta_1\beta)(k^2 + \alpha^2 e^{4\beta T})}{b(1+\omega)^2 e^{3(1-\omega)\beta T}}. \tag{51}$$

The model has no initial singularity. The expansion scalar θ is constant throughout the evolution of the universe and therefore the model represents uniform expansion. Also $q = -1$ shows that the expansion of the model is accelerating at a constant rate. At $T = 0$, R, σ, ζ, ρ, G and Λ all are constant. The energy density decreases as time increases and becomes constant at late times. Also G and Λ become infinite for very large values of T whereas they tend to zero for $\omega < -\frac{1}{3}$ at late times. We observe that the gravitational constant G is positive for $b < 0$ whereas G is negative for $b > 0$.
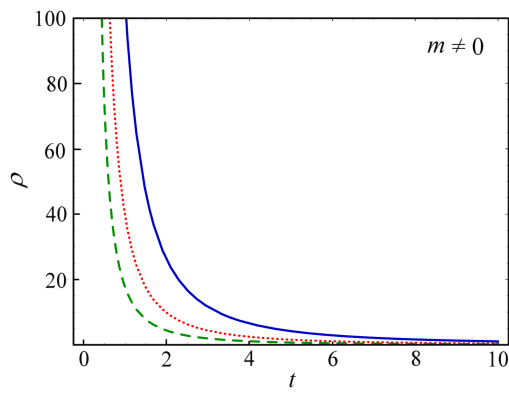
Fig. 1 Variation of matter energy density ρ with cosmic time t in case of $m \neq 0$ for $\omega = 0$ (*solid line*), $\omega = 1/3$ (*dotted line*) and $\omega = 1$ (*dashed line*).
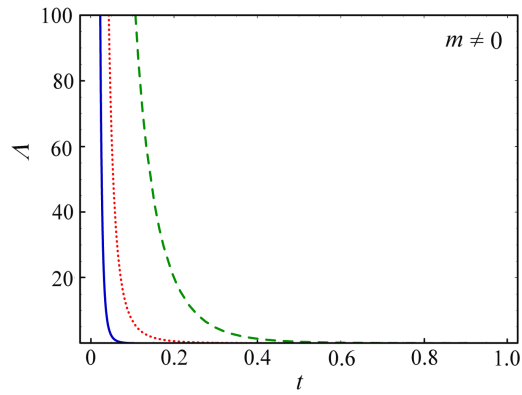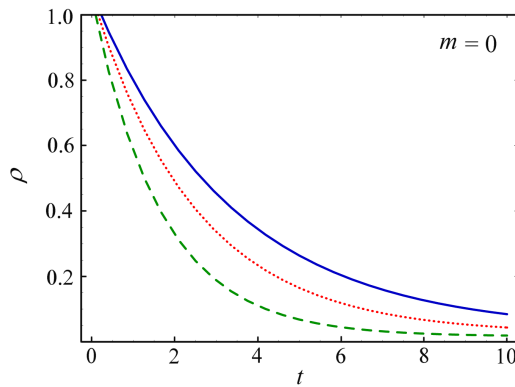

Fig. 2 Variation of vacuum energy density Λ with cosmic time t in case of $m \neq 0$ for $\omega = 0$ (*solid line*), $\omega = 1/3$ (*dotted line*) and $\omega = 1$ (*dashed line*).

Fig. 3 Variation of matter energy density ρ with cosmic time t in case of $m = 0$ for $\omega = 0$ (*solid line*), $\omega = 1/3$ (*dotted line*) and $\omega = 1$ (*dashed line*).
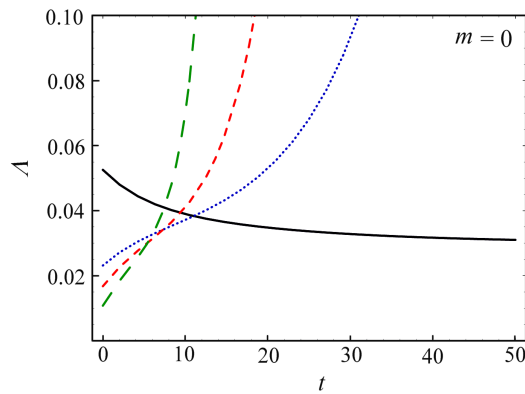

Fig. 4 Variation of vacuum energy density Λ with cosmic time t in case of $m = 0$ for $\omega = -1/2$ (*solid line*), $\omega = 0$ (*dotted line*), $\omega = 1/3$ (*short-dashed line*) and $\omega = 1$ (*long-dashed line*).

The possibility of a negative gravitational constant G has been discussed by Starobinskii (1981), who concluded that the effective gravitational constant may have changed sign in the early universe. As $T \to \infty$, the ratio σ/θ becomes zero. Therefore the model approaches isotropy for large values of T. In this case behaviors of the matter energy density ρ and vacuum energy density Λ with respect to cosmic time t are plotted in Figures 3 and 4.

5 CONCLUSIONS

In this paper, we have investigated spatially homogeneous and anisotropic Bianchi type V space-time with bulk viscous matter and a time-dependent gravitational constant G and cosmological term Λ used in general relativity. The field equations have been solved exactly by using a law of variation

for the generalized Hubble parameter H. Two universe models have been obtained and the physical behavior of the models is discussed. The coefficient of bulk viscosity ζ is assumed to be a simple linear function of the Hubble parameter H (i.e. $\zeta = \zeta_0 + \zeta_1 H$, where ζ_0 and ζ_1 are constants). For $\zeta_0 = 0 = \zeta_1$, we recover perfect fluid models. In the case of cosmology for $m \neq 0$, the universe starts from a singular state whereas cosmology for $m = 0$ follows a non-singular start. We observe that the presence of bulk viscosity increases the value of matter density. For the models obtained, $\sigma/\theta \rightarrow 0$ as $T \rightarrow \infty$. Thus the models approach isotropy at late times. From Equation (2), one concludes that for $m > 1$, the model represents a decelerating universe whereas for $0 \leq m < 1$, it gives rise to an accelerating universe. When $m = 1$, we obtain $H = \frac{1}{T}$ and $q = 0$, so that every galaxy moves with a constant speed.

References

Abdel-Rahman, A.-M. M. 1990, General Relativity and Gravitation, 22, 655

Abdel-Rahman, A.-M. M. 1992, Phys. Rev. D, 45, 3497

Abdussattar, & Vishwakarma, R. G. 1997, Classical and Quantum Gravity, 14, 945

Arbab, A. I. 1997, General Relativity and Gravitation, 29, 61

Arbab, A. I. 1998, General Relativity and Gravitation, 30, 1401

Bali, R., & Tinker, S. 2009, Chinese Physics Letters, 26, 029802

Beesham, A. 1986a, International Journal of Theoretical Physics, 25, 1295

Beesham, A. 1986b, Nuovo Cimento B, 96, 17

Beesham, A., Ghosh, S. G., & Lombard, R. G. 2000, General Relativity and Gravitation, 32, 471

Berman, M. S. 1983, Nuovo Cimento B, 74, 182

Berman, M. S. 1991, General Relativity and Gravitation, 23, 465

Bertolami, O. 1986, Nuovo Cimento B, 93, 36

Canuto, V. M., & Narlikar, J. V. 1980, ApJ, 236, 6

Carneiro, S. 2003, International Journal of Modern Physics D, 12, 1669

Coble, K., Dodelson, S., & Frieman, J. A. 1997, Phys. Rev. D, 55, 1851

Coley, A. A. 1990, General Relativity and Gravitation, 22, 3

Dirac, P. A. M. 1937, Nature, 139, 323

Freese, K., Adams, F. C., Frieman, J. A., & Mottola, E. 1987, Nuclear Physics B, 287, 797

Frieman, J. A., Hill, C. T., Stebbins, A., & Waga, I. 1995, Physical Review Letters, 75, 2077

Garnavich, P. M., Jha, S., Challis, P., et al. 1998, ApJ, 509, 74

Hawking, S. W., & Ellis, G. F. R. 1975, in The Large Scale Structure of Space-Time (Cambridge: Cambridge Univ. Press), 88

Heller, M., & Klimek, Z. 1975, Ap&SS, 33, L37

Kalligas, D., Wesson, P., & Everitt, C. W. F. 1992, General Relativity and Gravitation, 24, 351

Klimek, Z. 1976, Nuovo Cimento B, 35, 249

Knop, R. A., Aldering, G., Amanullah, R., et al. 2003, ApJ, 598, 102

Krauss, L. M., & Turner, M. S. 1995, General Relativity and Gravitation, 27, 1137

Landsberg, P. T., & Evans, D. A. 1977, Mathematical Cosmology. an Introduction (Oxford: Clarendon Press)

Lau, Y.-K. 1985, Australian Journal of Physics, 38, 547

Levitt, L. S. 1980, Nuovo Cimento Lettere, 29, 23

Maharaj, S. D., & Naidoo, R. 1993, Ap&SS, 208, 261

Meng, X.-H., Ren, J., & Hu, M.-G. 2007, Communications in Theoretical Physics, 47, 379

Misner, C. W. 1967, Nature, 214, 40

Murphy, G. L. 1973, Phys. Rev. D, 8, 4231

Özer, M., & Taha, M. O. 1987, Nuclear Physics B, 287, 776

Padmanabhan, T., & Chitre, S. M. 1987, Physics Letters A, 120, 433

Perlmutter, S., Aldering, G., Goldhaber, G., et al. 1999, ApJ, 517, 565

Pradhan, A., Yadav, L., & Yadav, A. K. 2004, Czechoslovak Journal of Physics, 54, 487

Pradhan, A., Yadav, A. K., & Yadav, L. 2005, Czechoslovak Journal of Physics, 55, 503

Reddy, D. R. K., Naidu, R. L., & Adhav, K. S. 2007, Ap&SS, 307, 365

Riess, A. G., Filippenko, A. V., Challis, P., et al. 1998, AJ, 116, 1009

Riess, A. G., Strolger, L.-G., Tonry, J., et al. 2004, ApJ, 607, 665

Saha, B. 2005, Modern Physics Letters A, 20, 2127

Saha, B. 2006, Ap&SS, 302, 83

Sahni, V., & Starobinsky, A. 2000, International Journal of Modern Physics D, 9, 373

Schmidt, B. P., et al. 1998, ApJ, 507, 46

Singh, C. P., & Beesham, A. 2010, International Journal of Modern Physics A, 25, 3825

Singh, C. P., & Kumar, S. 2009, International Journal of Theoretical Physics, 48, 2401

Singh, J. P., & Baghel, P. S. 2009a, Electronic Journal of Theoretical Physics, 6, 85

Singh, J. P., & Baghel, P. S. 2009b, International Journal of Theoretical Physics, 48, 449

Singh, J. P., & Baghel, P. S. 2010, International Journal of Theoretical Physics, 49, 2734

Singh, J. P., Pradhan, A., & Singh, A. K. 2008, Ap&SS, 314, 83

Starobinskii, A. A. 1981, Soviet Astronomy Letters, 7, 36

Tonry, J. L., Schmidt, B. P., Barris, B., et al. 2003, ApJ, 594, 1

Weinberg, S. 1989, Reviews of Modern Physics, 61, 1

Wesson, P. S. 1980, Gravity, particles, and astrophysics - A review of modern theories of gravity and G-variability, and their relation to elementary particle physics and astrophysics, Astrophysics and Space Science Library (Reidel, Dordrecht), 79

Yadav, A. K. 2010, International Journal of Theoretical Physics, 49, 1140

Yadav, A. K., Pradhan, A., & Singh, A. K. 2012, Ap&SS, 337, 379