

# Latest cosmological constraints on Cardassian expansion models including the updated gamma-ray bursts \*

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**Abstract** We constrain the Cardassian expansion models from the latest observations, including the updated Gamma-ray bursts (GRBs), which are calibrated using a cosmology independent method from the Union2 compilation of type Ia supernovae (SNe Ia). By combining the GRB data with the joint observations from the Union2 SNe Ia set, along with the results from the Cosmic Microwave Background radiation observation from the seven-year Wilkinson Microwave Anisotropy Probe and the baryonic acoustic oscillation observation galaxy sample from the spectroscopic Sloan Digital Sky Survey Data Release, we find significant constraints on the model parameters of the original Cardassian model  $\Omega_{M0} = 0.282^{+0.015}_{-0.014}$ ,  $n = 0.03^{+0.05}_{-0.05}$ ; and  $n = -0.16^{+0.25}_{-3.26}$ ,  $\beta = 0.76^{+0.34}_{-0.58}$  of the modified polytropic Cardassian model, which are consistent with the  $\Lambda$ CDM model in a  $1\text{-}\sigma$  confidence region. From the reconstruction of the deceleration parameter  $q(z)$  in Cardassian models, we obtain the transition redshift  $z_T = 0.73 \pm 0.04$  for the original Cardassian model and  $z_T = 0.68 \pm 0.04$  for the modified polytropic Cardassian model.

**Key words:** gamma-rays: bursts — cosmology: cosmological parameters

## 1 INTRODUCTION

Recent years, the cosmological observations from type Ia supernovae (SNe Ia; Riess et al. 1998; Perlmutter et al. 1999), cosmic microwave background radiation (CMB; Spergel et al. 2003) and large-scale structures (LSS; Tegmark et al. 2004; Eisenstein et al. 2005) have been used to extensively explore cosmology and support the theory that the present expansion of our universe is accelerating. In order to explain the accelerating expansion of the universe, many cosmological models have been proposed. The first categories were proposed by introducing an energy component called dark energy with negative pressure in the universe, which dominates the universe to drive the acceleration of expansion in recent times. Many candidates for dark energy have been taken into account, such as the cosmological constant with equation of state  $w = -1$  (Carroll et al. 1992), the scalar field

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models with dynamical equation of state, e.g., quintessence (Ratra & Peebles 1988; Caldwell et al. 1998), phantom (Caldwell 2002), k-essence (Armendariz-Picon et al. 2001), tachyon (Padmanabhan 2002; Sen 2005), quintom (Feng et al. 2005; Guo et al. 2005; Liang et al. 2009), as well as the Chaplygin gas (Kamenshchik et al. 2001) and the generalized Chaplygin gas model (GCG, Bento et al. 2002), holographic dark energy (Cohen et al. 1999; Li 2004), the agegraphic dark energy (Cai 2007; Wei & Cai 2008a,b), the Ricci dark energy (Gao et al. 2009) and so on. On the other hand, many alternative models in which gravity is modified to drive the universe's acceleration have been proposed, e.g., the  $f(R)$  theory in which the non-linear gravity Lagrangian ( $L \sim R + f(R)$ , where  $R$  is the scalar curvature) has been taken into account (Capozziello & Fang 2002), the braneworld models such as the Dvali-Gabadadze-Porrati (DGP) model which consider that our observable universe might be a surface or a brane embedded in a higher dimensional bulk spacetime (Dvali et al. 2000), as well as the Cardassian expansion model in which the Friedmann equation is modified (Freese & Lewis 2002; Wang et al. 2003).

In 2002, Freese and Lewis (Freese & Lewis 2002) proposed the Cardassian expansion model as a possible explanation for the acceleration by modifying the Friedmann equation without introducing dark energy. The modified Friedmann equation for the original Cardassian model is

$$H^2 = \frac{8\pi G}{3}(\rho + B\rho^n). \quad (1)$$

The Cardassian term which is proportional to  $\rho^n$  may show that our observable universe is a  $3 + 1$  dimensional brane embedded in extra dimensions. The first term on the right side of the equation dominates initially, so the equation becomes the usual Friedmann equation in the early universe. Then the two terms become equal at redshift  $z = z_{\text{card}} \sim O(1)$  (Freese & Lewis 2002) and thereafter the Cardassian term begins to dominate the universe. Here  $n$  is assumed to satisfy  $n < 2/3$  to give rise to a positive acceleration of the universe. If  $n = 0$ , the Cardassian term becomes the cosmological constant. If  $B = 0$ , the equation becomes the usual FRW equation without the cosmological constant. Furthermore, the modified polytropic Cardassian model can be obtained by introducing an additional parameter  $\beta$  into the original Cardassian model (Wang et al. 2003). The corresponding modified Friedmann equation is

$$H^2 = \frac{8\pi G}{3}(\rho^\beta + C\rho^{n\beta})^{1/\beta}. \quad (2)$$

When  $\beta = 1$ , the modified polytropic Cardassian model reduces to the original model.

Gamma-ray bursts (GRBs) are likely to occur in the high-redshift range beyond the SNe Ia redshift limit. Up to now, the farthest GRB detected has been GRB 090423 at  $z = 8.2$  (Tanvir et al. 2009; Salvaterra et al. 2009). Recently, several empirical GRB luminosity relations have been proposed as distance indicators (Amati et al. 2002; Norris et al. 2000; Fenimore & Ramirez-Ruiz 2000; Reichart et al. 2001; Schaefer 2003a; Yonetoku et al. 2004; Ghirlanda et al. 2004a; Liang & Zhang 2005; Firmani et al. 2006b; Yu et al. 2009). Therefore, GRBs could be regarded as the standard candles that act as a complementary cosmological probe to the universe at high redshift (Schaefer 2003b; Takahashi et al. 2003; Bloom et al. 2003; Dai et al. 2004; Ghirlanda et al. 2004b; Friedman & Bloom 2005; Firmani et al. 2005, 2006a; Liang & Zhang 2005; Di Girolamo et al. 2005; Bertolami & Silva 2006; Ghirlanda et al. 2006; Schaefer 2007; Wright 2007; Wang et al. 2007; Amati et al. 2008; Basilakos & Perivolaropoulos 2008; Mosquera Cuesta et al. 2008a,b; Daly et al. 2008; Qi et al. 2008a,b; Vitagliano et al. 2010). Due to the lack of a low-redshift sample, these luminosity relations have usually been calibrated by assuming a particular cosmological model (e.g., the  $\Lambda$ CDM model with particular model parameters according to the concordance cosmology). Therefore, most of the calibrations of GRBs use cosmology-dependent methods which lead to the circularity problem in cosmological research. Many works have treated the circularity problem with statistical approaches,

such as simultaneous fitting of the parameters in the calibration curves and the cosmology (Li et al. 2008; Wang 2008; Samushia & Ratra 2010; Xu 2010; Graziani 2011). However, it is noted that an input cosmological model is still required for simultaneous fitting.

In our previous paper (Liang et al. 2008), we presented a new method to calibrate GRB luminosity relations in a completely cosmology-independent way. The luminosity distance of GRBs in the redshift range of SNe Ia can be obtained by interpolating directly from the SNe Ia Hubble diagram and GRB data at high redshift can be obtained by utilizing the calibrated relations (Liang et al. 2008). Similar to the interpolation method, the luminosity distance of GRBs could be obtained by another mathematical approach, such as empirical formula fitting (Kodama et al. 2008), the non-parametric reconstruction method (Liang & Zhang 2008), local regression (Cardone et al. 2009) and cosmographic fitting (Gao et al. 2010; Capozziello & Izzo 2010).

Following the GRB calibration method directly from SNe Ia, the derived cosmology-independent GRB data at high redshift can be used to constrain cosmological models by using the standard Hubble diagram method (Liang & Zhang 2008; Capozziello & Izzo 2008; Izzo et al. 2009; Capozziello & Izzo 2009; Wei & Zhang 2009; Wei 2009; Wang et al. 2009a,b; Qi et al. 2009; Wang & Liang 2010; Liang et al. 2010; Wei 2010; Freitas et al. 2010; Liang et al. 2011; Liang & Zhu 2011). Capozziello & Izzo (2008) first used the GRB relations calibrated with the so-called Liang method to derive the cosmography parameters at high redshift. Liang et al. (2010) combined the GRB data with the joint data to constrain the cosmological parameters and reconstructed the acceleration history of the universe.

Here we consider the Cardassian model viewed as purely phenomenological modifications of the Friedmann equation to drive the universe's acceleration and focus on the latest cosmological constraints including GRBs. Until now, the Cardassian model has been constrained from many observational data, such as the angular size of the compact radio sources (Zhu & Fujimoto 2002), SNe Ia (Wang et al. 2003; Zhu & Fujimoto 2003; Szydlowski & Czaja 2004; Godłowski et al. 2004; Frith 2004; Bento et al. 2005), the X-ray gas mass fraction of clusters (Zhu & Fujimoto 2004; Zhu et al. 2004; Zhu & Fujimoto 2004; Zhu et al. 2004), CMB (Sen & Sen 2003; Savage et al. 2005), the large scale structure (Multamäki et al. 2003; Amarzguioui et al. 2005; Fay & Amarzguioui 2006), the gravitational lensing (Alcaniz et al. 2005), the baryonic acoustic oscillation (BAO) (Wang et al. 2007), the Hubble parameter versus redshift data (Yi & Zhang 2007), as well as the different combined data (Bento et al. 2006; Davis et al. 2007; Wang 2007; Wang & Wu 2009; Feng & Li 2010). Also, constraints from GRBs with the joint analysis on the Cardassian model can be obtained in Wang et al. (2007); Mosquera Cuesta et al. (2008a); Wang et al. (2009a); Wang & Liang (2010). Very recently, the Union2 compilation of the SNe Ia data set, which consists of 557 SNe Ia, has been released (Amanullah et al. 2010), whereas the seven-year data of the Wilkinson Microwave Anisotropy Probe (WMAP7) have also been released (Komatsu et al. 2011).

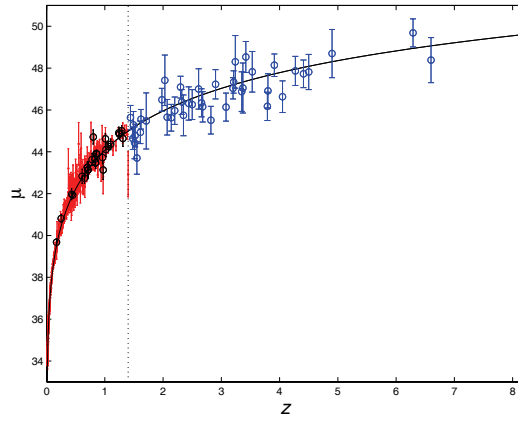
In this paper, with the updated GRB data calibrated directly from the Union2 set, we constrain the Cardassian model and the modified polytropic Cardassian model from the latest observations by combining the GRB data with the joint observations from the Union2 set, along with the CMB observation from the WMAP7 result and the BAO observation from the spectroscopic Sloan Digital Sky Survey (SDSS) Data Release galaxy sample (Eisenstein et al. 2005). We also reconstruct the deceleration parameter  $q(z)$  in Cardassian expansion models and obtain the transition redshift  $z_T$ . We find that tighter and more stringent constraints can be provided with the combined data by including GRBs in this work.

This paper is organized as follows. In Section 2, we introduce the analysis for the observation data. In Section 3, we present constraint results on Cardassian models from the joint observations including GRBs, as well as SNe Ia, CMB and BAO. Conclusions and discussions are given in Section 4.

## 2 OBSERVATIONAL DATA ANALYSIS

In our previous papers (Liang et al. 2008, 2010), we used the 192 SNe Ia compiled by Davis et al. (2007) and the 397 SNe Ia set (Hicken et al. 2009) in the interpolation procedure to calibrate GRB luminosity relations from the 69 GRBs compiled by Schaefer (2007). A larger number of SNe Ia in the sample could bring a more accurate result using an interpolation procedure. Very recently, the Union2 compilation (Amanullah et al. 2010) of 557 SNe Ia data set has been released by the Supernova Cosmology Project Collaboration (SCP). In this paper, we use the Union2 set to calibrate GRB luminosity relations with the GRB sample at  $z \leq 1.4$  by using the linear interpolation method and we update the distance moduli of the GRBs at  $z > 1.4$  obtained by utilizing the new calibrated relations. For more details on the calculation, see Liang et al. (2008, 2010).

We plot the Hubble diagram of Union2 SNe Ia and the GRBs obtained using the interpolation methods in Figure 1. The distance moduli of the 27 GRBs at  $z \leq 1.4$  are obtained by using the linear interpolation method directly from the Union2 SNe set; the 42 GRB data at  $z > 1.4$  are obtained by utilizing the five relations calibrated with the sample at  $z \leq 1.4$ .



**Fig. 1** Hubble Diagram of 557 SNe Ia (*red dots*; color online) and the 69 GRBs (*circles*) obtained using the interpolation method. The 27 GRBs at  $z \leq 1.4$  are obtained by linear interpolation from SNe Ia data (*black circles*) and the 42 GRBs at  $z > 1.4$  (*blue circles*) are obtained with the five relations calibrated with the sample at  $z \leq 1.4$ . The curve is the theoretical distance modulus in the concordance model ( $w = -1$ ,  $\Omega_{M0} = 0.27$ ) and the vertical dotted line represents  $z = 1.4$ .

The position of the first acoustic peak in the power spectrum of CMB favors a spatially flat Universe, therefore we assume a flat universe prior throughout this work. Constraints from SNe Ia and GRB data can be obtained by fitting the distance moduli  $\mu(z)$ . A distance modulus can be calculated as

$$\mu = 5 \log \frac{d_L}{\text{Mpc}} + 25 = 5 \log_{10} D_L - \mu_0, \quad (3)$$

where  $\mu_0 = 5 \log_{10} h + 42.38$ ,  $h = H_0 / (100 \text{ km s}^{-1} \text{ Mpc}^{-1})$  and  $H_0$  is the Hubble constant. For a flat universe, the luminosity distance  $D_L$  can be calculated by

$$D_L \equiv H_0 d_L = (1+z) \int_0^z \frac{dz'}{E(z')}, \quad (4)$$

where  $E(z) = H/H_0$ , which is determined by the choice of the specific cosmological model. The  $\chi^2$  values of the observed distance moduli can be calculated by

$$\chi_\mu^2 = \sum_{i=1}^N \frac{[\mu_{\text{obs}}(z_i) - \mu(z_i)]^2}{\sigma_{\mu,i}^2}, \quad (5)$$

where  $\mu_{\text{obs}}(z_i)$  are the observed distance moduli for the SNe Ia and/or GRBs at redshifts  $z_i$  with error  $\sigma_{\mu_i}$ ;  $\mu(z_i)$  are the theoretical values of distance moduli from cosmological models. Following an effective approach (Nesseris & Perivolaropoulos 2005), we marginalize the nuisance parameter  $\mu_0$  by minimizing

$$\hat{\chi}_\mu^2 = A - B^2/C, \quad (6)$$

where

$$\begin{aligned} A &= \sum [\mu_{\text{obs}}(z_i) - 5 \log_{10} D_L]^2 / \sigma_{\mu_i}^2, \\ B &= \sum [\mu_{\text{obs}}(z_i) - 5 \log_{10} D_L] / \sigma_{\mu_i}^2, \\ C &= \sum 1 / \sigma_{\mu_i}^2. \end{aligned}$$

For the CMB observation, the shift parameters  $R$  provide an efficient summary of CMB data to constrain cosmological models. For a flat universe, the shift parameter can be expressed as (Bond et al. 1997)

$$R = \sqrt{\Omega_{M0}} \int_0^{z_{\text{rec}}} \frac{dz}{E(z)}, \quad (7)$$

where  $z_{\text{rec}}$  is the redshift of recombination. From the WMAP7 result, the shift parameter is constrained to be  $R = 1.725 \pm 0.018$  and  $z_{\text{rec}} = 1091.3$  (Komatsu et al. 2011). The  $\chi^2$  value of the shift parameter can be calculated by

$$\chi_{\text{CMB}}^2 = \frac{(R - 1.725)^2}{0.018^2}. \quad (8)$$

For the BAO observation, we use the distance parameter  $A$  which, for a flat universe, can be expressed as (Eisenstein et al. 2005)

$$A = \frac{\sqrt{\Omega_{M0}}}{E(z_{\text{BAO}})^{1/3}} \left[ \frac{1}{z_{\text{BAO}}} \int_0^{z_{\text{BAO}}} \frac{dz}{E(z)} \right]^{2/3}, \quad (9)$$

where  $z_{\text{BAO}} = 0.35$ . From the SDSS spectroscopic sample of luminous red galaxies, the distance parameter is measured to be  $A = 0.469(n_s/0.98)^{-0.35} \pm 0.017$  (Eisenstein et al. 2005), with the scalar spectral index  $n_s = 0.963$  from the WMAP7 data (Komatsu et al. 2011). The  $\chi^2$  value of the distance parameter can be calculated by

$$\chi_{\text{BAO}}^2 = \frac{(A - 0.4666)^2}{0.017^2}. \quad (10)$$

### 3 CONSTRAINTS FROM COMBINING GRBS, SNE IA, CMB AND BAO

In order to combine GRB data into the joint observational data analysis to constrain cosmological models, we follow a simple method in order to avoid any correlation between the SNe Ia data and the GRB data (Liang et al. 2010). The 40 SNe points used in the interpolating procedure are excluded

from the Union2 SNe Ia sample used to compute the joint constraints. The best fit values for model parameters can be determined by minimizing

$$\chi^2 = \hat{\chi}_{\mu\{517\text{SNe}+42\text{GRBs}\}}^2 + \chi_{\text{CMB}}^2 + \chi_{\text{BAO}}^2. \quad (11)$$

From the modified Friedmann equation of the original Cardassian model, if only considering the matter term without considering the radiation for simplification, using  $\rho_{\text{M}} = \rho_{\text{M}0}(1+z)^3 = \Omega_{\text{M}0}\rho_{\text{c}}(1+z)^3$  with the present critical density of the universe  $\rho_{\text{c}} = 3H_0^2/8\pi G$ , we obtain

$$f_{\text{X}}(z) \equiv \frac{\rho_{\text{X}}}{\rho_{\text{X}0}} = (1+z)^{3n}, \quad (12)$$

where subscript ‘X’ refers to any component providing an additional term in the Friedmann equation. The corresponding  $E(z)$  of the Cardassian model is

$$E(z) = [\Omega_{\text{M}0}(1+z)^3 + (1 - \Omega_{\text{M}0})(1+z)^{3n}]^{1/2}. \quad (13)$$

For the modified polytropic Cardassian model, we obtain

$$f_{\text{X}}(z) = \frac{\Omega_{\text{M}0}}{1 - \Omega_{\text{M}0}}(1+z)^3 \left[ \left( 1 + \frac{\Omega_{\text{M}0}^{-\beta} - 1}{(1+z)^{3(1-n)\beta}} \right)^{1/\beta} - 1 \right]. \quad (14)$$

The corresponding  $E(z)$  of the modified polytropic Cardassian model is

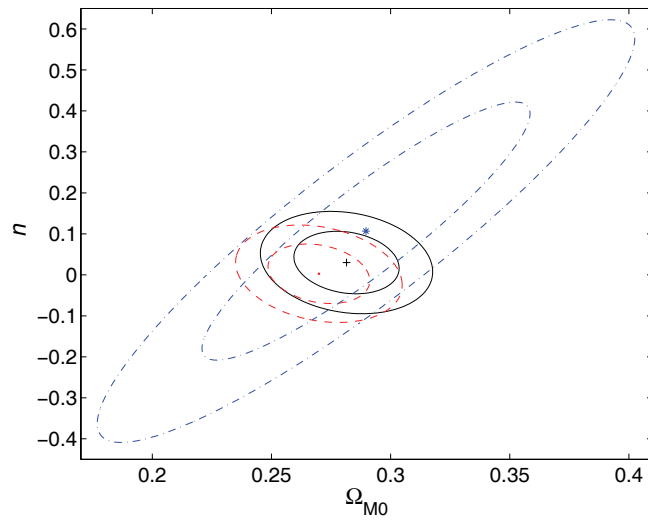
$$E(z) = \left\{ \Omega_{\text{M}0}(1+z)^3 \left[ 1 + \frac{\Omega_{\text{M}0}^{-\beta} - 1}{(1+z)^{3(1-n)\beta}} \right]^{1/\beta} \right\}^{1/2}. \quad (15)$$

The joint confidence regions in the  $\Omega_{\text{M}0} - n$  plane with the combined observational data for the original Cardassian expansion model are shown in Figure 2. With SNe Ia + GRBs + CMB + BAO, the best-fit values at the 1- $\sigma$  confidence level are  $\Omega_{\text{M}0} = 0.282_{-0.014}^{+0.015}$ ,  $n = 0.03_{-0.05}^{+0.05}$ . For comparison, fitting results from the joint data without GRBs are also given in Figure 2. The best-fit values with SNe Ia + CMB + BAO are  $\Omega_{\text{M}0} = 0.270_{-0.014}^{+0.014}$ ,  $n = 0.00_{-0.05}^{+0.05}$ . With GRBs + CMB + BAO without SNe Ia, the best-fit values are  $\Omega_{\text{M}0} = 0.290_{-0.046}^{+0.045}$ ,  $n = 0.11_{-0.25}^{+0.18}$ . We present the best-fit value of  $\Omega_{\text{M}0}$ ,  $n$  with 1- $\sigma$  uncertainties,  $\chi_{\text{min}}^2$  and  $\chi_{\text{min}}^2/\text{dof}$  for the original Cardassian model in Table 1.

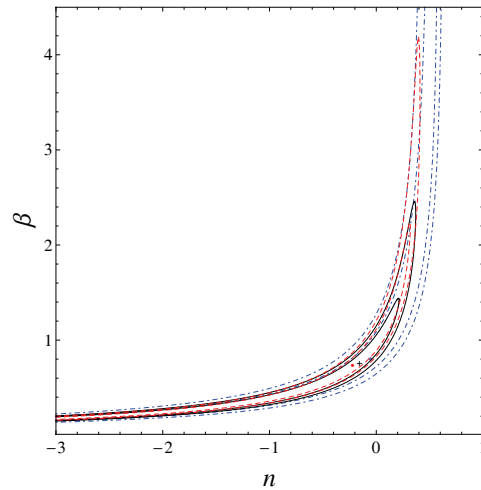
**Table 1** Best-fit value of parameters  $\Omega_{\text{M}0}$ ,  $n$  and  $\beta$  for the original Cardassian model and the modified polytropic Cardassian model with 1 $\sigma$  uncertainties, as well as  $\chi_{\text{min}}^2$  and  $\chi_{\text{min}}^2/\text{dof}$ , with SNe+GRBs+CMB+BAO, SNe+CMB+BAO and GRBs+CMB+BAO.

	Original Cardassian Model			Modified polytropic Cardassian Model		
	SN+GRB+CMB+BAO	SN+CMB+BAO	GRB+CMB+BAO	SN+GRB+CMB+BAO	SN+CMB+BAO	GRB+CMB+BAO
$\Omega_{\text{M}0}$	$0.282_{-0.014}^{+0.015}$	$0.270_{-0.014}^{+0.014}$	$0.290_{-0.046}^{+0.045}$	$0.285_{-0.015}^{+0.014}$	$0.271_{-0.015}^{+0.015}$	$0.285_{-0.045}^{+0.045}$
$n$	$0.03_{-0.05}^{+0.05}$	$0.00_{-0.05}^{+0.05}$	$0.11_{-0.25}^{+0.18}$	$-0.16_{-3.26}^{+0.25}$	$-0.22_{-3.27}^{+0.34}$	$-0.05_{-5.11}^{+0.59}$
$\beta$	$\beta \equiv 1$	$\beta \equiv 1$	$\beta \equiv 1$	$0.76_{-0.58}^{+0.34}$	$0.74_{-0.56}^{+1.15}$	$0.81_{-0.51}^{+3.80}$
$\chi_{\text{min}}^2$	538.10	542.92	34.76	537.46	542.81	34.76
$\chi_{\text{min}}^2/\text{dof}$	0.96	0.98	0.83	0.96	0.98	0.83

For the modified polytropic Cardassian model, we find that the best-fit values at the 1- $\sigma$  confidence level with SNe Ia + GRBs + CMB + BAO are  $\Omega_{\text{M}0} = 0.285_{-0.014}^{+0.015}$ ,  $n = -0.16_{-3.26}^{+0.25}$  and  $\beta = 0.76_{-0.58}^{+0.36}$ .



**Fig. 2** Joint confidence regions in the  $\Omega_{M0} - n$  plane for the original Cardassian model in a flat universe. The contours correspond to  $1-\sigma$  and  $2-\sigma$  confidence regions. The black solid lines, red dashed lines and the blue dash-dotted lines, represent the results of SNe+GRBs+CMB+BAO, SNe+CMB+BAO and GRBs+CMB+BAO, respectively. The black plus, red point and blue star correspond to the best-fit values of SNe+GRBs+CMB+BAO, SNe+CMB+BAO and GRBs+CMB+BAO, respectively.



**Fig. 3** Joint confidence regions in the  $n - \beta$  plane for the modified polytropic Cardassian model in a flat universe. The contours correspond to  $1-\sigma$  and  $2-\sigma$  confidence regions. The black solid lines, red dashed lines and the blue dash-dotted lines (color online) represent the results of SNe+GRBs+CMB+BAO, SNe+CMB+BAO and GRBs+CMB+BAO, respectively. The black plus, red point and blue star correspond to the best-fit values of SNe+GRBs+CMB+BAO, SNe+CMB+BAO and GRBs+CMB+BAO, respectively.

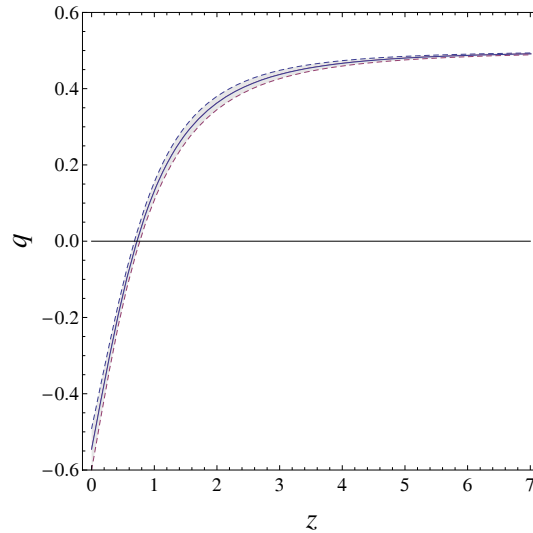


Figure 3 shows the joint confidence regions with the combined observational data for the modified polytropic Cardassian model in the  $n - \beta$  plane, while fixing  $\Omega_{M0}$  with the best-fit values. With SNe Ia + CMB + BAO, the best-fit values are  $\Omega_{M0} = 0.271^{+0.015}_{-0.014}$ ,  $n = -0.22^{+0.34}_{-3.27}$  and  $\beta = 0.74^{+1.15}_{-0.56}$ , while with GRBs + CMB + BAO, the best-fit values are  $\Omega_{M0} = 0.285^{+0.015}_{-0.014}$ ,  $n = -0.06^{+0.59}_{-5.11}$  and  $\beta = 0.81^{+3.80}_{-0.51}$ . We also present the best-fit value of  $\Omega_{M0}$ ,  $n$  and  $\beta$  with  $1-\sigma$  uncertainties,  $\chi^2_{\min}$  and  $\chi^2_{\min}/\text{dof}$  for the modified polytropic Cardassian model in Table 1.

From Figures 2 and 3 and Table 1, we can find that GRBs can also give strong constraints when combined with CMB and BAO data without SNe Ia. By comparing the joint constraints with GRBs and without GRBs, we can see that the contribution of GRBs to the joint cosmological constraints is a slight shift which adds the best-fit value of  $\Omega_{M0}$  and significantly narrows the parameters' confidence ranges of the modified polytropic Cardassian model. We also find that the  $\Lambda$ CDM model ( $n \equiv 0$ ,  $\beta \equiv 1$ ) is consistent with all the joint data in the  $1-\sigma$  confidence region and combining these observational data can tighten the model parameters significantly compared to results from former works (Wang 2007; Wang & Wu 2009). We also investigate the deceleration parameter for Cardassian expansion models. The deceleration parameter  $q(z)$  can be calculated by

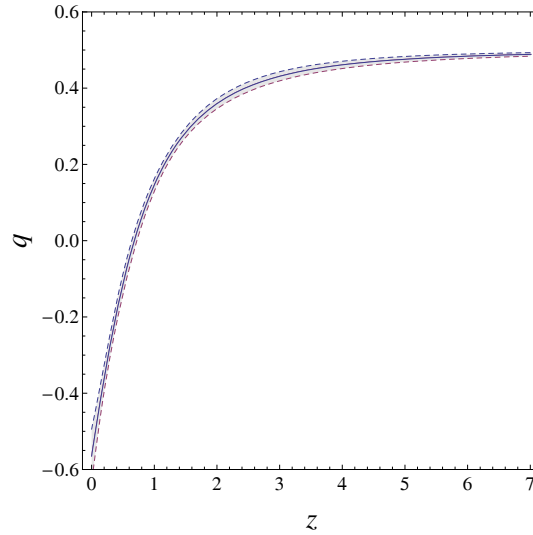
$$q = -1 + (1+z)E(z)^{-1} \frac{dE(z)}{dz}. \quad (16)$$

In Figure 4, we show the evolution of  $q(z)$  for the original Cardassian expansion model. We obtain  $q_0 = -0.55 \pm 0.054$  and the transition redshift is  $z_T = 0.73 \pm 0.04$  at the  $1\sigma$  confidence level, which is more stringent and comparable with the former result ( $z_T = 0.70 \pm 0.05$ ) by Wang (2007), but is slightly later than the former result ( $z_T = 0.55 \pm 0.05$ ) by Wang & Wu (2009). We show the evolution of  $q(z)$  for the polytropic Cardassian expansion model in Figure 5 and we find the transition redshift  $z_T = 0.68 \pm 0.04$  and  $q_0 = -0.57 \pm 0.07$ .



**Fig. 4** Evolution of the deceleration parameter  $q(z)$  from fitting results in the original Cardassian model. The solid line is drawn by using the best fit parameters. The shaded region shows the  $1-\sigma$  uncertainties.





**Fig. 5** Evolution of the deceleration parameter  $q(z)$  from fitting results in the polytropic Cardassian model. The solid line is drawn by using the best fit parameters. The shaded region shows the  $1-\sigma$  uncertainties.

#### 4 CONCLUSIONS AND DISCUSSION

In this paper, by using the Union2 set of 557 SNe Ia, we calibrate GRB data in a completely cosmology-independent way. When combining the GRB data with the Union2 set, we avoid any correlation between the SNe Ia data and the GRB data (Liang et al. 2010). From the GRB data to the joint observations with the Union2 set, along with the CMB from WMAP7 and the BAO observation from the SDSS Data Release galaxy sample, we find significant constraints on model parameters of the original Cardassian model  $\Omega_{M0} = 0.282^{+0.015}_{-0.014}$ ,  $n = 0.03^{+0.05}_{-0.05}$ , and  $n = -0.16^{+0.25}_{-3.26}$ ,  $\beta = 0.76^{+0.34}_{-0.58}$  from the modified polytropic Cardassian model, which are consistent with the  $\Lambda$ CDM model in the  $1-\sigma$  confidence region. From the reconstruction of the deceleration parameter  $q(z)$  in Cardassian expansion models, we obtain the transition redshift  $z_T = 0.73 \pm 0.04$  for the original Cardassian model and  $z_T = 0.68 \pm 0.04$  for the modified polytropic Cardassian model, which are more stringent compared to the former results (Wang 2007; Wang & Wu 2009). It is found that GRBs can give strong constraints when combined with CMB and BAO data without SNe data and we can see the contribution of GRBs to the joint cosmological constraints by comparing to the joint constraints with GRBs and without GRBs. Hereafter, along with more and more observed data, GRBs could be used as an optional choice to set tighter constraints on the Cardassian model and even other cosmological models.

Recently, some works pointed out that there are observational selection biases in the GRB relations (Butler et al. 2007; Shahmoradi & Nemiro 2009) and possible evolution effects in GRB relations (Li 2007; Tsutsui et al. 2008). However, it is found that there is no sign of evolution with redshift of the Amati relation and the instrumental selection effects do not dominate for GRB relations (Ghirlanda et al. 2008, 2009). Nevertheless, further examinations of possible evolutionary effects and selection biases should be required for considering GRBs as standard candles for cosmological use.

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