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Pulsating magneto-dipole radiation of a quaking neutron star powered by energy of Alfvén seismic vibrations *

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Abstract We compute the characteristic parameters of the magneto-dipole radiation of a neutron star undergoing torsional seismic vibrations under the action of Lorentz restoring force about an axis of a dipolar magnetic field experiencing decay. After a brief outline of the general theoretical background of the model of a vibration-powered neutron star, we present numerical estimates of basic vibration and radiation characteristics, such as frequency, lifetime and luminosity, and investigate their time dependence on magnetic field decay. The presented analysis suggests that a gradual decrease in frequencies of pulsating high-energy emission detected from a handful of currently monitored AXP/SGR-like X-ray sources can be explained as being produced by the vibration-powered magneto-dipole radiation of quaking magnetars.

Key words: neutron stars — torsion Alfvén vibrations — vibration powered radiation — magnetic field decay — magnetars

1 INTRODUCTION

The last two decades have seen an increasing interest in Soft Gamma Repeaters (SGRs) and Anomalous X-ray Pulsars – commonly referred to as magnetars (Duncan & Thompson 1992), the seismic and radiative activity of which is fairly different from that of rotation-powered radio pulsars (e.g. Kouveliotou 1999; Harding 1999; Woods & Thompson 2006; Mereghetti 2008; Qiao et al. 2010). The most popular idea is that the energy supply of the long-period pulsating radiation of these fairly young neutron stars comes from a process involving decay of an ultra-strong magnetic field. One such process could be magneto-mechanical vibrations driven by forces of magnetic-field-dependent stresses (Bastrukov et al. 2002). In the development of this line of reasoning, particular attention has been given to the torsion vibrations of perfectly conducting stellar matter about the magnetic axis of the star under the action of magnetic Lorentz force, with a focus on the discrete frequency spectra of the toroidal Alfvén mode (a-mode). In the past, the standing-wave regime of such vibrations has been the subject of several investigations (Ledoux & Walraven 1958). In works (Bastrukov et al. 2009b,a, 2010) focus concentrated on the non-investigated regime of node-free vibrations in the static (time-independent) field. The prime purpose of these latter works was to gain

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some insight into the difference between spectra of discrete frequencies of toroidal a-modes in neutron star models having the same mass M and radius R, but different shapes of constant-in-time poloidal magnetic fields. By use of the Rayleigh energy method, it was found that each specific form of spatial configuration of static magnetic field, about the axis of which the neutron star matter undergoes node-free differentially rotational oscillations, is uniquely reflected in the discrete frequency spectra in the form of dependence of frequency on overtone ℓ of vibrations. The subject of our present study is the radiative activity of a quaking neutron star powered by the energy of Alfvén vibrations in its own magnetic field which is experiencing decay. Part of this project has been reported in recent workshops and conferences and in a short paper (Bastrukov et al. 2011). Taking this into account, only a brief overview of the theoretical background of the model is given in this paper. The focus is placed on numerical computation of the basic characteristics of vibration and radiation, which are of interest in the observational search for such objects.

2 GENERAL BACKGROUND OF THE VIBRATION POWERED NEUTRON STAR MODEL

Only a brief outline of this model is given here and more details can be found elsewhere (Bastrukov et al. 2009b, 2010, 2011a). The Lorentz-force-driven differentially rotational vibrations of perfectly conducting matter in a neutron star about the axis of poloidal internal and dipolar external magnetic fields, which are evolving in time, are properly described in terms of material displacements $\boldsymbol{u}$ obeying the equation of magneto-solid-mechanics

$$\rho(r)\,\ddot{\boldsymbol{u}}(\boldsymbol{r},t) = \frac{1}{4\pi}[\nabla\times[\nabla\times[\boldsymbol{u}(\boldsymbol{r},t)\times\boldsymbol{B}(\boldsymbol{r},t)]]]\times\boldsymbol{B}(\boldsymbol{r},t), \tag{1}$$

$$\dot{\boldsymbol{u}}(\boldsymbol{r},t) = [\boldsymbol{\omega}(\boldsymbol{r},t)\times\boldsymbol{r}], \quad \boldsymbol{\omega}(\boldsymbol{r},t) = A_t[\nabla\chi(r)]\,\dot{\alpha}(t). \tag{2}$$

The field $\dot{\boldsymbol{u}}(\boldsymbol{r},t)$ is identical to that for torsion node-free vibrations restored by Hooke's force of elastic stresses (Bastrukov et al. 2007, 2010) with $\chi(\boldsymbol{r}) = A_\ell\, f_\ell(r)\, P_\ell(\cos\theta)$ where $f_\ell(r)$ is the nodeless function of distance from the star's center and $P_\ell(\cos\theta)$ is the Legendre polynomial of degree ℓ specifying the overtone of the toroidal mode. In Equation (2), the amplitude $\alpha(t)$ is the basic dynamical variable describing time evolution of vibrations and is different for each specific overtone. The bulk density can be represented in the form $\rho(r) = \rho\phi(r)$, where ρ is the density at the star's center and $\phi(r)$ describes the radial profile of density, which can be taken from computations of neutron star structure relying on realistic equations of state accounting for non-uniform mass distribution in the star's interior (e.g., Weber 1999). Central to the subject of our study is the following representation of the time-evolving internal magnetic field $\boldsymbol{B}(\boldsymbol{r},t) = B(t)\,\boldsymbol{b}(\boldsymbol{r})$, where $B(t)$ is the time-dependent intensity and $\boldsymbol{b}(\boldsymbol{r})$ is a dimensionless vector function of the field distribution over the star's volume. The gist of the energy variational method of computing the frequency of node-free Alfvén vibrations is found in the following separable representation of material displacements $\boldsymbol{u}(\boldsymbol{r},t) = \boldsymbol{a}(\boldsymbol{r})\,\alpha(t)$. The scalar product of Equation (1) with this form of $\boldsymbol{u}$ and integration over the star's volume leads to the equation for amplitude $\alpha(t)$ having the form of an equation of an oscillator with a time-dependent spring constant

$$\mathcal{M}\ddot{\alpha}(t) + \mathcal{K}(t)\alpha(t) = 0, \tag{3}$$

$$\mathcal{M} = \rho\, m, \quad m = \int \phi(r)\,\boldsymbol{a}(\boldsymbol{r})\cdot\boldsymbol{a}(\boldsymbol{r})\,d\mathcal{V}, \quad \boldsymbol{a} = A_t\nabla\times[\boldsymbol{r}\, f_\ell(r)\, P_\ell(\cos\theta)],$$

$$\mathcal{K} = \frac{B^2(t)}{4\pi}\,k, \quad k = \int \boldsymbol{a}(\boldsymbol{r})\cdot[\boldsymbol{b}(\boldsymbol{r})\times[\nabla\times[\nabla\times[\boldsymbol{a}(\boldsymbol{r})\times\boldsymbol{b}(\boldsymbol{r})]]]]\,d\mathcal{V}.$$

As far as the general asteroseismology of compact objects is concerned, the above equations seem to be appropriate not only for neutron stars, but also for white dwarfs (Molodtsova et al. 2010;

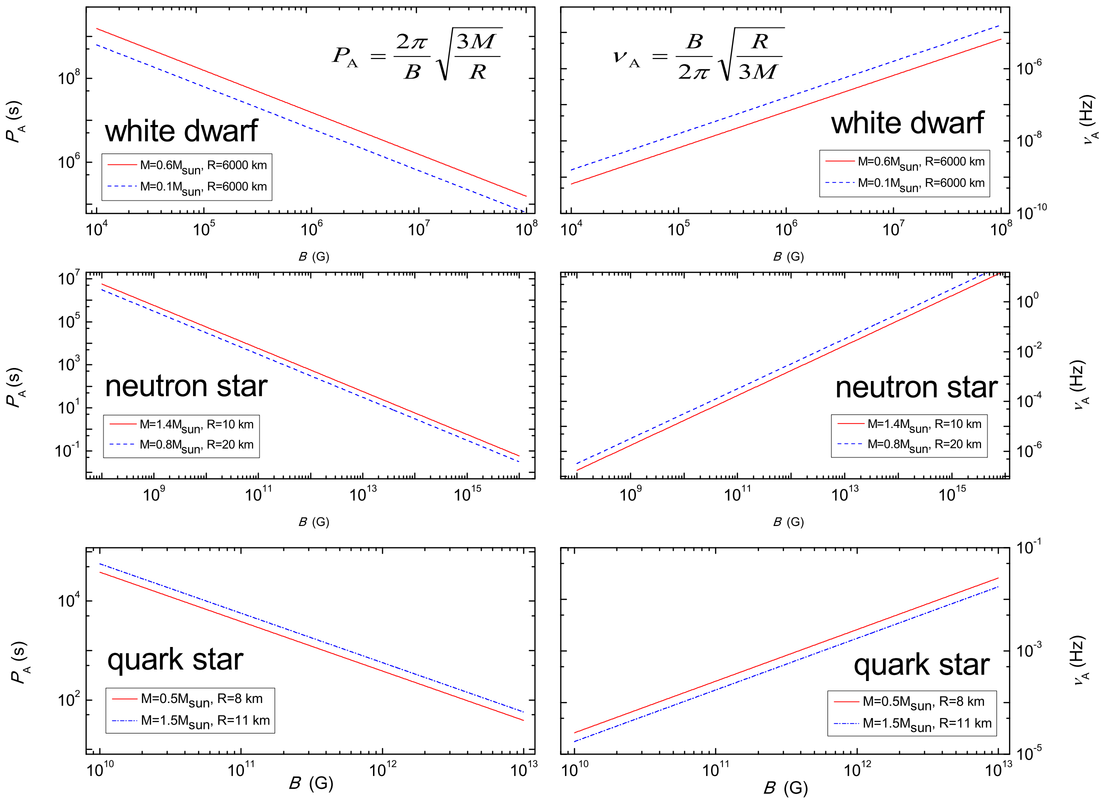

Fig. 1 Basic frequencies and periods of global Alfvén oscillations as functions of magnetic field for the typical mass and radius of white dwarfs, neutron stars and quark stars. The difference in mass and radius can be regarded as reflecting the difference in the underlying equations of state of matter in these compact objects.

Bastrukov et al. 2010) and quark stars. The superdense material of strange quark stars is also expected to be in solid state (Xu 2003, 2009).

In Figure 1 we plot the fundamental frequency $\nu_A = \omega_A/2\pi$ (where $\omega_A = v_A/R$) and the period $P_A = \nu_A^{-1}$ of global Alfvén oscillations

$$\nu_A = \frac{B}{2\pi}\sqrt{\frac{R}{3M}}, \quad P_A = \frac{2\pi}{B}\sqrt{\frac{3M}{R}} \tag{4}$$

as functions of intensity of magnetic field B which is constant in time for solid star models with masses and radii of typical white dwarfs, neutron stars and quark stars.

The total vibration energy and frequency are given by

$$E_A(t) = \frac{\mathcal{M}\dot{\alpha}^2(t)}{2} + \frac{\mathcal{K}(B(t))\alpha^2(t)}{2}, \quad \omega(t) = \sqrt{\frac{\mathcal{K}(t)}{\mathcal{M}}} = B(t)\kappa, \quad \kappa = \sqrt{\frac{R}{3M}}\,s. \tag{5}$$

Here M and R are the neutron star's mass and radius, respectively, and s is the parameter depending on the overtone of the Alfvén toroidal mode and the depth of the seismogenic layer. The energy conversion of the above magneto-mechanical vibrations into magneto-dipole radiation is governed by equation

$$\frac{dE_{\rm A}(t)}{dt} = -\mathcal{P}(t), \quad \mathcal{P}(t) = \frac{2}{3c^3}\delta\ddot{\boldsymbol{\mu}}^2(t). \tag{6}$$

The axisymmetric torsional oscillations of matter around the magnetic axis of the star are accompanied by fluctuations of total magnetic moment preserving its initial (in a seismically quiescent state) direction: $\boldsymbol{\mu} = \mu\,\boldsymbol{n} = \mathrm{constant}$. The frequency $\omega(t)$ of such oscillations must be the same for both fluctuations of magnetic momenta $\delta\boldsymbol{\mu}(t)$ and the above magneto-mechanical oscillations, which are described in terms of $\alpha(t)$, namely

$$\delta\ddot{\boldsymbol{\mu}}(t) + \omega^2(t)\delta\boldsymbol{\mu}(t) = 0, \quad \ddot{\alpha}(t) + \omega^2(t)\alpha(t) = 0, \quad \omega^2(t) = B^2(t)\kappa^2. \tag{7}$$

This suggests $\delta\boldsymbol{\mu}(t) = \boldsymbol{\mu}\,\alpha(t)$. On account of this, the equation of energy conversion is reduced to the following law of magnetic field decay

$$\frac{dB(t)}{dt} = -\gamma\, B^3(t), \quad \gamma = \frac{2\mu^2\kappa^2}{3\mathcal{M}c^3} = \mathrm{constant}, \tag{8}$$

$$B(t) = \frac{B(0)}{\sqrt{1+t/\tau}}, \quad \tau^{-1} = 2\gamma B^2(0). \tag{9}$$

Thus, the magnetic field decay resulting in the loss of total energy of Alfvén vibrations in the star causes its vibration period $P \sim B^{-1}$ to lengthen at a rate proportional to the rate of magnetic field decay. The time evolution of vibration amplitude (the solution of the equation for $\alpha(t)$ (see Fig. 2), obeying two conditions $\alpha(t=0) = \alpha_0$ and $\alpha(t=\tau) = 0$) reads[1](e.g., Polyanin & Zaitsev 2004)

$$\alpha(t) = C\, s^{1/2}\{J_1(z(t)) - \eta\, Y_1(z(t))\}, \; z(t) = 2\beta\, s^{1/2}(t), \quad s = 1 + t/\tau, \tag{10}$$

where $J_1(z)$ and $Y_1(z)$ are Bessel functions (e.g., Abramowitz & Stegun 1972) and

$$\alpha_0^2 = \frac{2\bar{E}_{\rm A}(0)}{M\omega^2(0)} = \frac{2\bar{E}_{\rm A}(0)}{K(0)}, \quad \omega^2(0) = \frac{K(0)}{M}. \tag{11}$$

Here $\bar{E}_{\rm A}(0)$ is understood to be the average energy stored in the torsional Alfvén vibrations of a magnetar. If all the detected energy $E_{\rm burst}$ of an X-ray outburst goes into the quake-induced vibrations, $E_{\rm burst} = E_{\rm A}$, then the initial amplitude α_0 is determined unambiguously.

Thus, the magnetic field decay is crucial to the energy conversion from Alfvén vibrations to electromagnetic radiation, whose most striking feature is the gradual decrease of frequency of pulses (period lengthening). The magnetic-field-decay induced lengthening of period in pulsating radiation (equal to a period of vibration) is described by

$$P(t) = P(0)\,[1 + (t/\tau)]^{1/2}, \; \dot{P}(t) = \frac{1}{2\tau}\frac{P(0)}{[1+(t/\tau)]^{1/2}},$$
$$\tau = \frac{P^2(0)}{2P(t)\dot{P}(t)}, \quad P(0) = \frac{2\pi}{\kappa\, B(0)}. \tag{12}$$

On comparing τ given by Equations (9) and (12), one finds that the interrelation between the equilibrium value of the total magnetic moment μ of a neutron star of mass $M = 1.2\, M_\odot$ and radius $R = 15$ km vibrating in the quadrupole overtone of the toroidal a-mode, pictured in Figure 3, is given by

$$\mu = 5.5 \times 10^{37} \sqrt{P(t)\,\dot{P}(t)}\, \mathrm{G\,cm^3}. \tag{13}$$

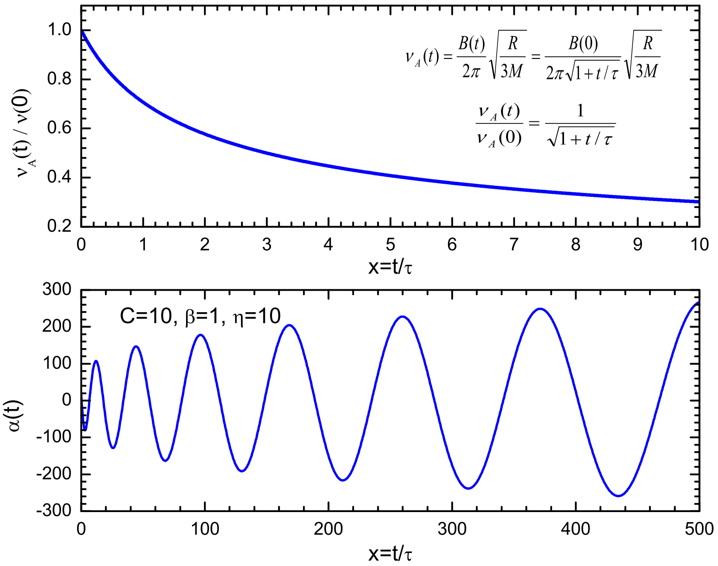

Fig. 2 The figure illustrates the effect of magnetic field decay on the vibration frequency and amplitude of quadrupole toroidal a-mode presented as functions of $x = t/\tau$.

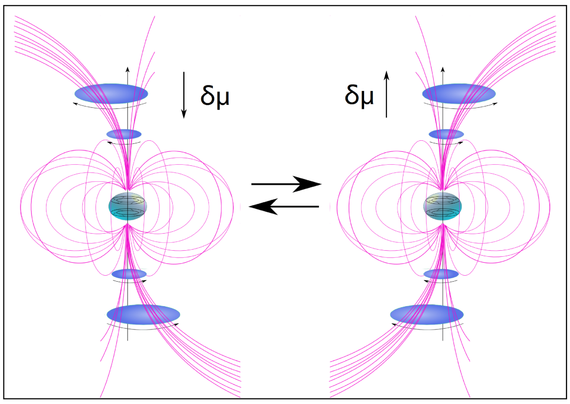

Fig. 3 Artist's view of the quadrupole overtone of the torsional Alfvén vibrations of a magnetar about a magnetic axis producing oscillations of lines of a dipolar magnetic field defining the beam direction for an outburst of X-ray emission.

This illustrative picture shows that the pulsating radiation in question belongs to the periodic changes of polarization of magneto-dipole radiation powered by Alfvén seismic vibrations of a neutron star. In the remainder of this paper, we present numerical estimates of the characteristic parameters of pulsating magneto-dipole radiation produced by a quaking neutron star at the expense of

[1] The authors are indebted to Alexey Kudryashov (Saratov, Russia), for the helpful assistance in solving this equation.

Table 1 Alfvén frequency of Lorentz-force-driven torsion vibrations, $\nu_{\rm A}$, and their lifetime, which is equal to the decay time of magnetic field, τ, in neutron stars with typical magnetic fields for pulsars and magnetars.

	M ($M_\odot$)	R (km)	B (G)	$\nu_{\rm A}$ (Hz)	τ (yr)
Pulsars	0.8	20	10^{12}	3.25×10^{-3}	4.53×10^{10}
	1.0	15	10^{13}	2.52×10^{-2}	2.98×10^{7}
Magnetars	1.1	13	10^{14}	0.22	7.4×10^{3}
	1.2	12	10^{15}	2.06	1.31
	1.3	11	10^{15}	1.89	2.38
	1.4	10	10^{16}	17.4	4.44×10^{-4}

energy of Lorentz-force-driven differentially rotational vibrations about the axis of a dipole magnetic moment.

3 NUMERICAL ANALYSIS

To get an idea of the magnitude of the characteristic parameter of vibrations providing an energy supply of magneto-dipole radiation from a neutron star, in Table 1 we present results of numerical computations of the fundamental frequency of neutron star oscillations in quadrupole toroidal a-mode and time of decay of magnetic field τ as functions of increasing magnetic field.

As was emphasized earlier, the most striking feature of the considered model of vibration-powered radiation is the lengthening of periods of pulsating emission caused by the decay of an internal magnetic field. This suggests that this model is relevant to the electromagnetic activity of magnetars - neutron stars endowed with a magnetic field of extremely high intensity, the radiative activity of which is ultimately related to the magnetic field decay. Such a view is substantiated by estimates of Alfvén frequency presented in the table. For the magnetic fields of typical rotation-powered radio pulsars, $B \sim 10^{12}$ G, the computed frequency $\nu_{\rm A}$ is much smaller than the detected frequency of pulses whose origin is attributed to the lighthouse effect. In addition, for neutron stars with magnetic fields $B \sim 10^{14}$ G, the estimates of $\nu_{\rm A}$ are in the realm of observed frequencies of high-energy pulsating emission of soft gamma repeaters (SGRs), anomalous X-ray pulsars (AXPs) and sources exhibiting similar features. According to common belief, these are magnetars - highly magnetized neutron stars whose radiative activity is related with magnetic field decay (e.g., Woods & Thompson 2006). The amplitude of vibration is estimated as

$$\alpha_0 = \left[\frac{2\bar{E}_{\rm A}(0)}{\mathcal{M}\omega^2(0)}\right]^{1/2} = 3.423 \times 10^{-3} \bar{E}_{\rm A,40}^{1/2} B_{14}^{-1} R_6^{-3/2}, \quad (14)$$

where $\bar{E}_{\rm A,40} = \bar{E}_{\rm A}/(10^{40}\,{\rm erg})$ is the energy stored in the vibrations. Here $R_6 = R/(10^6\,{\rm cm})$ and $B_{14} = B/(10^{14}\,{\rm G})$. The presented computations show that the decay time of the magnetic field (equal to the duration time of the vibration-powered radiation in question) strongly depends on the intensity of the initial magnetic field of the star: the larger the magnetic field B, the shorter the time of radiation τ, at the expense of the energy of vibration. The effect of the equation of state of neutron star matter (which is most strongly manifested in different values for mass and radius of the star) on frequency $\nu_{\rm A}$ is demonstrated by numerical values of this quantity for magnetars with one and the same value of magnetic field $B = 10^{15}$ G but different values of mass and radius.

According to the analytic results presented above, the neutron star luminosity powered by neutron star vibrations in the quadrupole toroidal a-model is given by

$$\mathcal{P} = \frac{\mu^2}{c^3}\kappa^4 B^4(t)\alpha^2(t), \quad \mu = (1/2)B(0)R^3, \quad B(t) = B(0)[1 + t/\tau]^{-1/2}, \quad (15)$$

$$\alpha(t) = C\, s^{1/2}\{J_1(z(t)) - \eta\, Y_1(z(t))\}, \quad z = 2\omega(0)\tau\,(1 + t/\tau). \quad (16)$$

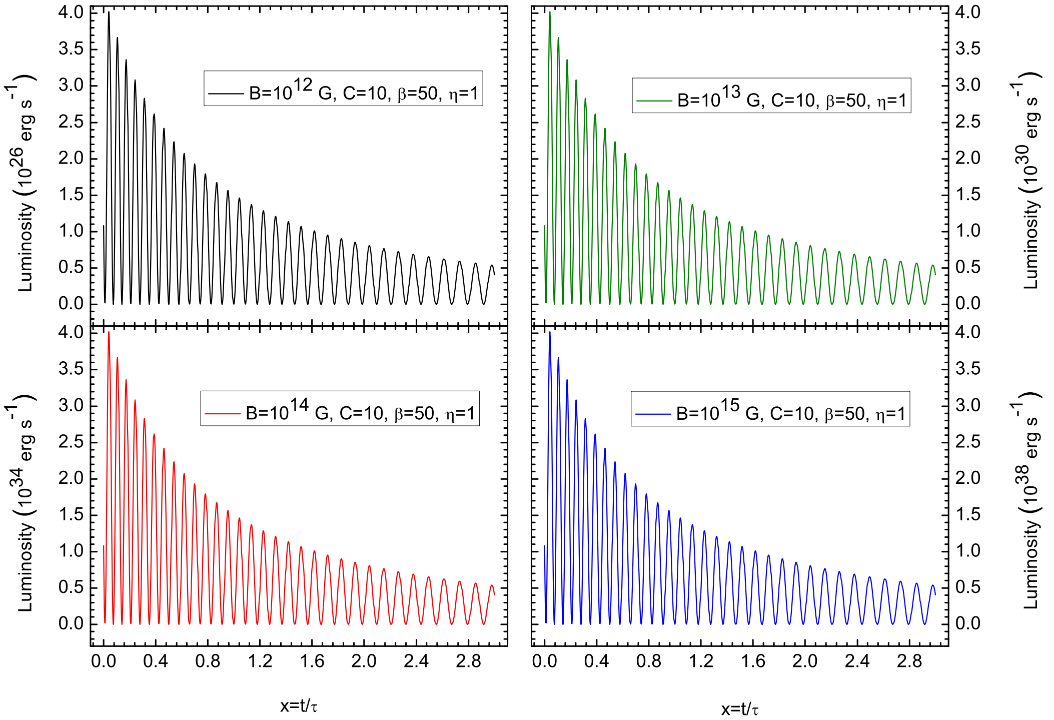

Fig. 4 Time evolution of luminosity of magneto-dipole radiation powered by energy of torsional Alfvén seismic vibrations of a neutron star with mass $M = 1.2M_{\odot}$ and radius $R = 15$ km with different intensities of the magnetic field.

Figure 4 presents computations of the power of magneto-dipole radiation of a neutron star (of one and the same mass and radius but different values of magnetic fields) exhibiting an oscillating luminosity character. The frequency of these oscillations is equal to that of torsional Alfvén seismic vibrations of a neutron star. The practical usefulness of the presented computations is that they can be used as a guide in the observational quest of vibration-powered neutron stars among the currently monitored AXP/SGR-like sources.

4 SUMMARY

It is generally realized today that the standard model of an inclined rotator, which forms the basis of our understanding of radio pulsars (Manchester & Telor 1977, Lorimer & Kramer M. 2004), faces serious difficulties in explaining the long-period ($2 < P < 12$ s) pulsed radiation of soft gamma repeaters (SGRs) and anomalous X-ray pulsars (AXPs). Observations show that the persistent X-ray luminosity of these sources ($10^{34} < L_{\rm X} < 10^{36}$ erg s^{-1}) is appreciably (10–100 times) larger than expected from a neutron star deriving radiation power from the energy of rotation with frequency of detected pulses. It is believed that this discrepancy can be resolved assuming that AXP/SGR-like sources are magnetars – young, isolated and seismically active neutron stars whose energy supply of pulsating high-energy radiation comes not from rotation (as is the case of radio pulsars) but from a different process involving the decay of an ultra-strong magnetic field, $10^{14} < B < 10^{16}$ G. Adhering to this theory, we have investigated the model of a neutron star deriving radiation power from the energy of torsional Lorentz-force-driven oscillations in its own magnetic field which is experiencing decay. It is worth noting that such an idea is not new and was first discussed by Hoyle et al. (1964), before the discovery of pulsars (e.g., Pacini 2008). What is newly disclosed here is that such radiation is possible when the magnetic field has decayed. Since the magnetic field decay is one of the most conspicuous features distinguishing magnetars from rotation-powered pulsars, it seems meaningful to expect that at least some of the AXP/SGR-like sources can be vibration-powered magnetars. Working from this, we have presented an extended numerical analysis of the model of a vibration-powered neutron star whose results can be efficiently, it is hoped, utilized as a guide for the discrimination of vibration-powered neutron stars from rotation powered ones. The application of these analytic and numerical results to the analysis of specific currently monitored sources will be be presented in forthcoming papers.

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