*R*esearch in
*A*stronomy and
*A*strophysics

Fractal dimension and thermodynamic fluctuation properties of IDV light curves

Chun-Sing Leung[1,2], Jian-Yan Wei[1], Zoltan Kovács[3] and Tiberiu Harko[3]

[1] National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012, China; *astrosinghk@yahoo.com.hk*
[2] Graduate University of Chinese Academy of Sciences, Beijing 100049, China
[3] Department of Physics and Center for Theoretical and Computational Physics, The University of Hong Kong, Pok Fu Lam Road, Hong Kong, Hong Kong SAR, China

Abstract Fractals are a basic tool for phenomenologically describing natural objects having a high degree of temporal or spatial variability. From a physical point of view the fractal properties of natural systems can also be interpreted by using the standard formalism of thermodynamical fluctuations. We introduce and analyze the fractal dimension of the Intra-Day-Variability (IDV) light-curves of BL Lac objects, in the optical, radio and X-ray bands. A general description of the fluctuation spectrum of these systems based on general thermodynamical principles is also proposed. Based on the general fractal properties of a given physical system, we also introduce the predictability index for the IDV light curves. We have explicitly determined the fractal dimension for the R-band observations of five blazars, as well as for the radio band observations of the compact extragalactic radio source J 1128+5925 and of several X-ray sources. Our results show that the fractal dimension of the optical and X-ray observations indicates an almost pure "Brownian noise" (random walk) spectrum, with a very low predictability index, while in the radio band the predictability index is much higher. We have also studied the spectral properties of the IDV light curves and we have shown that their spectral index is very closely correlated with the corresponding fractal dimension.

Key words: galaxies — quasars: emission lines — instabilities — gravitation

1 INTRODUCTION

Fractals are a basic tool to phenomenologically describe natural objects. According to Mandelbrot (1983), a fractal is a set whose Hausdorff dimension is not an integer. The fractal dimension is one possible parameter that characterizes chaotic systems and the analysis of time series is one of the most common means to find the fractal dimension from observable quantities. Fractals are also very useful for the analysis of waveforms, a term that describes the shape of a wave, usually drawn as instantaneous values of a variable quantity versus time. The fractal analysis of waveforms was

introduced by Katz (1988), who considered the possibility that the complexity of a waveform may be represented by its fractal dimension. Katz (1988) proposed that the fractal dimension can be measured empirically by sampling the waveform at N points evenly spaced on the abscissa, which discretized the waveform into $N' = N - 1$ segments. Then the fractal dimension can be obtained as

$$D_F = \log N' / \left[\log N' + \log (d/L) \right],$$

where $d = \max \left[\mathrm{dist}(i, j) \right]$ is the planar extent of the curve and $L = \sum_{i=0}^{N'} \mathrm{dist}(i, i+1)$ is the length of the curve, where "max" stands for the maximum $\mathrm{dist}(i, j)$, the distance between points i and j of the curve. An alternative and more efficient algorithm for the computation of the fractal dimension was proposed by Sevcik (1998, 2006), which we will consider in the following.

The fractal analysis has been extensively used in astronomy and astrophysics for the study of the fractal properties of gamma ray bursts (Shakura et al. 1994), the critical properties of spherically symmetric accretion in a fractal medium (Roy & Ray 2007), for the analysis of fractal structures in the photospheric and the coronal magnetic field of the Sun (Ioshpa et al. 2008; Dimitropoulou et al. 2009), as a measure of the scale of homogeneity (Yadav et al. 2010), for the study of the star forming regions (Sánchez et al. 2010) and for the analysis of the dark matter and gas distributions in the Mare-Nostrum universe (Gaite 2010). The quasar distribution on the celestial sphere is characterized by power laws as well with correlation dimension values ranging from 1.49 to 1.58 for different redshift layers in the same range (Rozgacheva & Agapov 2011a). A fractal cosmological model, which accounts for the observable fractal properties of the large-scale structure of the Universe, was discussed in Rozgacheva & Agapov (2011b).

Many classes of Active Galactic Nuclei have variability properties. The time scale of variability can be classified as yearly, monthly or daily. If the sources vary within one night or day, the time variation is called "Intra-day Variability," or simply IDV. There is one main method for confirming whether or not the object shows IDV in the differential light curve. This is the so-called statistical method. In this method one checks if the variability amplitude is greater than three times the sigma value. The best criterium is, however, five times the sigma criterium (Fan 2011). Another method, based on the so called C-parameter, was introduced by Romero et al. (1999). The criterium for confirming if the object shows IDV is having the "C" value greater than or equal to 2.576. If this is the case, the IDV variability is confirmed at the 99% confidence level. Furthermore, the object is said to be variable if the "C" value is greater than or equal to 2.576 at two different bands. The IDV can be checked by the formula proposed by Heidt & Wagner (1996). For checking the IDV for the source in radio band, a statistical parameter called modulation index is used. Usually, for the IDV light curve of blazars, there is no significant pattern or period. Roland et al. (2009) proposed that the formation and the rotation of a warp in the inner part of the accretion disk can lead to some perturbations of the beam that finally produce the observed IDV.

It is the purpose of the present paper to analyze the IDV light curves by using their fractal dimension properties and to try to interpret the actual value of the fractal dimension by using the general formalism of thermodynamic fluctuations. The actual IDV signal is always fluctuating. Profiles of IDV signals are also remarkably varied. The possible mechanisms for generating IDV emissions could also be different. The fractal dimension could provide a hint into the physical processes that could trigger the IDV processes. In the present paper, we have explicitly determined the fractal dimension for the R-band optical emission of five blazars (Gupta et al. 2008) and for the radio band emission of the compact extragalactic radio source J 1128+5925 (Gabányi et al. 2007). The fractal dimension allows us to introduce the predictability index, which represents a powerful indicator of the nature of the IDV signal. The general fractal properties of IDV signals can be interpreted in the framework of the general theory of thermodynamic fluctuations for self-gravitating systems. The fractal dimension of the optical observations indicates an almost pure "Brownian noise" (random walk) spectrum, with a very low predictability index, while in the radio band the predictability index is much higher. We have also studied the spectral properties of the IDV light curves by using the

periodogram method and we have shown that their spectral index is very closely correlated with the corresponding fractal dimension.

The present paper is organized as follows. In Section 2 we consider the theory of the thermodynamic fluctuations of the self-gravitating systems and obtain the general theoretical predictions for the fractal dimension of the fluctuation spectrum. The methods for the calculation of the fractal dimension and of the power spectrum are presented in Section 3. The fractal dimensions and the power spectra of several IDV signals in optical, radio and X-ray bands are obtained in Section 4. The correlation between the fractal dimension and the spectral index is also discussed in detail. We discuss and conclude our results in Section 5.

2 FRACTAL PROPERTIES OF FLUCTUATING SELF-GRAVITATING SYSTEMS

In the following we consider a system of self-gravitating particles (an astrophysical accretion disk, for example), embedded in a heat bath (an external medium) of gravitationally non-interacting particles. The system interacts with the surrounding bath through a friction force, due to which the particles in Brownian motion lose energy to the medium and simultaneously gain energy from the random kicks of the thermal bath, which can be described by a stochastic force (for a description of the dynamics of the system via a Langevin equation see Leung et al. 2011). As a result of the interaction between the system and the heat bath, the thermodynamical quantities of the system fluctuate. In the following we consider the standard thermodynamic fluctuation formalism (Landau & Lifshitz 1980; Kandrup 1983) as applied to self-gravitating systems and we derive the time dependence of the fluctuation spectrum.

According to the fundamental principles of statistical mechanics, the probability w for a quantity to have a value in the interval from r to $r + dr$ is proportional to $e^{\Delta S_t(r)}$, where $\Delta S_t(r)$ is the change in the entropy during the fluctuation (Landau & Lifshitz 1980). The entropy change can be written as $\Delta S_t = -W_{\min}/T$, where $W_{\min}$ is the minimum work needed to reverse the given change in the thermodynamic quantities in a small part of the system. Thus $w \propto e^{-W_{\min}/T}$. For $W_{\min}$ we can adopt the expression

$$W_{\min} = \Delta E - T\Delta S + P\Delta V + \Delta R_{\min}, \tag{1}$$

where ΔE, ΔS and ΔV are the fluctuations in energy, entropy and volume, respectively, while $\Delta R_{\min}$ is the minimal work necessary for reversible removal of mass ΔM from a distance δr in the gravitational field of a mass M. Thus we have

$$w \propto \exp\left(-\frac{\Delta E - T\Delta S + P\Delta V}{T} - \frac{\Delta R_{\min}}{T}\right). \tag{2}$$

Equation (2) can be applied to any fluctuation, small or large. However, in the following we only consider the case of small fluctuations. In this case, we can expand ΔE in a series, obtaining

$$\Delta E = \frac{\partial E}{\partial S}\Delta S + \frac{\partial E}{\partial V}\Delta V + \frac{1}{2}\left[\frac{\partial^2 E}{\partial S^2}(\Delta S)^2 + 2\frac{\partial^2 E}{\partial S\partial V}\Delta S\Delta V + \frac{\partial^2 E}{\partial V^2}(\Delta V)^2\right]. \tag{3}$$

By taking into account that $\frac{\partial E}{\partial S} = T$, $\frac{\partial E}{\partial V} = -P$, we obtain

$$\Delta E - T\Delta S + P\Delta V = \frac{1}{2}\left[\frac{\partial^2 E}{\partial S^2}(\Delta S)^2 + 2\frac{\partial^2 E}{\partial S\partial V}\Delta S\Delta V + \frac{\partial^2 E}{\partial V^2}(\Delta V)^2\right]$$

$$= \frac{1}{2}\left(\Delta S\Delta T - \Delta P\Delta V\right), \tag{4}$$

giving

$$w \propto \exp\left(\frac{\Delta S\Delta T - \Delta P\Delta V}{2T} - \frac{\Delta R_{\min}}{T}\right). \tag{5}$$

By taking V and T as independent variables, we first obtain

$$\Delta S = \left(\frac{\partial S}{\partial T}\right)_V \Delta T + \left(\frac{\partial S}{\partial V}\right)_T \Delta V = \frac{C_v}{T}\Delta T + \left(\frac{\partial P}{\partial T}\right)_V \Delta V, \tag{6}$$

$$\Delta P = \left(\frac{\partial P}{\partial T}\right)_V \Delta T + \left(\frac{\partial P}{\partial V}\right)_T \Delta V, \tag{7}$$

where C_v is the specific heat at constant volume. Therefore for the fluctuation probability we obtain

$$w \propto \exp\left[-\frac{C_v}{2T^2}(\Delta T)^2 + \frac{1}{2T}\left(\frac{\partial P}{\partial V}\right)_T (\Delta V)^2 - \frac{\Delta R_{\min}}{T}\right]. \tag{8}$$

In order to estimate $R_{\min}$ we consider that it can be obtained as (Rozgacheva & Agapov 2011b; Rozgacheva 1984)

$$\Delta R_{\min} \approx G\frac{M\Delta M}{r^2}\Delta r = \frac{4\pi}{3}G\rho r\Delta M\Delta r, \tag{9}$$

where $M = 4\pi\rho r^3/3$ is the mass at radius r and $\Delta M = 4\pi\rho r^2\Delta r$ is the mass of a spherical shell of thickness Δr. Therefore the probability of thermal fluctuations in a self-gravitating system takes the form (Rozgacheva & Agapov 2011b; Rozgacheva 1984)

$$w \propto \exp\left[-\frac{1}{2\delta_T^2}\left(\frac{\Delta T}{T}\right)^2 - \frac{1}{2\delta_r^2}\left(\frac{\Delta r}{r}\right)^2\right], \tag{10}$$

where $\delta_T^2 = C_v^{-1}$ and

$$\delta_r^2 = \left\{\frac{12\pi\rho}{kT}\left[\left(\frac{\partial P}{\partial V}\right)_T r^3 + \frac{8\pi G\rho}{9}r^5\right]\right\}^{-1}, \tag{11}$$

where k is Boltzman's constant. The root-sum-square relative density fluctuation (fluctuation spectrum) is given by (Rozgacheva & Agapov 2011b)

$$\sqrt{\left\langle\left(\frac{\Delta\rho}{\rho}\right)^2\right\rangle} = 3\sqrt{\left\langle\left(\frac{\Delta r}{r}\right)^2\right\rangle} = 3\left\{\frac{12\pi\rho}{kT}\left[\left(\frac{\partial P}{\partial V}\right)_T r^3 + \frac{8\pi G\rho}{9}r^5\right]\right\}^{-1/2}. \tag{12}$$

The fluctuations of the density are related to the fluctuations of the radial coordinate by

$$\frac{\Delta\rho}{\rho} = -3\frac{\Delta r}{r}. \tag{13}$$

Since our main goal is to study the time variability of some astrophysical phenomena, we have to convert the radial variability into time variability. There are two possible time-scales for astrophysical phenomena, namely, the oscillation time scale and the free-fall time scale. For a homogeneous uniform self-gravitating system that behaves as a perfect gas throughout, the pulsation period t_p is $t_p = 2\pi/\sqrt{3\pi G\rho}$, while the free-fall time is $t_{ff} = \sqrt{4\pi/3G\rho}$. In both cases, the time-scale is proportional to $1/\sqrt{\rho}$. Therefore, assuming such a general time - density dependence, we obtain

$$\frac{\Delta t}{t} = -\frac{1}{2}\frac{\Delta\rho}{\rho}. \tag{14}$$

With the use of Equation (13) we obtain

$$\frac{2}{3}\frac{\Delta t}{t} = \frac{\Delta r}{r}, \tag{15}$$

giving

$$r \propto t^{2/3}. \tag{16}$$

We can now discuss the limiting cases of the time-spectrum in a self-gravitating system. At spatial scales when the pressure gradients dominate, we have

$$\sqrt{\left\langle \left(\frac{\Delta\rho}{\rho}\right)^2 \right\rangle} \approx 3 \left\{ \frac{12\pi\rho}{kT} \left[\left(\frac{\partial P}{\partial V}\right)_T r^3 \right] \right\}^{-1/2} \propto r^{-3/2} \propto t^{-1}. \tag{17}$$

At large scales, when gravitational effects are important, we obtain

$$\sqrt{\left\langle \left(\frac{\Delta\rho}{\rho}\right)^2 \right\rangle} \approx 3 \left\{ \frac{12\pi\rho}{kT} \left[\frac{8\pi G\rho}{9} r^5 \right] \right\}^{-1/2} \propto r^{-5/2} \propto t^{-5/3}. \tag{18}$$

However, the above discussion is incomplete in the sense that in a typical fluctuating system we have two independent modes, one growing and one decaying. A better estimation of the time dependence of the fluctuations can be obtained as follows (Rozgacheva 1988). According to Chebyshev's inequality from probability theory, the probability that a density fluctuation $\Delta\rho/\rho$ exceeds $\delta > 0$ is given by (Feller 1971)

$$P\left(|\Delta\rho/\rho| \geq \delta\right) \leq \frac{C}{\delta^2}, \tag{19}$$

where C is a constant. Now let us assume that for the independent growing mode we have $\delta_1 \sim t^{\alpha}$, while the decaying mode behaves as $\delta_2 \sim t^{-\gamma}$, where $\alpha, \gamma > 0$ are constants. Then from Chebyshev's inequality we find $P_1\left(|\Delta\rho/\rho| = \delta_1\right) \sim t^{-2\alpha}$ and $P_2\left(|\Delta\rho/\rho| = \delta_2\right) \sim t^{2\gamma}$. We now take into account the collective effects and look for the probability of the existence of a growing density contrast if there are neighboring regions of space in which the density inhomogeneity decreases. From a mathematical point of view, we need to find the conditional probability P_{12}, which, according to the rules of probability theory, is given by $P_1 = P_{12}P_2$. Therefore

$$P\left(|\Delta\rho/\rho| = \delta\right) = P_{12} = \frac{P_1}{P_2} \sim t^{-2\alpha - 2\gamma}. \tag{20}$$

By again using Chebyshev's inequality we find

$$\frac{\Delta\rho}{\rho} \sim t^{\alpha + \gamma}. \tag{21}$$

3 FRACTAL DIMENSION AND PREDICTABILITY INDEX

The fractal dimension can be defined by the self-similar power law scaling function (Mandelbrot 1983; Rangarajan & Sant 2004)

$$y(x) = a_s x^D, D > 0, \tag{22}$$

where $y(x)$ denotes the number of self-similar objects in the sphere or circle of radius x; a_s and D stand for the scaling factor and for the spatial fractal dimension, respectively. Physical systems evolve not only in space but also in time. Therefore many time dependent natural processes can be successfully fitted by the temporal counterpart of the fractal function given by Equation (22),

$$y(t) = a_t t^{D_t}, t > 0, \tag{23}$$

in which $y(t)$ characterizes the time-evolution of the system, D_t is its temporal fractal dimension and a_t is a scaling factor.

In the following, we will obtain an expression for the calculation of the fractal dimension by starting from the definition of the Haussdorf dimension (Sevcik 1998, 2006). The Haussdorf D_{H} dimension of a set in a metric space is defined as

$$D_{\mathrm{H}} = -\lim_{\epsilon \to 0} \frac{\ln [N(\epsilon)]}{\ln \epsilon}, \tag{24}$$

where $N(\epsilon)$ is the number of open balls of a radius ϵ needed to cover the entire set. In a metric space, given any point P, an open ball of center P and radius ϵ, there is a set of all points x for which $\mathrm{dist}(P, x) < \epsilon$. A line of length L may be divided into $N(\epsilon) = L/(2\epsilon)$ segments of length 2ϵ and may be covered by N open balls of radius ϵ. Thus, Equation (24) may be rewritten as

$$D_{\mathrm{H}} = \lim_{\epsilon \to 0} \left[\frac{-\ln(L) + \ln(2\epsilon)}{\ln(\epsilon)} \right] = \lim_{\epsilon \to 0} \left[1 - \frac{\ln(L) - \ln(2)}{\ln(\epsilon)} \right] = \lim_{\epsilon \to 0} \left[1 - \frac{\ln(L)}{\ln(\epsilon)} \right]. \tag{25}$$

Waveforms are planar curves in a space with coordinates which usually have different units. Since the topology of a metric space does not change under linear transformation, it is convenient to linearly transform a waveform into another normalized space, where all axes are equal. Sevcik (1998) proposed to use two linear transformations that map the original waveform into another form, embedded in an equivalent metric space. The first transformation normalizes every point in the abscissa as $x_i^* = x_i/x_{\max}$, where x_i are the original values of the abscissa and $x_{\max}$ is the maximum x_i. The second transformation normalizes the ordinate as $y_i^* = (y_i - y_{\max})/(y_{\max} - y_{\min})$, where y_i are the original values of the ordinate and $y_{\min}$ and $y_{\max}$ are the minimum and maximum y_i, respectively. These two linear transformations map the N points of the waveform into another space that is a subset of a unit square. This unit square may be visualized as covered by a grid of $N \times N$ cells, with N of them containing one point of the transformed waveform. Calculating L of the transformed waveform and taking $\epsilon = 1/(2 \times N')$, where $N' = N - 1$, Equation (25) becomes (Sevcik 1998)

$$D_{\mathrm{H}} \approx D = 1 + \frac{\ln(L)}{\ln(2 \times N')}. \tag{26}$$

The approximation for the calculation of D_{H} improves as $N' \to \infty$.

If the fractal dimension D for the time series is 1.5, there is no correlation between amplitude changes corresponding to two successive time intervals. Therefore, no trend in amplitude can be discerned from the time series and hence the process is unpredictable. However, as the fractal dimension decreases to 1, the process becomes more and more predictable since it exhibits "persistence." That is, the future trend is more and more likely to follow an established trend. As the fractal dimension increases from 1.5 to 2, the process exhibits "anti-persistence." That is, a decrease in the amplitude of the process is more likely to lead to an increase in the future. Hence, the predictability again increases (Rangarajan & Sant 2004).

Based on the fractal dimension, we can introduce the predictability index P of a given physical system, defined as (Rangarajan & Sant 1997),

$$PI = 2\,|D - 1.5|, \tag{27}$$

where $|\ \ |$ denotes the absolute value of the number D. We use absolute values since predictability increases in both of the following cases - when the fractal dimension becomes less than 1.5 and when it becomes greater than 1.5. In the former case, we have correlation (persistence) behavior and in the latter case, anti-correlation (anti-persistence) behavior. However, in either case, the process becomes more predictable. Thus, the use of absolute values ensures that a process with $D = 1.3$ has the same predictability index as a process with $D = 1.7$.

4 FRACTAL DIMENSION, PREDICTABILITY INDEX AND POWER SPECTRUM INDEX OF IDV LIGHT CURVES

In order to analyze the fractal properties of IDV light curves, we consider a sample of several blazars, which have been observed in a simultaneous multi-wavelength observing campaign in their outburst phase by Gupta et al. (2008). Optical photometric monitoring of nine blazars was carried out in 13 observing nights during the observing run of 2006 October 27–2007 March 20, using the 1.02 m optical telescope equipped with a CCD detector and BVRI Johnson broadband filters at Yunnan Astronomical Observatory, Kunming, China. The R-band observations of five blazars are represented in Figure 1.

There are many ways to characterize different noise sources. One possibility, which we have also explored in the present paper, is to consider the spectral density, that is, the mean square fluctuation at any particular frequency f and its variation with frequency. In many natural processes, the power spectrum $P(f)$ is proportional to $1/f^\beta$, where the power index $\beta \geq 0$. When $\beta = 0$, the noise is referred to as white noise, when it is 2 it is referred to as a Brownian noise and when it is 1 it is called $1/f$ noise (Havlin et al. 1995; Denisov 1998).

In order to obtain the power spectral index β we have used the periodogram method, as developed in Vaughan (2005, 2010). To fit the data we have used power law fitting for all our bands. In fact, we can also use other options. However, since the number of available data points in the optical and radio bands is limited, for the sake of uniqueness we can only use the power law fitting method. Although the X-ray data can be fitted by different functions, the optical data and the radio data cannot be fitted by other methods, such as the bending power method. Therefore, we fitted the data in all bands by using a power law. With the use of the above procedures, we can obtain the power spectrum, the power spectrum index and all the other useful parameter information, such as the p-value. The error values after fitting have also been obtained and we present them in Tables 1 and 2 as a percentage error.

The fractal dimensions, the predictability index PI_{IDV} and the power index of the observed blazar optical time series are presented in Table 1.

Table 1 Fractal dimensions, predictability indices and power indices of a sample of R-band light curves (Gupta et al. 2008).

Object	Date of Observation	D	PI_{IDV}	β	Fit Error of β
S2 0109+224	2007–01–11	1.52591	0.051	0.909647	0.1723201
S5 0716+714	2007–01–11	1.44996	0.100	1.41473	0.1404088
S5 0716+714	2007–01–12	1.45699	0.086	1.389167	0.07811043
S5 0716+714	2007–02–23	1.35088	0.298	1.672849	0.2028525
S5 0716+714	2007–03–19	1.57989	0.159	0.2951006	0.1540101
S5 0716+714	2007–03–20	1.44484	0.110	1.21574	0.08695967
PKS 0735+178	2007–01–11	1.56557	0.131	0.6411014	0.1548084
ON 231	2007–01–11	1.45285	0.094	0.7989009	0.1536858
1ES 2344+514	2007–01–12	1.42142	0.157	1.148679	0.2762177

As a second example of the determination of the fractal dimension, we consider the short time-scale radio variations of the compact extragalactic radio source J 1128+5925 (Gabányi et al. 2007). The flux density variability of J 1128+5925 was monitored with dense time sampling between 2.7 and 10.45 GHz. For our analysis we use the data observed with the 100 m Effelsberg radio telescope of the Max-Planck-Institut fur Radioastronomie at 2.70, 4.85 and 10.45 GHz. Ten observing sessions that lasted several days during the period between 2004–2006 were performed.

Apart from the optical and radio data, we also tried to use short time-scale IDV data from the XMM database for calculating the fractal dimension. We reduced the data and followed the result
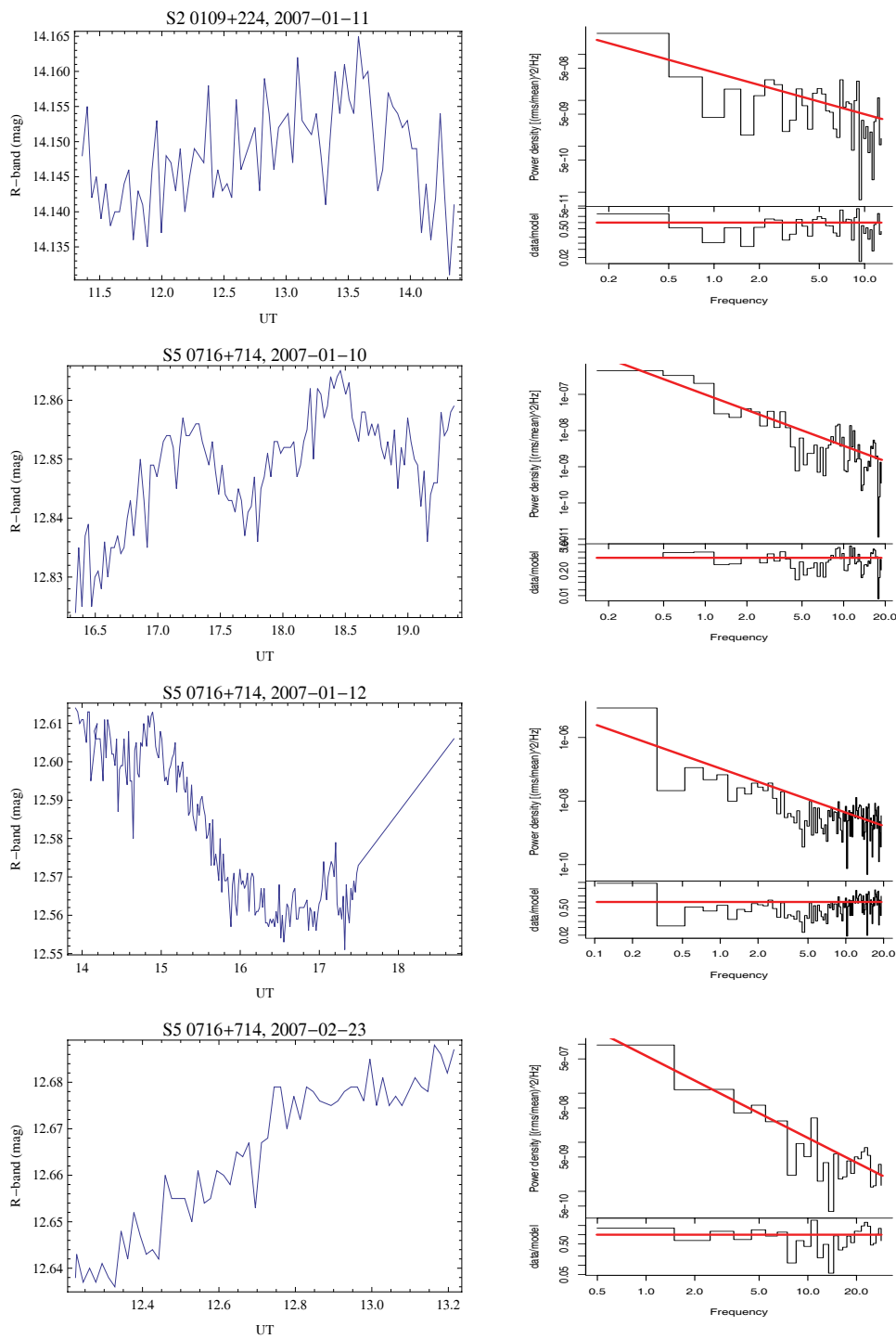
Fig. 1 *R*-band variability and power spectrum of the sample of blazars observed in Gupta et al. (2008).
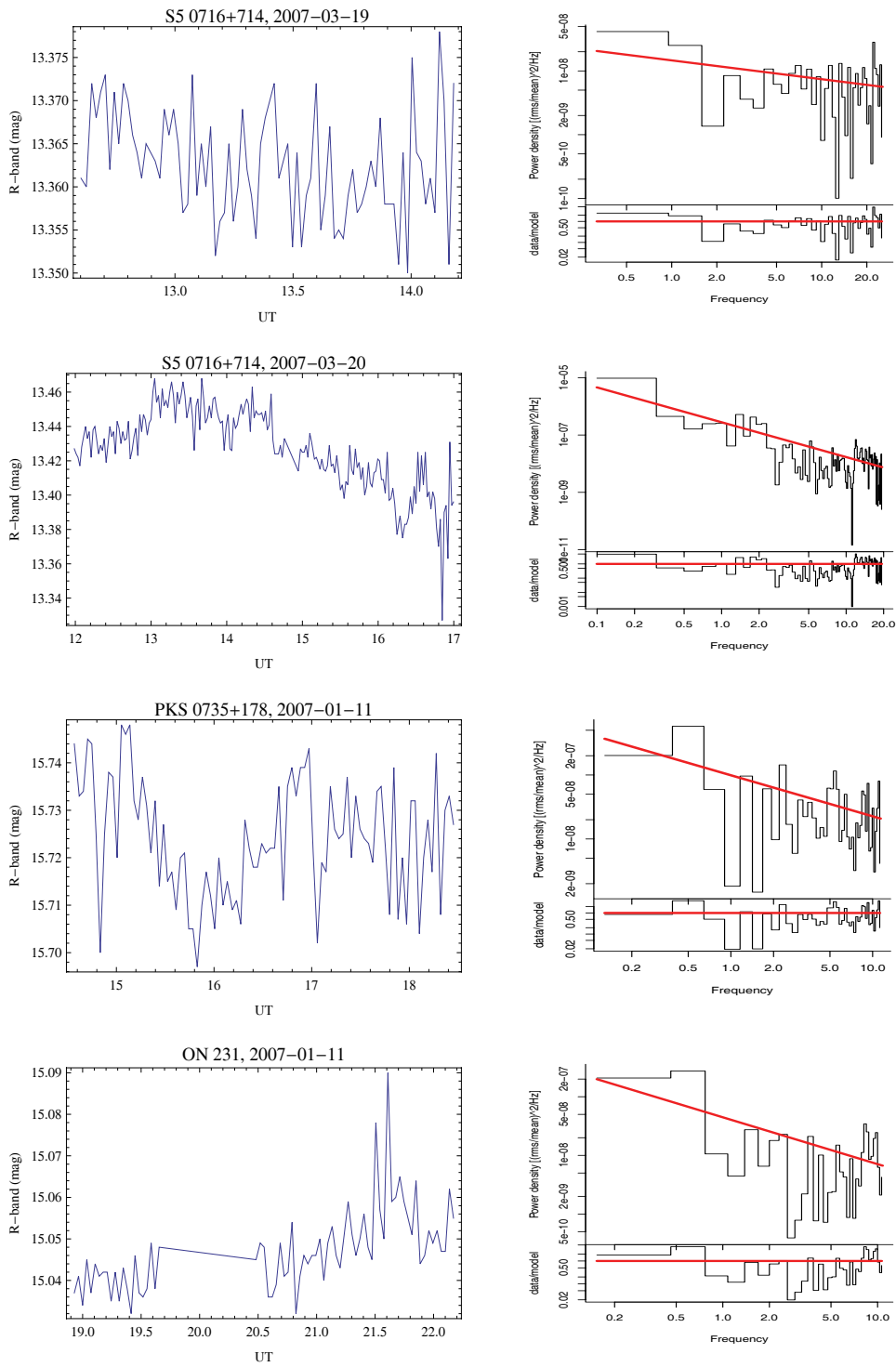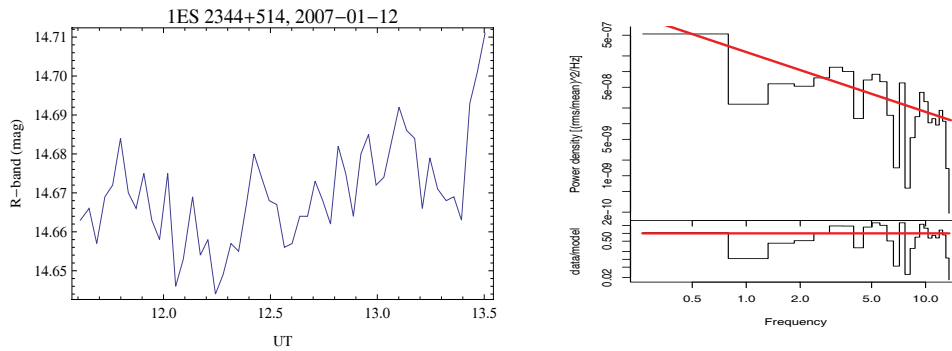
Fig. 1 — *Continued.*

Fig. 1 — *Continued.*

Table 2 Fractal dimensions, predictability indices and power spectral indices of the compact extragalactic radio source J 1128+5925 (Gabányi et al. 2007) and of the X-ray sources ON 231, 1ES 2344+514 and PKS 2155–304 (Gaur et al. 2010).

Object	Date of Observation	D	PI_{IDV}	β	Fit Error of β
J 1128+5925	2005–09–16	1.28642	0.42716	2.188192	0.1472077
J 1128+5925	2006–04–28	1.25587	0.48826	2.042647	0.1100852
ON 231	2002–06–26	1.43877	0.122	1.621563	0.06751574
1ES 2344+514	2005–06–19	1.60232	0.204	0.63486	0.06634579
PKS 2155–304	2002–05–24	1.44012	0.11976	1.055890	0.0554563
PKS 2155–304	2002–05–24	1.56946	0.13892	0.6418146	0.06857538
PKS 2155–304	2002–05–24	1.33378	0.33244	1.615948	0.06764645
PKS 2155–304	2000–05–31	1.57255	0.1451	1.023354	0.04338111
PKS 2155–304	2000–11–21	1.51078	0.02156	1.171058	0.04761672

from the published targets as shown in Gaur et al. (2010). From the XMM database we have reduced seven blazars.

We used the *XMM-Newton* Science Analysis System (SAS) version 11.0 to reduce the EPIC-pn data taken from the archive. We followed the same procedure outlined in Gaur et al. (2010) to extract the lightcurves. In brief, we excluded all the high background due to flaring and restricted all the events to the 0.3–10 keV energy band. We used a 45 arcsec radius as the extraction region for both the source and the background (from a source-free region). In addition, the background subtracted light curves were corrected for vignetting, bad pixels, point spread function (PSF) variation, quantum efficiency and dead time. All the light curves were binned with 100-second timing resolution.

The radio and X-ray data, together with their corresponding power spectra are shown in Figure 2, while the fractal dimension, predictability indices and the power spectral indices are presented in Table 2.

There is a relation between the fractal dimension D of the Brownian noise and the power index β, namely (Havlin et al. 1995; Denisov 1998)

$$D = \frac{5 - \beta}{2}. \tag{28}$$

If the fractal dimension of the considered IDV sources is around $D = 1.5$, then the power spectral index is around $\beta = 2$. We have studied the correlation between the fractal dimension D and the power spectral index of the optical and X-ray IDV sources - we did not include the radio data in our study, due to their small number. There is a very good correlation between D and β and for these 15 sources the relation between the fractal dimension and the power index can be given as

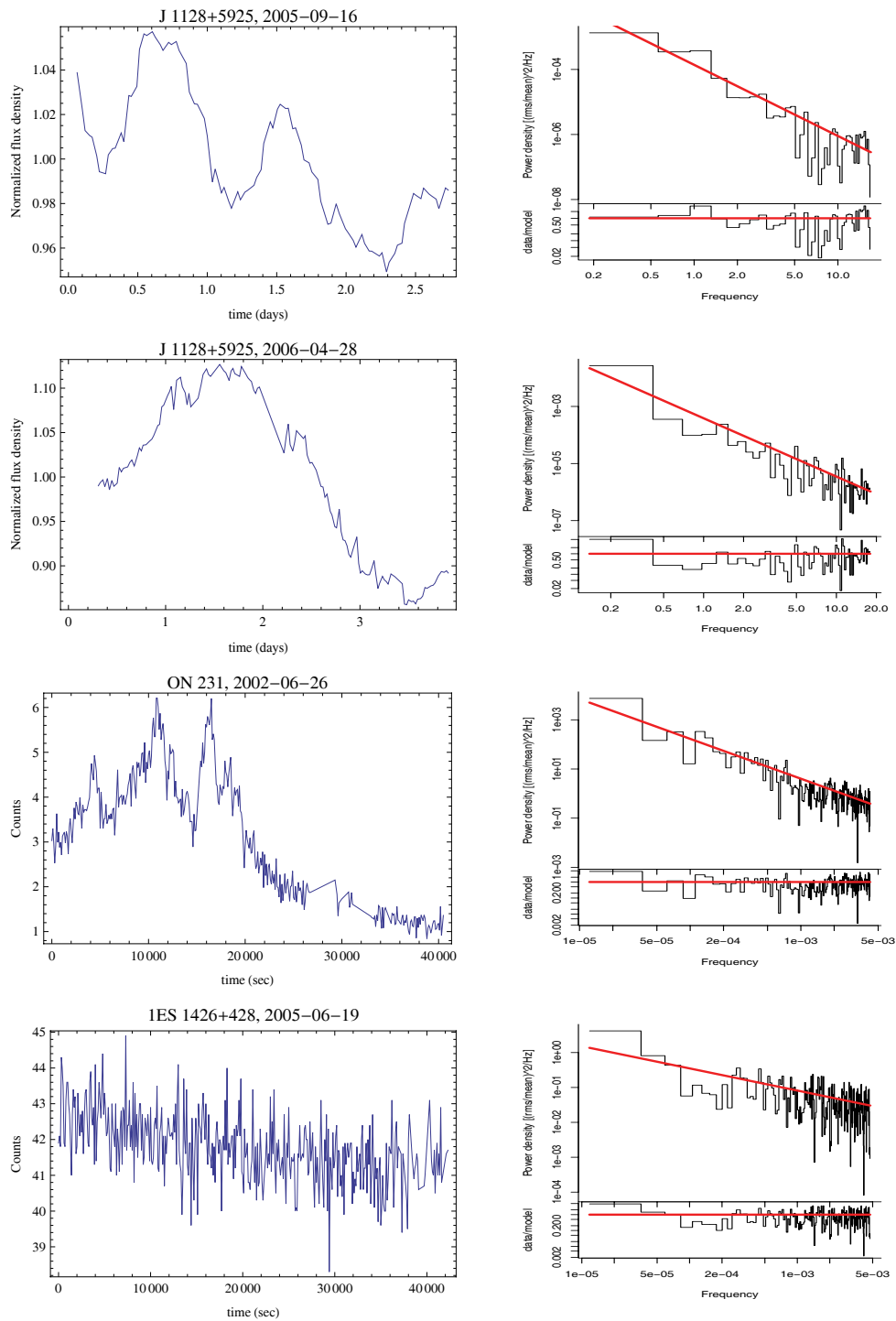$$D = 1.66229 - 0.16699 \times \beta. \tag{29}$$

Fig. 2 Radio-band variability and power spectrum of the compact extragalactic radio source J 1128+5925 (Gabányi et al. 2007) and X-ray variability and power-spectrum of the sources ON 231 and 1ES 2344+514.

Table 3 Regression statistics for the correlation between fractal dimensions and power spectral indices of optical and X-ray sources. The correlation coefficient is denoted by C.C.

C.C.	C.C.^{2}	Standard Error	Total Number of Cases
0.8428	0.7104	0.0469	15

Table 4 ANOVA Table for the correlation coefficient of the fractal dimensions and power spectral indices of optical and X-ray sources.

Name	d.o.f	SS	MS	F	p-value
Regression	1	0.07041	0.07041	31.89023	0.00008
Residual	13	0.0287	0.00221	–	–
Total	14	0.09912	–	–	–

Notes: d.o.f represents the number of degrees of freedom, SS is the sum of squares, MS is the mean sum of squares value, F represents the value of the F-statistics and p-value represents the measure of the degree of the insignificance of the regression model.

Equation (29) can also be written, to a very good approximation, as

$$D = \frac{10 - \beta}{6}. \tag{30}$$

The correlation statistics of the $D - \beta$ correlation is represented in Table 3. For the chosen set of data the correlation coefficient between these two parameters is 0.8428.

The statistical significance of the correlation coefficient is presented in Table 3 by means of the ANOVA analysis. In Table 3, C.C. represents the correlation coefficient. It represents the fitting accuracy. With the C.C. equal to 1 and –1, the scatter diagram can be fitted by one single straight line. C.C.^{2} represents the square of C.C. In statistics, the number of degrees of freedom is the number of values in the final calculation of a statistic that are free to vary. If the F-statistic values computed in the ANOVA table are less than the F-table statistics, or the p-value is greater than the alpha level of significance, then there is no reason to reject the null hypothesis that all the means are the same. For our p-value, we obtained 0.00008 and thus we conclude that the null hypothesis should not be rejected at the 1% significance level, since the p-value is much less than 0.01.

The relation between the theoretical data fit and the observational data, given by Equation (29), is presented graphically in Figure 3. The presence of the strong Brownian noise in the IDV spectra also strongly support the idea that random stochastic processes play a significant role in their formation and evolution, as suggested in Leung et al. (2011).

5 DISCUSSION AND FINAL REMARKS

Since the determination of the fractal dimension of the time series is relatively simple, it can provide useful information about the signal fluctuation from variable astrophysical sources. In the present paper, we have explicitly obtained the fractal dimensions for several IDV emissions in both optical and radio bands. From the point of view of the fractal analysis, the optical and the radio signals show some remarkable differences. In the optical and X-ray bands, the fractal dimension of the IDV signals is very close to the value $D = 1.5$, indicating an almost perfect "Brownian noise" (random walk, or Wiener process) fluctuation spectrum (Brownian noise can be generated by integrating white noise) (Havlin et al. 1995; Denisov 1998). The predictability index of this signal is very low, showing that the emission in the optical band is dominated by purely stochastic processes. On the other hand, the radio band emission pattern of the compact extragalactic radio source J 1128+5925 shows a relatively low fractal dimension on the order of $D = 1.25$, with a much higher predictability index. This result
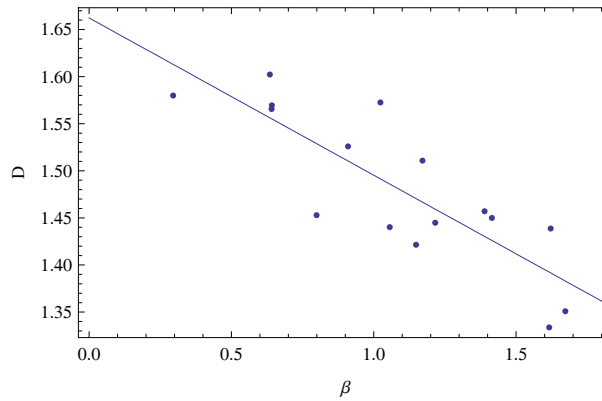
Fig. 3 Correlation between the fractal dimension D and the spectral index β.

can be interpreted from a physical point of view in the framework of the thermodynamic theory of fluctuations as follows. Assuming that in both fluctuating phases the self-gravitating fluctuations have a time dependence given by Equation (16), we have $\alpha = \gamma = 2/3$. Then from Equation (21) it follows that the time dependence of the fluctuation spectrum is on the order of $\sim t^{\alpha+\gamma} \sim t^{4/3} \sim t^{1.33}$, which gives a fractal dimension $D = 1.33$, very close to the observed values of $D = 1.25$ and $D = 1.28$. Therefore, this implies that the dominant time scale in the emission process is on the order of the gravitational free fall time scale. On the other hand, in order to obtain the fractal dimension ($D = 1.5$) in the optical phase of the IDV through the same method, the assumption of a $t^{\pm 0.75}$ dependence in the fluctuation spectrum from the fluctuating phases is necessary. This shows that the characteristic time scale of the density fluctuations cannot be on the same order of magnitude as the free fall time and other physical processes, like, for example, viscous dissipation, which may be responsible for the dynamics.

We have also obtained the general time dependence of the fluctuation spectrum in a self-gravitating system, as given by Equations (17) and (18). The density fluctuations can be related to the luminosity fluctuations of the emitting source. By assuming that the emission of the radiation is in the form of thermal radiation, from the Stefan-Boltzmann law we find that the luminosity (energy emitted per second) from a spherical surface with radius R is given by $L = 4\pi R^2 \sigma T_{\text{eff}}^4$, where σ is Stefan's constant and T_{eff} is the effective temperature of the emitting surface. With the use of the hydrostatic equilibrium equation and of the virial theorem one can show that $L \propto \rho^{1/3}$ (Choudhuri 2010). With the use of this relation, we immediately obtain $\Delta L/L \propto \Delta\rho/\rho$. Therefore, with the use of Equations (17) and (18) the radial distance and time spectrum of the fluctuations of the luminosity in self-gravitating systems can be written as

$$\sqrt{\left\langle \left(\frac{\Delta L}{L}\right)^2 \right\rangle} \propto r^{-D_r} \propto t^{-D_t}. \tag{31}$$

The constants D_r and D_t can be interpreted physically as the fractal dimensions, describing the fractal properties of the thermal radiation emission spectra from fluctuating sources. The comparison of the fractal dimension of these spectra with the IDV observational data could give a powerful indication about the nature of the physical processes (thermodynamical or gravitational, random or deterministic) that play the dominant role in the electromagnetic emission process.

In the present paper, we have also studied the correlation between the fractal dimension and the power spectral index β. We have found a very good correlation between these two physical parameters.

Fractal properties, or self-similarity, can be found in many aspects in nature. Our analysis has concentrated on the fractal properties of the IDV signal. Due to the limited amount of data used, our results can be considered only as a first step in the investigation of the fractal properties of the IDVs. It will be important in the future to investigate more signals at different wavelengths, in order to obtain a more accurate determination of the fractal dimension and to clarify the relation between the fractal dimensions of different bands and the spectral power index.

Appendix A:

To obtain the spectral index β and for fitting our data, we used the periodogram method as developed in Vaughan (2010). The periodogram is used for dealing with the evenly sampled time series x_k of K points at intervals ΔT. The definition of the periodogram is (Vaughan 2005)

$$I(f_j) = \frac{2\Delta T}{\langle x \rangle^2 N} |X_j|^2, \tag{A.1}$$

where $|X_j|^2$ is the modulus-squared of the discrete Fourier transform.

The unit of the normalization of the periodogram is $(\text{rms/mean})^2 \ \text{Hz}^{-1}$. The initial purpose of the periodogram was to search for "hidden periodicity" in time series analysis. However, for a single time series, the periodogram of a noise process shows a great deal of scatter around the underlying power spectrum. For the case of a given frequency, $I(f_j)$ is scattered around the true power spectrum, $P(f_j)$. It also follows a χ^2 distribution with two degrees of freedom,

$$I(f_j) = P(f_j) \chi_2^2/2, \tag{A.2}$$

where χ_2^2 is a random variable distributed as χ^2 with two degrees of freedom. It is equivalent to an exponential probability density, with a mean and variance of two and four, respectively,

$$p_{\chi^2}(x) = \exp(-x/2)/2. \tag{A.3}$$

For the case of a white noise flat spectrum, the power level is calculated by making use of Equations (A.2) and (A.3) to obtain the maximum likelihood of a given periodogram. Similarly, for the more general case of non-white noise, we need to examine the spurious peaks in the periodogram and regard them as Brownian (red) noise. A simple method to obtain a reasonable estimate of the power-law spectrum is $S(f) = \alpha f^{-\beta}$. α is the so called normalization of the power-law power spectrum and β is the power spectral index.

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