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Dark energy from logarithmically modified gravity and deformed Coleman-Weinberg potential

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Abstract Recent astrophysical measurements strongly suggest the existence of a missing energy component dubbed dark energy that is responsible for the current accelerated expansion of the universe. A new class of modified gravity theory is introduced which yields a universe accelerating in time and dominated by dark energy. The new modified gravity model constructed here concurrently includes a Gauss-Bonnet invariant term, barotropic fluid with a time-dependent equation of state parameter, a Coleman-Weinberg (CW) potential-like expression $V(\phi) = \xi\phi^m \ln\phi^n$ and a new Einstein-Hilbert term $f(R, \phi) = E(\phi)R$ which depends on both the scalar curvature and the scalar field ϕ through a generic logarithmic function $E(\phi) = \ln\phi$. Here m and n take different values from the standard CW potential and ξ is a real parameter. It was shown that the presence of these terms provides many useful features which are discussed in some detail.

Key words: dark energy — gravitation — equation of state

1 INTRODUCTION

There exists much evidence that quantum gravitational effects using the one-loop approximation play an important role in understanding the many cosmological inflationary models free from initial singularity, in particular in an early regime where the curvature is near the Planck-Wheeler range (Starobinsky 1980). From a mathematical point of view, the reduced Einstein equations do not contain higher than second derivatives but have the same physical content as the fourth order equations. Although in most cases the semi-classical quantum corrections play only a small role far from the Planck-Wheeler scale, there are some cosmological theoretical evidences in which the quantum corrections yield additional important information about the picture of the early inflationary universe departing from the classical solution (Barrow & Maeda 1990). For these reasons, inflation cosmology is considered the most undeniable explanation to most cosmological puzzles, e.g. flatness and horizon problems (Linde 1982; Guth 1981). However, the recent astronomical observations of the dynamics of galaxies, clusters of galaxies, COBE DMR measurements of large-angular scale anisotropy in the cosmic microwave background (CMB) radiation, Type Ia supernovae (SNIa) with redshift $z > 0.35$ and other cosmological tests provide important support not only for the hot Big

Bang model but also for a spatially flat universe currently undergoing a phase of accelerated expansion (Riess et al. 1998, 2004; Perlmutter et al. 1999; Schmidt et al. 1998; Steinhardt et al. 1999; Persic et al. 1996; Alcaniz 2004).

The first prominent conclusion to be drawn from the COBE DMR data and large scale structure is that it is consistent with a scale-invariant spectrum of primordial scalar (energy density) and tensor (stochastic gravitational wave background) perturbations extending outside the horizon at the epoch of the last scattering (Smith et al. 2008). This fact is in reality one of the most remarkable features of inflationary cosmology with different classes of inflationary potentials arising from spontaneous symmetry breaking (Higgs, Coleman-Weinberg, chaotic, power law, exponential, hybrid and so on) and is characterized by the curvature of the potential evaluated at the field value corresponding to CMB observations. This predicts perturbations generated by quantum fluctuations and also phenomenological models that generate perturbations by classical effects, such as theories with cosmic strings, textures, global monopoles, and non-topological excitations (Davis et al. 1992). In this paper, we shall deal with Coleman-Weinberg (CW) potential-like expressions in the same way that the cosmological consequences of the standard CW (new inflation) were widely studied in the past decade and where many interesting consequences can be revealed (Linde 1982; Steinhardt & Turner 1984; Sen & Sen 2001).

The second important conclusion to be drawn from observations and results obtained using combined WMAP data and data from the Supernova Legacy Survey of type Ia and galaxy studies favors a spatially flat accelerated universe whose energy density is dominated by some missing energy component with negative pressure and negative equation of state parameter (EoSP) at the 68% confidence level, which is dubbed dark energy (DE) whose nature is still a source of much debate (Riess et al. 1998, 2004; Perlmutter et al. 1999; Schmidt et al. 1998; Steinhardt et al. 1999; Persic et al. 1996; Alcaniz 2004).

There are a number of cosmological models which have been put forward in recent years by high energy physicists and cosmologists to explain the nature of DE. These models include the cosmological constant (Ostriker & Steinhardt 1995), quintessence which is similar to the inflation field with the difference that it evolves on a much lower energy scale (Peebles & Ratra 2003), K-essence (Brax & Martin 1999), Chaplygin gas and Generalized Chaplygin gas (Fabris et al. 2002, 2006; Kamenshchik et al. 2001; Bilić et al. 2002; El-Nabulsi 2010d), holographic dark energy (Setare & Saridakis 2008a,b) and so on. More recent DE models include the one containing a negative kinetic scalar field and a normal scalar field (Feng et al. 2005), or a single scalar field model (Li et al. 2005) and interacting holographic DE models (Wang et al. 2005). Possible candidates for DE also include the Higgs field (electro-weak phase transition at 200 GeV), quark-antiquark bilinears (chiral-symmetry breaking in the strong interaction) and further particles responsible for unknown phase transitions in the early universe (Klapdor-Kleingrothaus & Zuber 1997). The cosmological scenarios of these effective models in four dimensions have been extensively studied.

On the other hand, scalar-tensor gravity theories have been widely applied in modern cosmology since the pioneering work of Brans and Dicke several decades ago. In recent years, there have also been attempts at modeling the missing energy of the universe and explaining its delayed accelerated expansion in view of these scalar tensor theories where the scalar field is non-minimally coupled to the gravity term (low energy effective string action) (Nojiri et al. 2006; Nojiri & Odintsov 2007). It has long been expected that a better understanding of modern cosmology needs to include a higher order curvature derivative term (modified gravity or $f(R)$ theories of gravity) motivated from string/M-theories. These theories are appealing since they yield a unification of the early-time inflation and late-time acceleration. Moreover, the coincidence problem may be solved in such a theory simply by the expansion of the universe. However, one possible way to deal with this problem is to add higher derivative terms in the Lagrangian that depend on quadratic combinations of the Riemann tensor yielding higher than second-order field equations which are also responsible for early inflation. One exceptional quadratic combination of the Riemann curvature tensor is known as the Gauss-

Bonnet (GB) combination that, if added to the standard Einstein-Hilbert action, does not increase the differential order of the equations of motion. The GB term (G $= R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$; R is the scalar curvature, $R_{\mu\nu}$ is the Ricci tensor and $R_{\mu\nu\rho\sigma}$ is the Riemann tensor) is the topological invariant in four dimensions and it was revealed to lead to many interesting cosmological effects. The coupling of the GB term with the scalar field was shown to contribute to the creation of the effective quintessence and phantom era (Nojiri et al. 2006; Nojiri & Odintsov 2007). Additional aspects of modified GB gravity, such as its possibility to describe the inflationary era, transition from the deceleration phase to acceleration phase, crossing the phantom-divide-line and passing the solar system tests have largely been explored in literature (El-Nabulsi 2008, 2009a,b, 2010a,b,c,e,f; Gasperini et al. 2002; Piazza & Tsujikawa 2004; Calcagni et al. 2005, 2006; Nojiri et al. 2005a,b; Nojiri & Odintsov 2006; Bamba et al. 2007; Srivastava 2008, and references therein).

The present paper is devoted to the study of some features of the modified gravity model $f(R,\phi) = E(\phi)R$ ($E(\phi)$ is a generic function of the scalar field ϕ) in the presence of the CW potential-like expression $V(\phi) = \xi\phi^m \ln\phi^n$ with m and n taken here to have different values and ξ is a constant. Such a term appears in the CW potential for new inflation. This kind of potential has previously been studied for the inflationary model with a minimally coupled scalar field by Barrow and Parsons (1995). Most of the generic functions explored in literature are power-law functions of the scalar field. In this paper, we will choose, for reasons that will be clarified later, the special logarithmic form $E(\phi) = \ln\phi$.

We will ignore the non-minimal couplings to the matter terms since our major intent is to investigate the new cosmological features arising from the presence of the GB curvature term in the new logarithmic modified gravity. Furthermore, in order that the equivalence principle is satisfied, we will categorize the scalar field as a run-away modulus devoid of direct matter couplings, though gravitational dynamics are modified due to the presence of modulus-dependent loop corrections. We shall focus our attention on the scaling behaviors of the dynamical parameters, which is one of the simplest solutions.

2 ACTION EQUATIONS OF MOTION AND COSMOLOGICAL SOLUTIONS

In particular, we shall assume the following four-dimensional generalized gravity action (in units $8\pi G = \hbar = c = 1$)

$$\begin{aligned} S_4 \ &= \int d^4\chi\sqrt{-g_4}\left(\frac{E(\phi)}{2}R - \frac{\omega(\phi)}{2}\partial_\mu\phi\partial^u\phi - V(\phi)\right) \\ &+ \int d^4\chi\sqrt{-g_4}F(\phi)G + \int d^4\chi\sqrt{-g_4}L_m\,, \end{aligned} \tag{1}$$

where $\omega(\phi)$ is another generic function of the scalar field, L_m is the matter Lagrangian, $F(\phi)$ is the GB coupling function of the scalar field and g is the metric. We assume a usual spatially flat spacetime described by the Friedmann-Robertson-Walker (FRW) metric

$$ds_4^2 = -dt^2 + a^2(t)\left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{2}$$

where $a(t)$ is the scale factor. The field equations are usually obtained by varying the action (1) with respect to the metric $g_{\mu\nu}$ and the scalar field ϕ. After long algebraic manipulation we obtain

$$\begin{aligned} &E(\phi)\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) - \omega\nabla_\mu\phi\nabla_\nu\phi - \nabla_\mu\nabla_\nu E(\phi) \\ +\ &g_{\mu\nu}\left(\Box E(\phi) + 4R\Box F(\phi) - 8R^{\sigma\tau}\nabla_\sigma\nabla_\tau F(\phi) + \frac{\omega(\phi)}{2}\nabla_\sigma\phi\nabla^\sigma\phi + V(\phi)\right) \\ -\ &4\left(R\nabla_\mu\nabla_\nu F(\phi) + 2R_{\mu\nu}\Box F(\phi) + 2R_(\mu_\nu^{\sigma\tau})\nabla_\sigma\nabla_\tau F(\phi) - 4R_\sigma(_\mu\nabla^\sigma\nabla_\nu)F(\phi)\right) \\ =\ &T_{\mu\nu}, \end{aligned} \tag{3}$$

$$\begin{aligned}&\omega(\phi)\Box\phi-\frac{dV(\phi)}{d\phi}+\frac{1}{2}\frac{d\omega(\phi)}{d\phi}\nabla_\mu\phi\nabla^\mu\phi+\frac{dE(\phi)}{d\phi}\frac{R}{2}\\&+\ \frac{dF(\phi)}{d\phi}\left(R^2-4R_{\mu\nu}R^{\mu\nu}+R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)=0.\end{aligned}\tag{4}$$

In Equation (3), we recognize the stress-energy tensor $T_{\mu\nu}=(p+\rho)u_\mu u_\nu+pg_{\mu\nu}$ where p and ρ are the pressure and density of the perfect fluid, respectively, and u_μ is the fluid rest-frame four velocity. The dynamical equations of motion, namely the modified Friedmann equations and the modified Klein-Gordon equations, are correspondingly

$$3\ln\phi H^2+3\frac{\dot{\phi}}{\phi}H+24\dot{\phi}\frac{dF(\phi)}{d\phi}H^3=\frac{\omega(\phi)}{2}\dot{\phi}^2+\xi\phi^m\ln\phi^n+\rho,\tag{5}$$

$$\begin{aligned}&\omega(\phi)(\ddot{\phi}\ +\ 3H\dot{\phi})+\xi\left(m\phi^{m-1}\ln\phi^n+n\phi^{m-1}\right)+\frac{1}{2}\frac{d\omega(\phi)}{d\phi}\dot{\phi}^2\\&\quad-\ 24\frac{dF(\phi)}{d\phi}(\dot{H}H^2+H^4)-\frac{3}{\phi}(2H^2+\dot{H})=0,\end{aligned}\tag{6}$$

where $H=\dot{a}/a$ is the Hubble parameter.

In fact, we are interested in power-law solutions given that they can be viewed as approximations to more realistic cosmological models and, moreover, they are consistent with nucleosynthesis (Kaplinghat et al. 1999, 2000; Sethi et al. 1999) and with the age of objects found in high-redshift globular clusters (Kaplinghat et al. 1999; Lohiya & Sethi 1999; Sethi et al. 2005). In addition, power-law cosmologies are successful in describing the gravitational lensing statistics (Buchbinder et al. 1992), the angular size-redshift data of compact radio sources (Dev et al. 2008) and the SNIa magnitude-redshift relation (Dev et al. 2001; Jain et al. 2003). In summary, they are compatible with observations although they exhibit the usual phantom features, such as the Big Rip singularity (Kaeonikhom et al. 2011). For these reasons, we assume that the scale factor evolves as $a=a_0(t/t_0)^q$ and the scalar field evolves as $\phi=\phi_0(t/t_0)^p$. Here q and p are constants; a_0 and ϕ_0 are the values of the parameters at the present time t_0 (in units $\hbar=c=1$). Obviously, according to the value of q, this model of the universe can describe the radiation epoch, the dark matter epoch and the accelerating expansion epoch. We choose as well $F(\phi)=F_0\phi^r$ where F_0 is the value of the scalar field functions F at the present time.

Moreover, we believe that the conservation equation for the stress energy-momentum tensor $T_{\mu\nu}$, i.e. $\nabla_\nu T^{\mu\nu}=0$, holds for the underlying spacetime geometry and took the common form $\dot{\rho}+3H(\rho+p)=0$. When the universe contains a perfect fluid with a barotropic equation of state $p=w\rho=(\gamma-1)\rho$, $w(t)=\gamma(t)-1$ is expected to be a time-dependent parameter. It is noteworthy that many quintessence-based cosmological models involving scalar fields give rise to a time-dependent equation of state parameter (EoSP) (Serna & Alimi 1996; Sahoo & Singh 2004; Diaz-Rivera & Pimentel 2003; Farajollahi & Mohamadi 2010). The continuity equation straightforwardly gives $\dot{\rho}+3H\gamma\rho=0$.

We can now write Equations (5) and (6) respectively as explicit functions of the cosmic time

$$\begin{aligned}&3pq^2\left(\frac{t}{t_0}\right)^{-2}\ln\left(\frac{t}{t_0}\right)+3pq\left(\frac{t}{t_0}\right)^{-2}+24prq^3\left(\frac{t}{t_0}\right)^{pr-4}\\&\quad=\frac{1}{2}\omega p^2\left(\frac{t}{t_0}\right)^{2p-2}+\xi np\left(\frac{t}{t_0}\right)^{pm}\ln\left(\frac{t}{t_0}\right)+\rho,\end{aligned}\tag{7}$$

$$\omega p(p-1+3q)\left(\frac{t}{t_0}\right)^{p-2}+\xi\left[nmp\ln\left(\frac{t}{t_0}\right)+n\right]\left(\frac{t}{t_0}\right)^{p(m-1)}+\frac{1}{2}pt^{p-1}\frac{d\omega}{dt}$$
$$-24rq^3\left(\frac{t}{t_0}\right)^{pr-p-4}(q-1)-3q\left(\frac{t}{t_0}\right)^{-p-2}(2q-1)=0. \tag{8}$$

One interesting choice of a consistent solution is obtained if we set $p(m-1)=pr-p-4=-p-2$, $\xi n-24rq^3(q-1)-3q(2q-1)=0$ and we conjecture that $\omega(t)$ obeys the following differential equation

$$\frac{1}{2}p\frac{d\omega}{dt}+\omega p(p-1+3q)\left(\frac{t}{t_0}\right)^{-1}+\xi nmp\left(\frac{t}{t_0}\right)^{-1-2p}\ln\left(\frac{t}{t_0}\right)=0. \tag{9}$$

By setting $p=-1/2$, i.e. $m=4$, $r=-4$ and $\xi n=3q(2q-1)-96q^3(q-1)$, it is easy to check that the solution of Equation (9) is given by

$$\omega(t)=\left\{\omega(t_0)-\frac{12\left[3q(2q-1)-96q^3(q-1)\right]t_0}{\left[3t_0(2q-1)+1\right]^2}\right\}\left(\frac{t}{t_0}\right)^{-3t_0(2q-1)}$$
$$-\frac{12\left[3q(2q-1)-96q^3(q-1)\right]t_0}{\left[3t_0(2q-1)+1\right]^2}\left(\frac{t}{t_0}\right)\left\{\left[3t_0(2q-1)+1\right]\ln\left(\frac{t}{t_0}\right)-1\right\}. \tag{10}$$

Interestingly, $\omega(t)$ depends on the value of q and hence changes at every epoch of time. After inserting Equation (10) into Equation (7), we obtain

$$\rho(t) = \underbrace{\frac{3q}{2}\left\{\left[2q-1-32q^2(q-1)\right]\left[1+\frac{3t_0}{3t_0(2q-1)+1}\right]-q\right\}}_{A}\left(\frac{t}{t_0}\right)^{-2}\ln\left(\frac{t}{t_0}\right)$$
$$+\ \underbrace{-\frac{3q}{2}\left\{1-36q^2+\frac{\left[3(2q-1)-96q^2(q-1)\right]t_0}{(\left[3t_0(2q-1)+1\right]^2}\right\}}_{B}\left(\frac{t}{t_0}\right)^{-2}$$
$$+\ \underbrace{-\frac{1}{8}\left\{\omega(t_0)-\frac{12\left[3q(2q-1)-96q^3(q-1)\right]t_0}{\left[3t_0(2q-1)+1\right]^2}\right\}}_{C}\left(\frac{t}{t_0}\right)^{-3t_0(2q-1)-3}. \tag{11}$$

For now, we set $t_0=1$ for mathematical convenience. The continuity equation gives

$$\gamma=-\frac{\dot{\rho}}{3H\rho}=-\frac{-2A\ln T+(A-2B)-6qCT^{-3(2q-1)-1}}{3q\left[A\ln T+B+CT^{-3(2q-1)-1}\right]}, \tag{12}$$

where $T=t/t_0=t$. The EoSP is therefore time-dependent as expected. However, at the present time, i.e. $t=t_0=1$, the EoSP is given by

$$\gamma_0\equiv\gamma_{\text{present}}=-\frac{A-2B-6q_0C}{3q_0\,(B+C)}, \tag{13}$$

where $q_0\equiv q_{\text{present}}$ and

$$A=\frac{3q_0}{4(3q_0-1)}\left(-192q_0^4+160q_0^3+38q_0^2-2q_0-1\right), \tag{14}$$

$$B=-\frac{3q_0}{8\,(3q_0-1)^2}\left(-324q_0^4+120q_0^3+69q_0^2-2\right), \tag{15}$$

$$C = -\frac{1}{32\,(3q_0-1)^2}\left[4\omega(t_0)\,(3q_0-1)^2 + 1152q_0^4 - 1152q_0^3 - 72q_0^2 + 36q_0\right]. \tag{16}$$

We believe, on the other hand, that the EoSP must decrease in time, nevertheless, in our approach, when $t \to \infty$, $\gamma \to 2/3q < 2/3$ for $q > 1$ (accelerated phase) and hence the universe is dominated by dark energy after a very long time. Notice that the radiation-dominated epoch ($\gamma = 4/3$, $q \approx 1/2$) and the matter-dominated epoch ($\gamma = 1$, $q \approx 2/3$) start respectively at times

$$T_{\mathrm{radiation}} = \frac{C}{A} = \frac{72-\omega(t_0)}{93}, \tag{17}$$

and

$$T_{\mathrm{matter}} = \sqrt{\frac{2C}{A}} = 0.14\sqrt{30.42-\omega(t_0)}. \tag{18}$$

As we naturally expect, $T_{\mathrm{matter}} < T_{\mathrm{radiation}}$, and $-1.11 < \omega(t_0) < 30.42$ or $\omega(t_0) < -24.4$. The theory may then lead to a negative value of $\omega(t_0)$. This result is consistent with conclusions drawn by Bertolami and Martins (Bertolami & Martins 2000), Banerjee and Pavón (Banerjee & Pavón 2001), and Sen and Seshadri (Sen & Seshadri 2003) that $\omega(t_0)$ should possess a low negative value for reasonable assumptions about structure formation, cosmic acceleration, coincidence problem, and to avoid the problems of quintessence within the formalism of the Brans-Dicke theory so that it could agree with locally measured values (Will 1993). This argument is based on the scalar-tensor field theories in which $\omega(t_0)$ depends on the scale, which is very high in the weak field approximation of the Solar System that investigates only a limited range of space and time (Farajollahi & Mohamadi 2010). For $\omega(t_0) \approx -1/2$, we obtain from Equations (13)–(16)

$$\begin{aligned}\gamma_0 \equiv \gamma_{\mathrm{present}} &= -\frac{A-2B-6q_0C}{3q_0\,(B+C)}\\ &= \frac{2(3q_0-1)^2\left(128q_0^3-56q_0^2-4q_0+3\right)}{1944q_0^5+144q_0^4+990q_0^3+45q_0^2-36q_0+1}.\end{aligned} \tag{19}$$

However, some recent observational results coming from SNIa data (Starobinsky 1980; Barrow & Maeda 1990; Linde 1982; Guth 1981; Riess et al. 1998; Perlmutter et al. 1999; Schmidt et al. 1998; Steinhardt et al. 1999) and SNIa data which corroborated with CMBR anisotropy and galaxy clustering statistics are $-1.67 < \gamma - 1 < -0.62$ and $-1.33 < \gamma - 1 < -0.79$ respectively. The first case gives $0.17 < q_0 < 1.46$ whereas the second case gives $0.18 < q_0 < 1.05$. Consequently we argue that the expansion of the universe has accelerated with time.

It is noteworthy that the linear expansion of the scale factor with time ($a(t) = t$) does not suffer from the horizon problem or from the flatness problem (Ford 1987; Dev et al. 2001; Jain et al. 2003; John & Narlikar 2002; Hansen et al. 2002). Furthermore, the deduced age of the universe from a measurement of the Hubble parameter is about 50% greater than the age inferred from the measurement in standard Friedmann-Robertson-Walker cosmology in the absence of the cosmological constant. Accordingly, the age estimate is concordant with age estimates of old clusters. In addition, the relative matter density matches the observed value surprisingly well (Tegmark et al. 2004). The CW potential-like expression now takes the special form $V(\phi) = [3q(2q-1)-96q^3(q-1)]\phi^4\ln\phi = 3\phi^4\ln\phi$ for $q = 1$ and hence $V(t) = 3t^{-2}\ln t^{-1/2}$ The scalar curvature in our framework decays like $E(\phi)R \propto t^{-2}\ln t^{-1/2}$ whereas the GB part $F(\phi)\mathrm{G} = \phi^r\mathrm{G} = t^{-2}$. Notice that at $t = t_0 = 1$, the modified Einstein-Hilbert part $E(\phi)R$ vanishes. This already means that string curvature corrections play a crucial role in determining the dynamical evolution of the universe in contrast to what is generally believed; the contribution of the GB term to the gravitational field equation in four-dimensional spacetime is insignificant and its effects emerge only in extra dimensional theories (Charmousis & Dufaux 2002; Davis 2003; Gravanis & Willison 2003; Bostock et al. 2004).

3 CONCLUSIONS

In conclusion, we have studied the cosmological evolution of logarithmically modified gravity in the presence of the CW potential-like expression, namely $V(\phi) = \xi\phi^m \ln\phi^n$. It was shown that the accelerated expansion of the universe may occur if, for instance, the CW potential behaves like $V(\phi) = [3q(2q-1) - 96q^3(q-1)]\phi^4 \ln\phi = 3\phi^4 \ln\phi$. The equation of state parameter was found to be time-dependent. The universe is expanding (linearly) in time, and its expansion decelerated during the radiation- and matter-dominated epochs, and accelerated during the dark energy-dominated epoch. An interesting point to note here is that the minimum of the potential $V(\phi) = 3\phi^4 \ln\phi$ has a nonzero value even in the absence of the cosmological constant. The scaling solutions obtained can lead to a viable late-time cosmology with the accelerated expansion starting earlier ($z > 1$) than in common dark energy cosmological models, but are still in agreement with the recent SNIa data. The energy density of the universe was found to decay as $\rho(t) = 0.375t^{-2}\ln t + 51.21t^{-2} + 0.34t^{-6}$ and the Brans-Dicke parameter decays as $\omega(t) = -2.75t^{-3} - (9/4)t\,(4\ln t - 1)$. This model is therefore a viable one. Work on further details is in progress.

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