Research in Astron. Astrophys. **2011** Vol. **11** No. **3**, 286–292
http://www.raa-journal.org *http://www.iop.org/journals/raa*

Research in Astronomy and Astrophysics

Long term periodicity analysis of the spectral index of 2251+158 *

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Received 2009 December 17; accepted 2010 November 15

Abstract We analyze the radio spectral index of blazars from the University of Michigan Radio Astronomy Observatory database, and find that there exists quasi-periodic activity in 2251+158. The long-term periodicity analysis is accomplished by three methods, which are the Jurkevich method (Jurkevich), the discrete correlation function (DCF), and the Periodogram method (P). The results show that 2251+158 has two strong periodicities, which are $P_1 = 6.3 \pm 1.1$ yr and $P_2 = 3.8 \pm 1.2$ yr.

Key words: blazars: data analysis: individual: 2251+158

1 INTRODUCTION

Blazars are well-known for their violent optical variation and variable timescales, which can range from hours to years over wavelengths from radio to X-rays (Fan et al. 2004; Ulrich et al. 1997). The optical variability timescales mainly consist of two types, short-term variation and long-term variation. Generally, the short-term variation is non-periodic but the long-term variation in some cases is claimed to be periodic and can be called quasi-periodic. The long-term optical variation of blazars has been discussed in many papers (Kunkel 1967; Sillanpaa et al. 2008; Raiteri et al. 2001; Chertoprud et al. 1973; Liu et al. 1995; Xie et al. 2002; Fan et al. 1998, 2006; Fan & Lin 2000; Stickel et al. 1993) and references therein.

There are many methods used to calculate the long-term variation. The Periodogram method came from Deeming (1975), and was improved by Lomb (1976) and Scargle (1982). The Jurkevich method was first proposed by Jurkevich et al. (1971). The structure function was first proposed by Simonetti et al. (1985). The Data Compensated Discrete Fourier Transform (DCDFT) was put forward by Ferraz-Mello (1981), which was improved by Foster (1995). The discrete correlation function method (DCF) was developed by Edelson & Krolik (1988).

The case 2251+158 (3C454.3, $z = 0.859$) is among the most intense and variable blazars, and exhibits correlated behavior at optical and radio wavelengths (Pyatunina et al. 2007). The long-term optical variation can be up to several years (Djorgovski et al. 2008). Webb et al. (1988) obtained three periods, 6.4, 3.0 and 0.8 yr, using the B-band lightcurves during the timespan 1971–1985. Ciaramella et al. (2004) found a period of 6.3 ± 0.2 yr using the radio data from the University of Michigan Radio Astronomy Observatory (UMRAO) database. Kudryavtseva & Pyatunina (2006)

* Supported by the National Natural Science Foundation of China.

show that there exist two periods, $P_1 = 12.4 \pm 0.6$ yr and $P_2 = 6.2 \pm 0.1$ yr, using the UMRAO and the Metsähovi Radio Observatory database (MRO). Fan et al. (2007) used the database of UMRAO and obtained the quasi-periods of $P_{14.5\,\mathrm{GHz}} = 6.3 \pm 0.2$ and 11.8 ± 1.1 yr, $P_{8\,\mathrm{GHz}} = 4.6 \pm 0.1$, 6.7 ± 0.1 and 13.6 ± 1.1 yr, and $P_{4.8\,\mathrm{GHz}} = 6.2 \pm 0.2$ and 12.3 ± 1.8 yr. The radio emission can be explained by the following model: There is a rotating black hole system surrounded by a massive accretion disk with an intense plasma jet which is closely aligned to the line of sight. The associated relativistic electrons produce the radio emission through synchrotron emissions (Ciaramella et al. 2004).

The spectral index is a very important parameter, which has wide application. For example, based on the flat spectrum feature, flat spectrum radio quasars (FSRQs) form a subdivision of blazars; the multi-band spectral index has a correlation with the accretion rate (Zhang et al. 2008). In this paper, we use the preliminary radio data from UMRAO to carry out our periodic analysis of the spectral index. This paper is arranged as follows: in Section 2, we obtain the spectral index from the database of UMRAO and select 2251+158 to calculate the long term variation; in Section 3, we introduce the methods to analyze the periodicity of 2251+158; and in Section 4, we discuss the results and draw the conclusion.

2 SAMPLE

Blazars generally show a flat and inverted radio spectrum. However, it is not easy to obtain a typical spectral index for a given source since the flux can be variable. Here, we use the University of Michigan Radio Astronomy Observatory (UMRAO) database to calculate the spectral index α ($F_\nu \propto \nu^\alpha$) by using the averaged flux densities at the three frequencies (4.8, 8 and 14.5 GHz). The detailed process is as follows: for sources with densely sampled data, we average the data every week (seven days). Therefore, we can get N sets of data for each source using the time bins (seven days for the entire data set); each set has three pairs of flux densities for the corresponding frequencies, so for the ith set, we fit the three pairs of data using linear regression to get the spectral index, α_i, and obtain the corresponding correlation coefficient. The UMRAO archive contains 141 blazars. We analyze those sources and find that 2251+158 has a quasi-periodic character. After the calculation, 2251+158 has 162 points. The spectral index (α) is in the range of $\alpha = -0.52 \sim 0.34$. We use $\langle\alpha\rangle = \sum_{i=1}^{N} \alpha_i/N$ to obtain the averaged value, and $\sigma_\alpha = \sqrt{\sum(\alpha_i - \alpha)^2/N}$ to obtain the standard deviation, which yields $\langle\alpha\rangle = -0.17 \pm 0.22$. Regarding the correlation coefficient (r), among the 162 points, only nine points have $r < 0.5$, and the other points show very good correlation. The averaged lightcurves (8GHz), the variable spectra, and the correlation coefficients are shown in Figure 1.

3 METHODS

3.1 Periodogram Method

The most common tool for periodicity analysis of both evenly and unevenly sampled signals is the Periodogram method, which is an estimator of the signal energy in the frequency domain (Deeming 1975). Lomb (1976) introduced a modified form of this method, which can be described as follows. Consider a series $x(n)$ with N points, and let f be the frequency and τ be a variable timescale. Their mean and variance are given by: $\overline{x} = \frac{1}{N}\sum_{n=1}^{N} x(n)$ and $\sigma^2 = \frac{1}{N}\sum_{n=1}^{N}(x(n)-\overline{x})^2$. The normalized Lomb's P^L, i.e. the power spectrum as a function of the angular frequency $\omega \equiv 2\pi f > 0$, is defined as

$$P_N^L(\omega) = \frac{1}{2\sigma^2}\left\{\frac{\left[\sum_{n=0}^{N-1}(x(n)-\overline{x})\cos\omega(t_n-\tau)\right]^2}{\sum_{n=0}^{N-1}\cos^2\omega(t_n-\tau)}\right\}$$

$$+\frac{1}{2\sigma^2}\left\{\frac{[\sum_{n=0}^{N-1}(x(n)-\overline{x})\sin\omega(t_n-\tau)]^2}{\sum_{n=0}^{N-1}\sin^2\omega(t_n-\tau)}\right\} \tag{1}$$

and τ is defined by the equation:

$$\tan(2\omega\tau)=\frac{\sum_{n=0}^{N-1}\sin 2\omega t_n}{\sum_{n=0}^{N-1}\cos 2\omega t_n}. \tag{2}$$

We use 'P' to represent the periodicity, so when τ is equal to the periodicity, $P=\tau$. Using Lomb's Periodogram method to derive the period and using the half width at half maximum (HWHM) to calculate the corresponding error, we can get the following results. For 2251+158, there are two quasi-periodicities, $f_1 = 0.0027 \pm 0.00045$ ($P_1 = 6.3 \pm 1.1$ yr) with significance level $p < 0.001\%$ and $f_2 = 0.0045 \pm 0.00035$ ($P_2 = 3.8 \pm 1.2$ yr) with significance level $p < 0.001\%$. The other periodicity $f = 0.0013$ ($P = 12.9$ yr) should be two times the quasi-period $P = 6.29$ yr. The results are shown in Figure 2. The white noise and the red noise (Schulz & Mudelsee 2002), both with probability 95%, are shown in Figure 2.

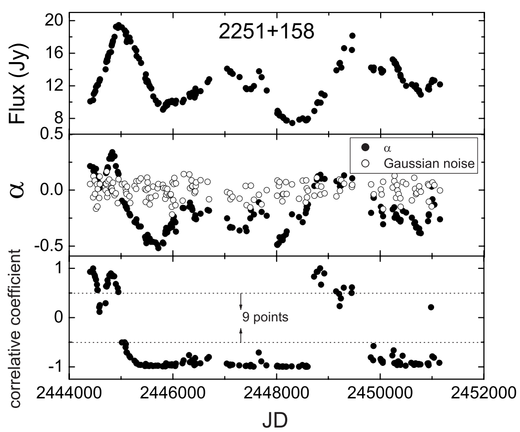

Fig. 1 *Upper Panel* represents the averaged lightcurves. In the *Middle Panel*, filled circles represent the variance of the spectral index, and open circles represent the Gaussian white noise. The *Lower Panel* shows the calculated spectral correlation coefficient.

3.2 DCF Method

The DCF method (Edelson & Krolik 1988; Hufnagel & Bregman 1992) can analyze the correlation between two-variable temporal data sets with a given time lag. If we only input one set, we can calculate the period of the set. In order to implement this method, firstly, we calculate the unbinned discrete correlation function (UDCF) between the two data streams a and b, i.e.

$$\mathrm{UDCF}_{ij}=\frac{(a_i-\langle a\rangle)\times(b_j-\langle b\rangle)}{\sqrt{\sigma_a^2\times\sigma_b^2}}, \tag{3}$$

where a_i and b_j are two data streams, $\langle a\rangle$ and $\langle b\rangle$ are the average values of the data sets, and σ_a and σ_b are the corresponding standard deviations. Secondly, we average the points by defining the same time lag through binning the UDCF_{ij} into suitably sized time-bins in order to get the DCF for each time lag τ,

$$\mathrm{DCF}(\tau) = \frac{1}{M}\sum \mathrm{UDCF}_{ij}(\tau), \tag{4}$$

where M is the total number of pairs. The standard error of each bin is

$$\sigma(\tau) = \frac{1}{M}\Big\{\sum[\mathrm{UDCF}_{ij} - \mathrm{DCF}(\tau)]^2\Big\}^{0.5}. \tag{5}$$

Using the DCF method and the Gaussian fitting, we can obtain that the periodicity of 2251+158 is 6.1 ± 0.084 yr, which is shown in Figure 3.

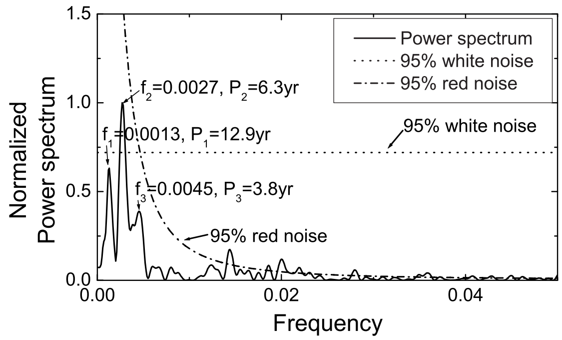

Fig. 2 Calculated result of the method of Lomb's Periodogram for 2251+158. The frequency is in units of $2\pi \times \mathrm{day}^{-1}$. The dotted line denotes the white noise with probability 95% and the dash-dotted line denotes the red noise with probability 95%.

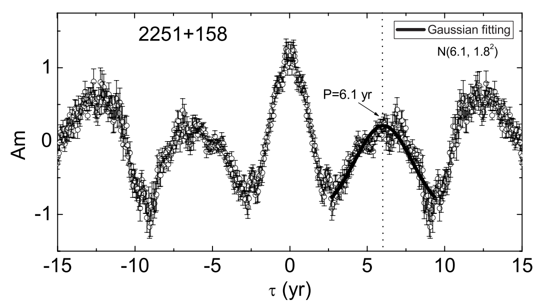

Fig. 3 Calculated result from using the method of DCF for 2251+158. The solid line stands for the result of the Gaussian fitting. Am stands for the amplitude of the result of DCF. N(6.1, 1.8^2) stands for the expected value being 6.1 and standard deviation being 1.8 for the Gaussian fitting.

3.3 Jurkevich Method

The Jurkevich method (Jurkevich et al. 1971) is based on the expected mean square deviation. It tests a run of trial periods around which the data are folded. All data are assigned to m groups according to their phases around each bin, and the entire V_m^2 for each bin is computed. If the trial period equals

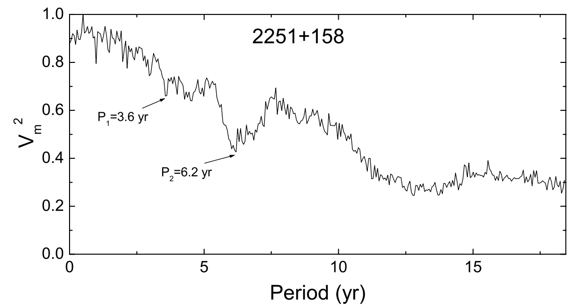

Fig. 4 Calculated result of the Jurkevich method for 2251+158.

the true one, then V_m^2 reaches its minimum. A 'good' period will give a much reduced variance relative to those given by 'false' trial periods and with almost constant values. Kidger et al. (1992) introduced a fraction of the variance of $\eta = (1 - V_m^2)/V_m^2$, where V_m^2 is the normalized value. In the normalized plot, a value of $V_m^2 = 1$ implies $\eta = 0$, and there is no periodicity. The best periods can be identified from the value of $\eta \geq 0.5$, which implies that there is a very strong periodicity, and a value of $\eta < 0.25$ implies that the periodicity, if genuine, is weak. A further test is the relationship between the depth of the minimum and the noise in the 'flat' section of the V_m^2 curve close to the adopted period. If the absolute value of the relative change of the minimum in the 'flat' section is larger than ten times the standard error of this 'flat' section, the periodicity in the data can also be considered to be significant, and the minimum can be regarded as highly reliable. The error in the period is estimated by calculating the HWHM of the minimum in the V_m^2.

We obtain the result by using $m = 20$, which is shown in Figure 4. There are two obvious minimum values of V_m^2, which indicate that there might be two quasi-periodicities in this sources. When $V_m^2 = 0.66$, i.e. $f = 0.52$, there is a strong periodicity, $P_1 = 3.6 \pm 0.1$ yr. When $V_m^2 = 0.43$, i.e. $f = 1.32$, there is another strong periodicity, $P_2 = 6.2 \pm 0.8$ yr.

4 DISCUSSION AND CONCLUSIONS

4.1 Discussion

Multi-wavelength observations of blazars suggest the presence of various emission components. For this reason, variable timescales, time delays and quantified spectral variability provide important information about the locations of different components (Trevese & Vagnetti 2002; Vagnetti & Trevese 2003).

From Figure 1, we can find that the lightcurve and the spectral variation have the same profile, so we use the DCF to analyze the time correlation between them. The result is shown in Figure 5 and the time delay is 329.4 ± 15.4 d (based on the Gaussian fitting). The existence of a time delay between the lightcurves and the spectral variance is a very special phenomenon. Many authors (Biermann & Strittmatter 1987; Kirk & Schneider 1987; Brindle 1996; Fritz 1989) have used the idea of a moving shock in a relativistic jet with a helical magnetic field to explain the variance of flux. In addition, they consider that the strong shocks are very important in blazar emission and suggest that the spectral variance is due to the superposition of a variable polarized cut-off component and an unpolarized component embedded in a steeper spectrum. The variance of the flux density is not necessarily synchronized with the spectral variance, so the lightcurves can have a delay in the spectral variance.

Because of the lack of papers referring to the periodicity analysis of the spectral index, we use the periodicity of the radio lightcurves to analyze our results. For 2251+158, our result shows that there are two strong quasi-periodicities, $P_1 = 6.3 \pm 1.1$ yr and $P_2 = 3.8 \pm 1.2$ yr; the former is

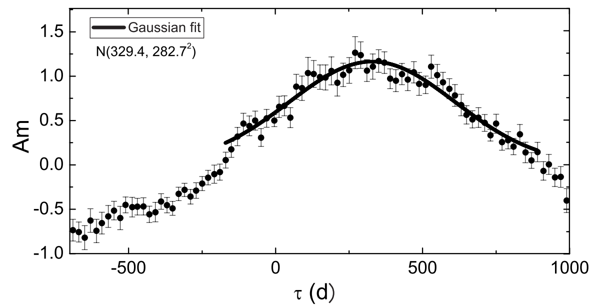

Fig. 5 Correlation analysis between the lightcurves and the spectral variation of 2251+158. The time delay is 329.4 d from the Gaussian fitting.

consistent with Fan et al. (2007), Kudryavtseva & Pyatunina (2006), and Ciaramella et al. (2004), and the later is consistent with Fan et al. (2007).

For blazars, the variability mechanism is not well understood. There are some models which have been proposed to explain the quasi-periodicity, for example, the binary black-hole model, the thermal instability model and the perturbation model (Fan et al. 2007). For 2251+158, the spectral variation shows quasi-periodicity, and this periodicity is consistent with the lightcurve. Furthermore, there is a time delay between the spectral variation and the lightcurves. Considering the discussion about the time delay, we can see that the periodicity of spectral variation can be explained by the perturbation model.

Fan et al. (2009) introduced a method to calculate the short term timescales, which can be summarized as follows: For a certain source, there are n sets of data $(t_i, S_i, i = 1, 2, ..., n)$. We calculate the time difference (Δt_{jk}), the variability (ΔF_{jk}), and the standard deviation (σ_{jk}) between the jth set and the kth set of data

$$\Delta t_{jk} = |t_j - t_k|, \quad \Delta F_{jk} = |F_j - F_k|, \quad \sigma_{jk} = \sqrt{\sigma_j^2 + \sigma_k^2}, \tag{6}$$

where $j, k = 1, 2, ..., n$, so we have $n(n-1)/2$ sets of Δt_{jk}, ΔF_{jk} and σ_{jk} values. Then, if $\Delta F_{jk} > 5\sigma_{jk}$, we select the shortest Δt_{jk} to be the short term timescale for this source, namely $t_{\rm obs} = \min(\Delta t_{jk})$. After our calculation, we can get $t_{\rm obs} = 4.96$ d. In our former paper, Fan et al. (2009) used the lightcurves to derive that the timescale is 0.77 d. The difference between the two timescales might come from the different spacing between the spectral variation and the lightcurves. Not only in the radio band, but also in the other bands, a variation of time scales over a few days has been found. Giommi et al. (2006) found that the optical and ultraviolet flux doubled within only 1.7 d and Ogle et al. (2010) found that the soft X-ray flux slightly varied during the same time.

4.2 Conclusions

In this paper, we use three methods, the Periodogram method, the Jurkevich method and the DCF method, to analyze the quasi-periodicity of 2251+158. The results show that 2251+158 has two strong periodicities, which are $P_1 = 6.3 \pm 1.1$ yr and $P_2 = 3.8 \pm 1.2$ yr.

Acknowledgements This work was supported by the National Natural Science Foundation of China (Grant No. 10633010). This research has made use of data from the University of Michigan Radio Astronomy Observatory, which has been supported by the University of Michigan and the National Science Foundation.

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