Bianchi–V string cosmological model and late time acceleration

Anil Kumar Yadav

Department of Physics, Anand Engineering College, Keetham, Agra - 282 007, India;
abanilyadav@yahoo.co.in

Abstract We have searched for the existence of the late time acceleration of the universe with string fluid as the source of matter in Bianchi–V space-time. To derive a deterministic solution, we choose the scale factor to be an increasing function of time that yields a time dependent deceleration parameter, representing a model which generates a universe showing a transition from an early decelerating phase to a recent accelerating phase. The study reveals that strings dominate the early universe and eventually disappear from the universe for sufficiently large times, i.e. in the present epoch. This picture is consistent with current astronomical observations. The physical behavior of the universe is discussed in detail.

Key words: early universe — cosmological parameters

1 INTRODUCTION

In recent years, Einstein's theory of gravity has been the subject of intense study for its success in explaining the observed accelerated expansion of the universe at late times. This substantial theoretical progress in string theory has brought forth a diverse new generation of cosmological models, some of which are subject to direct observational tests. The present day observations of the universe indicate the existence of a large scale network of strings in the early universe (Kibble 1976, 1980). One key advance is the emergence of methods of moduli stabilization. From the point of view of the four dimensional theory, compactification of string theory from the total dimension D down to four dimensions introduces many gravitationally-coupled scalar field moduli. We have recently studied an inhomogeneous string cosmological model formed by geometric strings and used this model as a source of the gravitational field (Pradhan et al. 2007; Yadav et al. 2009). We had two main reasons to study the above mentioned model. First, as a test of consistency, for some particular field theories based on string models and second we point out that the universe can be represented by a collection of extended galaxies. It is generally assumed that after the Big Bang, the universe may have undergone a series of phase transitions as its temperature cooled below some critical temperature as predicted by grand unified theories (Zeldovich et al. 1975; Kibble 1976, 1980; Everett 1981; Vilenkin 1981). At the very early stage of evolution of the universe, it is believed that during the phase transition, the symmetry of the universe was spontaneously broken. That could have given rise to topologically-stable defects such as domain walls, strings and monopoles (Vilenkin 1981). Among all the three cosmological structures, only cosmic strings have generated the most interesting consequence (Vilenkin 1985), because they give rise to the density perturbations which lead to the formation of galaxies. The cosmic strings are important in the early stage of evolution of the

universe before the creation of particles because cosmic strings have one dimensional topological defects associated with spontaneous symmetry breaking whose plausible production site is only a cosmological phase transition in the early universe. Also, the present day observations reveal that the cosmic strings are not responsible for either the CMB fluctuations or the observed clustering of galaxies (Pogosian et al. 2006; Pogosian et al. 2003). The gravitational effect of cosmic strings has been extensively discussed by Letelier (1979) who considered the massive strings to be formed by geometric strings with particles attached along their extension. Later Letelier (1983) used this idea in obtaining a cosmological solution in Bianchi I and Kantowski-Sachs space-time.

Recently, Pradhan & Amirhashchi (2011) studied the law of variation of scale factor as an increasing function of time in Bianchi–V space-time, which generates a time dependent deceleration parameter (DP). This law provides an explicit form of scale factors governing the Bianchi–V universe and facilitates describing the transition of the universe from an early decelerating phase to the recent accelerating phase. Yadav et al. (2011) and Bali (2008) have obtained Bianchi–V string cosmological models in general relativity. The string cosmological models with a magnetic field are discussed by Chakraborty (1991), Tikekar & Patel (1992, 1994) and Patel & Maharaj (1996). Singh & Singh (1999) investigated a string cosmological model with magnetic fields in the context of space-time with G_3 symmetry. Saha & Visinescu (2010) studied a string cosmological model in the presence of magnetic flux and concluded that the presence of cosmic strings does not allow the anisotropic universe to evolve into an isotropic one. Recently Yadav (2010) studied a Bianchi I string cosmological model with a variable DP. The studies of the Bianchi–V cosmological models create more interest as these models contain special isotropic cases and permit arbitrarily small levels of anisotropy at some instant of cosmic time. This property makes them suitable as a model of our universe. The homogeneous and isotropic FRW cosmological models, which are used to describe standard cosmological models, are a particular case of Bianchi–V universes, depending on whether the constant curvature of physical three-space, t = constant, is negative.

In this paper, unlike other authors, we have established the existence of a string cosmological model with time varying DP in Bianchi–V space-time. The organization of the paper is as follows: The model and field equations are presented in Section 2. Section 3 deals with a solution of field equations. The physical and geometrical properties of the model are presented in Section 4. Finally conclusions are presented in Section 5.

2 BIANCHI–V MODEL AND THE FIELD EQUATIONS

The line element for the spatially homogeneous and anisotropic Bianchi–V space-time is given by

$$ds^2 = -dt^2 + A^2 dx^2 + \mathrm{e}^{2\alpha x}\left(B^2 dy^2 + C^2 dz^2\right),\qquad(1)$$

where $A(t)$, $B(t)$ and $C(t)$ are the scale factors in different spatial directions and α is a constant.

We define $a = (ABC)^{1/3}$ as the average scale factor of the space-time (1) so that the average Hubble's parameter reads as

$$H = \frac{\dot{a}}{a},\qquad(2)$$

where the overhead dot denotes derivative with respect to the cosmic time t.

The energy-momentum tensor T_j^i for a cloud of massive strings and the distribution of perfect fluid is taken as

$$T_j^i = (\rho + p)v^i v_j + p g_j^i - \lambda x^i x_j,\qquad(3)$$

where p is the isotropic pressure; ρ is the proper energy density for a cloud of strings with particles attached to them; λ is the string tension density; $v^i = (0,0,0,1)$ is the four-velocity of the particles,

and x^i is a unit space-like vector representing the direction of the string. The vectors v^i and x^i satisfy the conditions

$$v_i v^i = -x_i x^i = -1, \qquad v^i x_i = 0. \tag{4}$$

Choosing x^i parallel to $\partial/\partial x$, we have

$$x^i = (A^{-1}, 0, 0, 0). \tag{5}$$

If the particle density of the configuration is denoted by ρ_p, then

$$\rho = \rho_p + \lambda. \tag{6}$$

The Einstein's field equations (in gravitational units $c = 1$, $8\pi G = 1$) are read as

$$R^i_j - \frac{1}{2} g^i_j R = -T^i_j. \tag{7}$$

The Einstein's field Equations (7) for the line-element (1) lead to the following system of equations

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = -p + \lambda, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2} = -p, \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = -p, \tag{10}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{3\alpha^2}{A^2} = \rho, \tag{11}$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0. \tag{12}$$

The energy conservation equation $T^i_{j;j} = 0$ leads to the following expression

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) - \lambda \frac{\dot{A}}{A} = 0, \tag{13}$$

which is a consequence of the field equations.

3 SOLUTION OF FIELD EQUATIONS

Integrating Equation (12) and absorbing the constant of integration in B or C, without loss of generality, we obtain

$$A^2 = BC. \tag{14}$$

Subtracting Equation (9) from Equation (10) and taking the second integral, we get the following relation

$$\frac{B}{C} = d_1 \exp\left(x_1 \int \frac{dt}{ABC}\right), \tag{15}$$

where d_1 and x_1 are constants of integration.

Equations (8)−(12) are five independent equations in six unknowns A, B, C, p, ρ and λ. For the complete determination of the system, we need one extra condition.

Following Pradhan & Amirhashchi (2011), we assume that

$$a = \left(t^n \mathrm{e}^t\right)^{\frac{1}{m}} , \qquad (16)$$

where m and n are positive constants. It is important to note here that the ansatz for scale factor generalized the one proposed by Pradhan & Amirhashchi (2011) because for $m = 2$, we obtain the same expression for a as proposed by Pradhan & Amirhashchi (2011) to describe the dark energy model in Bianchi–V space-time. However in this paper, we considered string fluid as the source of matter.

Now the spatial volume V of the model is read as

$$V = a^3 = \left(t^n \mathrm{e}^t\right)^{\frac{3}{m}} . \qquad (17)$$

Equations (14), (16) and (17) lead to

$$A(t) = \left(t^n \mathrm{e}^t\right)^{\frac{1}{m}} . \qquad (18)$$

Inserting Equation (18) into Equations (14) and (15), we get

$$B = \left(t^n \mathrm{e}^t\right)^{\frac{1}{m}} \sqrt{d_1} \exp\left(\frac{x_1}{2} \int \frac{dt}{\left(t^n \mathrm{e}^t\right)^{\frac{3}{m}}}\right) , \qquad (19)$$

$$C = \frac{\left(t^n \mathrm{e}^t\right)^{\frac{1}{m}}}{\sqrt{d_1}} \exp\left(-\frac{x_1}{2} \int \frac{dt}{\left(t^n \mathrm{e}^t\right)^{\frac{3}{m}}}\right) . \qquad (20)$$

4 SOME PHYSICAL AND GEOMETRICAL PROPERTIES

The isotropic pressure (p), proper energy density (ρ), string tension density (λ) and particle density (ρ_p) are given by

$$p = \alpha^2 (t^n \mathrm{e}^t)^{-\frac{2}{m}} + \frac{x_1}{m}\left(\frac{n}{t}+1\right)(t^n \mathrm{e}^t)^{-\frac{3}{m}} - \left[\frac{1}{m}\left(\frac{n}{t}+1\right) + \frac{x_1}{2}(t^n \mathrm{e}^t)^{-\frac{3}{m}}\right]^2 , \qquad (21)$$

$$\rho = \frac{3}{m^2}\left(\frac{n}{t}+1\right)^2 - \frac{x_1^2}{4}(t^n \mathrm{e}^t)^{-\frac{6}{m}} - 3\alpha^2(t^n \mathrm{e}^t)^{-\frac{2}{m}} , \qquad (22)$$

$$\lambda = \frac{x_1}{m}\left(\frac{n}{t}+1\right)(t^n \mathrm{e}^t)^{-\frac{3}{m}} + \frac{1}{m^2}\left(\frac{n}{t}+1\right)^2 + \left[\frac{1}{m}\left(\frac{n}{t}+1\right) + \frac{x_1}{2}(t^n \mathrm{e}^t)^{-\frac{3}{m}}\right]^2$$
$$- \frac{2n}{mt^2} - \frac{x_1^2}{4}(t^n \mathrm{e}^t)^{-\frac{6}{m}} , \qquad (23)$$

$$\rho_p = \frac{2}{m^2}\left(\frac{n}{t}+1\right)^2 + \frac{2n}{mt^2} - \frac{x_1}{m}\left(\frac{n}{t}+1\right)(t^n \mathrm{e}^t)^{-\frac{3}{m}} - \left[\frac{1}{m}\left(\frac{n}{t}+1\right) + \frac{x_1}{2}(t^n \mathrm{e}^t)^{-\frac{3}{m}}\right]^2$$
$$- 3\alpha^2(t^n \mathrm{e}^t)^{-\frac{2}{m}} . \qquad (24)$$

The average Hubble's parameter (H), expansion scalar (θ), anisotropy parameter (A_m) and shear scalar (σ) of the model are given by

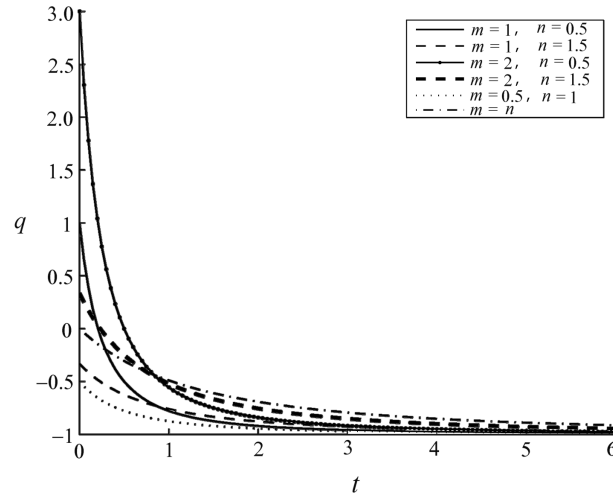$$H = \frac{1}{m}\left(\frac{n}{t}+1\right) , \qquad (25)$$

Fig. 1 The plot of DP (q) vs. time (t) .

$$\theta = 3H = \frac{3}{m}\left(\frac{n}{t} + 1\right) , \tag{26}$$

$$A_m = \frac{1}{9H^2}\left[\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)^2 + \left(\frac{\dot{C}}{C} - \frac{\dot{A}}{A}\right)^2\right] = \frac{m^2 x_1^2}{6\left(\frac{n}{t} + 1\right)}(t^n e^t)^{-\frac{6}{m}} , \tag{27}$$

$$\sigma^2 = \frac{3}{2}A_m H^2 = \frac{x_1^2}{4(t^n e^t)^{\frac{6}{m}}} . \tag{28}$$

The value of DP (q) is found to be

$$q = -1 + \frac{mn}{(n + t)^2} . \tag{29}$$

From Figure 1, we see that the dynamics of DP (q) depend on two free parameters: m and n. It is shown that for $m = n$, $m = 1$ and $n = 1.5$, and $m = 0.5$ and $n = 1$, the universe is accelerating from its birth, whereas for $m = 1$ and $n = 0.5$, $m = 2$ and $n = 0.5$, and $m = 2$ and $n = 1.5$, it goes through a transition from an early decelerating phase to the current accelerating phase. Since we are looking for a model that describes the universe from the early decelerating phase to the current accelerating phase, we choose $m = 1$ and $n = 0.5$ in the remaining discussions of the model, as these are the most appropriate choices in our case.

From Equation (29), it is clear that the DP (q) is time dependent. Also, the change in redshift from decelerated expansion to accelerated expansion is about 0.5. Now for a universe which was decelerating in the past and is accelerating at the present time, DP must show a signature of flipping (Amendola 2003; Riess et al. 2001).

It is observed that at $t = 0$, the spatial volume vanishes and other parameters H, θ and σ diverge. Hence the model starts with the Big Bang singularity at $t = 0$ and this singularity is point type because all directional scale factors vanish at the initial moment. Figure 2 depicts the variation of anisotropic parameter (A_m) versus cosmic time. It is shown that A_m decreases with time and
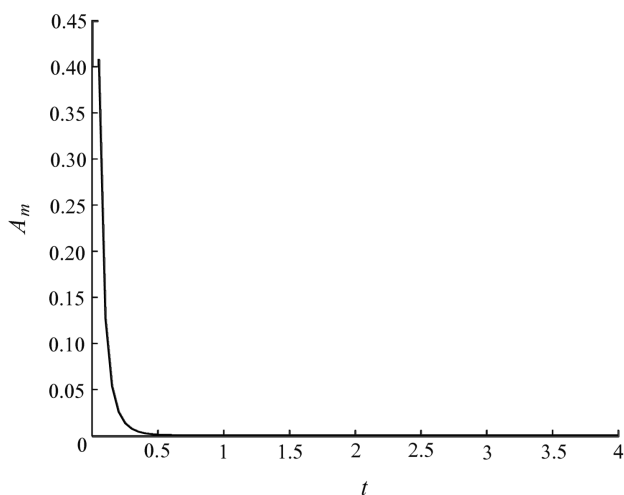
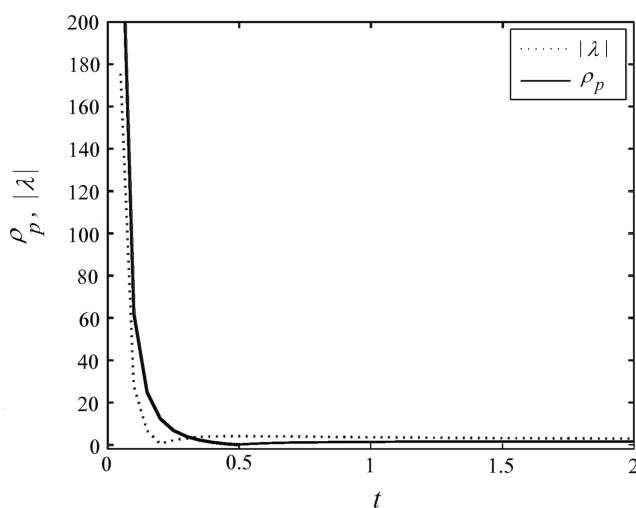**Fig. 2** The plot of anisotropic parameter A_m vs. time (t).

Fig. 3 The particle density (ρ_p) and string tension density (λ) vs. time (t).

tends to zero for sufficiently large times. Thus the anisotropic behavior of the universe dies out at later times and the observed isotropy of the universe can be derived by the model at the present epoch.

In this model, the universe starts with finite values of proper energy density (ρ), particle energy density (ρ_p) and string tension density (λ), but with evolution of the universe, ρ and λ becomes negligible for sufficiently long time (i.e. at the present epoch). The large values of ρ and $|\lambda|$ at the beginning suggest that strings dominate the early universe, but at later times, ρ_p and $|\lambda|$ become negligible. Thus the strings disappear from the universe for larger times and hence they are not observable today. This behavior of ρ_p and $|\lambda|$ is shown in Figure 3. Since there is no direct evidence
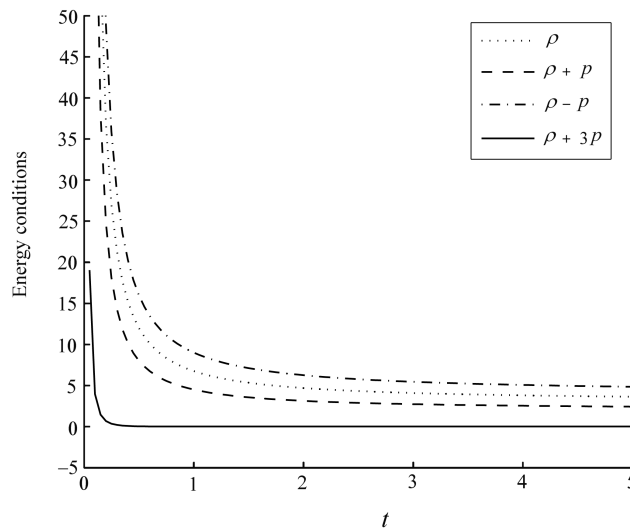
Fig. 4 The energy conditions vs. time (t).

of strings in the present day universe, we are, in general, interested in constructing a model of a universe that evolves purely from the era dominated by either geometric strings or massive strings and ends up in a particle dominated era with or without the remnants of strings. Therefore, the above model describes the evolution of the universe consistent with present day observations.

The left-hand side of the energy conditions has been graphed in Figure 4. From Figure 4, we observe that (i) $\rho > 0$, (ii) $\rho + p > 0$, (iii) $\rho - p > 0$.

Therefore, the weak energy condition (WEC) and the dominant energy condition (DEC) are satisfied in the derived model. It can also be observed that $\rho + 3p > 0$ at the initial moment and at later times $\rho + 3p \leq 0$, which in turn implies that the strong energy condition (SEC) is violated in the derived model at later times. The violation of SEC gives a reverse gravitational effect. Due to this effect, the universe is pushed and the transition from the earlier deceleration phase to the recent acceleration phase takes place (Caldwell et al. 2006). Thus the model presented in this paper turns out to be suitable for describing the late time acceleration of the universe.

5 CONCLUDING REMARKS

In this paper, a spatially homogeneous and anisotropic Bianchi–V string model has been investigated for which the string fluids are free of rotation but they have expansion and shear. We observe that $V \to \infty$ and $\rho \to 0$ as $t \to \infty$, i.e. spatial volume (V) increases with time and proper energy density (ρ) decreases with time as expected. The main features of the derived model are as follows:

– The model is based on an exact solution of Einstein's field equations for the anisotropic Bianchi–V space-time filled with string fluids as the source of matter.
– The dynamics of DP yield two different phases of the universe. Initially DP evolves with a positive sign that yields the decelerating phase of the universe, whereas at later times, it evolves with a negative sign which describes the present phase of acceleration of the universe. Thus the derived model shows a transition of the universe from the early decelerating phase to the current accelerating phase.

- In the present model, the WEC and DEC are satisfied, which in turn imply that the derived models are physically realistic while the violation of SEC is in agreement with current astrophysical observations.
- The string tension density (λ) starts off with extremely large values and tends to zero for sufficiently large times in the derived model. Thus the strings dominate the early universe and eventually disappear from the universe at later times (i.e. the present epoch).

Finally, the solution presented here can be one of the potential candidates to describe the observed universe. Therefore a physically viable Bianchi–V string cosmological model with a singular origin has been obtained.

References

Amendola, L. 2003, MNRAS, 342, 221

Bali, R. 2008, Electron. J. Theor. Phys., 5, 105

Caldwell, R. R., Komp, W., Parker, L., & Vanzella, D. A. T. 2006, Phys. Rev. D, 73, 023513

Chakraborty, S. 1991, Indian Journal of Pure and Applied Physics, 29, 31

Everett, A. E. 1981, Phys. Rev. D, 24, 858

Kibble, T. W. B. 1976, Journal of Physics A Mathematical General, 9, 1387

Kibble, T. W. B. 1980, Phys. Rep., 67, 183

Letelier, P. S. 1979, Phys. Rev. D, 20, 1294

Letelier, P. S. 1983, Phys. Rev. D, 28, 2414

Patel, L. K., & Maharaj, S. D. 1996, Pramana, 47, 33

Pogosian, L., Tye, S.-H. H., Wasserman, I., & Wyman, M. 2003, Phys. Rev. D, 68, 023506

Pogosian, L., Wasserman, I., & Wyman, M. 2006, arXiv: astro-ph/0604141

Pradhan, A., & Amirhashchi, H. 2011, Modern Physics Letters A, 26, 2261

Pradhan, A., Yadav, A. K., Singh, R. P., & Singh, V. K. 2007, Ap&SS, 312, 145

Riess, A. G., Nugent, P. E., Gilliland, R. L., et al. 2001, ApJ, 560, 49

Saha, B., & Visinescu, M. 2010, International Journal of Theoretical Physics, 49, 1411

Singh, G. P., & Singh, T. 1999, General Relativity and Gravitation, 31, 371

Tikekar, R., & Patel, L. K. 1992, General Relativity and Gravitation, 24, 397

Tikekar, R., & Patel, L. K. 1994, Pramana, 42, 483

Vilenkin, A. 1981, Phys. Rev. D, 24, 2082

Vilenkin, A. 1985, Phys. Rep., 121, 263

Yadav, A. K. 2010, arXiv: 1009.3867

Yadav, A. K., Yadav, V. K., & Yadav, L. 2009, International Journal of Theoretical Physics, 48, 568

Yadav, A. K., Yadav, V. K., & Yadav, L. 2011, Pramana, 76, 681

Zeldovich, I. B., Kobzarev, I. I., & Okun, L. B. 1975, Zhurnal Eksperimental noi i Teoreticheskoi Fiziki, 67, 3