An iterative wavefront sensing algorithm for high-contrast imaging systems [*]

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Abstract Wavefront sensing from multiple focal plane images is a promising technique for high-contrast imaging systems. However, the wavefront error of an optics system can be properly reconstructed only when it is very small. This paper presents an iterative optimization algorithm for the direct measurement of large static wavefront errors from only one focal plane image. We first measure the intensity of the pupil image to get the pupil function of the system and acquire the aberrated image on the focal plane with a phase error that will be measured. Then we induce a dynamic phase on the tested pupil function and calculate the associated intensity of the reconstructed image on the focal plane. The algorithm will then try to minimize the intensity difference between the reconstructed image and the aberrated test image in the focal plane, where the induced phase is a variable of the optimization algorithm. The simulation shows that the wavefront of an optical system can theoretically be reconstructed with high precision, which indicates that such an iterative algorithm may be an effective way to perform wavefront sensing for high-contrast imaging systems.

Key words: techniques: image processing — methods: numerical — planetary systems

1 INTRODUCTION

Discovering life on another planet would potentially be one of the most important scientific advances of this century. The search for life requires the ability to detect photons directly from an Earth-like planet and the use of spectroscopy to analyze its physical and atmospheric conditions. The direct imaging of an Earth-like low-mass planet orbiting its bright primary star is, however, extremely challenging. For NASA's Terrestrial Planet Finder Coronagraph, a contrast of 10^{-10} with the primary star, at an inner working angular (IWA) distance smaller than four diffraction beam widths, $4\lambda/D$, (where D and λ are the telescope aperture and working wavelength respectively) is required in the visible wavelengths (Brown & Burrows 1990). Recently, many high-contrast coronagraphs have

been proposed for the direct imaging of an Earth-like planet which can theoretically reach a contrast of 10^{-10} at a few λ/D from the bright star (Ren & Serabyn 2005; Guyon et al. 2006; Ren & Zhu 2007). However, most of the existing coronagraphs can only reach a contrast on the order of $10^{-5} \sim 10^{-7}$, even in the laboratory (Kasdin et al. 2004; Dou et al. 2010; Ren et al. 2010). One of the main factors limiting contrast comes from the wavefront error induced by imperfections in both the telescope optics and the coronagraph, which forms static speckle noise surrounding the star's image (Ren & Wang 2006).

The local speckles are much brighter than the planet's image because of the large brightness difference between the planet and its parent star, thus making the direct imaging of an Earth-like planet impossible. For space-based observations, the wavefront error changes very slowly and high S/N can be achieved by increasing the exposure time. For ground-based observations, the static wavefront error is one of the major sources of error, although dynamic wavefront error caused by atmospheric turbulence and which continuously changes dominates the imaging performance. For an extreme adaptive optics system dedicated to the direct imaging of an exoplanet, both errors need to be corrected efficiently. In Serabyn et al. (2010)'s recent paper, a phase retrieval wavefront sensing algorithm was proposed to correct the static speckle, and three planets around the star HR 8799 could be imaged.

To remove the speckles arising from wavefront error, one promising approach is called the speckle nulling technique, which can be achieved by using a deformable mirror (DM) to alter the phase properties in the pupil plane of the coronagraph system. With a specific phase provided by the DM, this system can theoretically create a local high-contrast zone on the focal plane that can be used as an area for exoplanet direct imaging (Malbet 1995). Based on the technique to remove speckles, an improvement of $10^{-2} \sim 10^{-3}$ in contrast is expected after eliminating the speckle noise surrounding the star image.

The most important procedure for the high-contrast zone correction is to precisely measure the wavefront error of the system. In recent papers, different algorithms were presented to reconstruct the wavefront directly from multiple images taken in the focal plane of an optics system. First, these algorithms can work only when the original wavefront error of an optics system is not too large, due to the approximation that was introduced in either the phase error to be measured or the phase provided by the DM (Borde & Traub 2006; Give'on et al. 2007; Dou et al. 2009). Second, the DM's deformation must be chosen carefully to create multiple images which are uncorrelated with each other, so that the denominator "D" in the above algorithms will not be zero. Otherwise, there will be no solution for the above algorithms, and the wavefront cannot be properly reconstructed. Third, in the above algorithms, the phase provided by the DM is calculated through the so-called influential function model, which may further lead to unnecessary errors in the actual process of wavefront sensing.

To overcome these problems, this paper presents an iterative optimization algorithm for wavefront measurement directly from one single focal plane image. First, the intensity of the image taken in the pupil plane of the optics system is measured and its pupil function can directly be calculated. Meanwhile, the aberrated image on the focal plane of the optics system, whose phase error needs to be measured, will be taken as the reference image. Then we introduce a dynamic phase to the tested pupil function and calculate the associated intensity of the reconstructed focal plane image including the induced phase. The algorithm will minimize the intensity difference between the reconstructed image and the aberrated focal plane image of the optics system, where the induced wavefront error is variable. In each step of the iterative optimization procedure, the induced phase, as the only variable of the algorithm, will change to make the the two images' intensity approximate each other. In the whole procedure, there is no approximation in the original phase, which may guarantee a high precision for the measurement of relatively large wavefront errors. For demonstration purposes, a numerical simulation is performed based on the graphical user interface (GUI) of the Optimization Toolbox in Matlab. Simulation results show that the wavefront of an optics system can theoreti-

cally be reconstructed with high precision, which shows that such an iterative algorithm may be an effective way for wavefront sensing in a high-contrast imaging system.

The outline of the paper is as follows. In Section 2, the principle of the wavefront sensing algorithm is proposed. In Section 3, we present the numerical simulation of the iterative optimization algorithm. The summary and conclusions are given in Section 4.

2 PRINCIPLE OF THE WAVEFRONT SENSING ALGORITHM

Recent laboratory tests have demonstrated that the actual coronagraph can provide contrast on the order of $10^{-5} \sim 10^{-7}$. Further improvement is limited by the speckle noise that is generated from the wavefront error of the coronagraph system. In this section, we consider a general optics system with a circular entrance pupil. For simplicity, the system will be operated at a monochromatic wavelength.

For an optics system with wavefront errors, the electric field of the electromagnetic wave in the pupil plane of the system can be expressed as

$$E_{\mathrm{pupil}}(u, v) = A(u, v)\mathrm{e}^{-i\phi(u,v)}, \tag{1}$$

where $A(u, v)$ represents the pupil function of the optics system; $\phi(u, v)$ represents the original wavefront or phase error of the optics system that will cause spot-like speckles surrounding the bright star image on the focal plane.

We first put a CCD camera in the pupil plane of the optics system and measure the intensity of the pupil image. Then the magnitude of the electric field on the pupil or the so-called pupil function can be measured directly from such an intensity and is given as

$$A_{\mathrm{t}}(u, v) = \sqrt{I_{\mathrm{pupil}}}, \tag{2}$$

where I_{pupil} represents the tested intensity of the pupil image.

Then we induce a dynamic phase on the tested pupil function and the reconstructed electric field in the pupil can be expressed as

$$E_{\mathrm{r}}(u, v) = A_{\mathrm{t}}(u, v)\mathrm{e}^{-i\Psi(u,v)}, \tag{3}$$

where Ψ is the induced dynamic phase.

Since the star's light is much brighter than that of the planet, the planet's image is much less intense than the star's image and is negligible during the wavefront sensing process without causing any significant error. The electric field of the starlight on the focal plane is the Fourier transform of the aberrated electric field on the pupil plane of the system. Then the reconstructed electric field on the focal plane can be expressed as

$$E_{\mathrm{focal\_r}}(x, y) = \boldsymbol{F}[E_{\mathrm{r}}(u, v)], \tag{4}$$

where $\boldsymbol{F}$ represents the Fourier transform of the associated function.

The point spread function (PSF) of the starlight on the focal plane is the square of the complex modulus of the aberrated electric field. The intensity of the reconstructed PSF image is given as

$$I_{\mathrm{r}}(x, y) = |E_{\mathrm{focal\_r}}(x, y)|^2. \tag{5}$$

Combining Equations (3), (4) and (5), the intensity of the reconstructed focal plane image becomes

$$I_{\mathrm{r}}(x, y) = |\boldsymbol{F}[A_{\mathrm{t}}(u, v)\mathrm{e}^{-i\Psi(u,v)}]|^2. \tag{6}$$

The principle of this algorithm is completely different from the previously proposed ones (Borde & Traub 2006; Give'on et al. 2007; Dou et al. 2009). In the iterative algorithm, only one focal plane image rather than three images is used to reconstruct the original wavefront. No approximation

is introduced in the original phase error, making it suitable for the measurement of much larger wavefront errors with high precision. Here we use the CCD camera to measure the intensity of the starlight image on the focal plane of the actual optics system. The intensity can be expressed as

$$I_{\mathrm{t}}(x, y) = |\boldsymbol{F}[A(u, v)\mathrm{e}^{-i\phi(u,v)}]|^2 \,, \tag{7}$$

where I_{t} is the tested intensity of the starlight image on the focal plane of the actual system; ϕ is the actual phase error of the optics system that will be measured.

To measure the original wavefront error of the optics system, the optimization algorithm needs to minimize the intensity difference between the reconstructed image and the actual focal plane image taken on the CCD. Here we subtract the intensity of each pixel on the two images and calculate the sum of the absolute value of the residual intensity. Once the sum of residual intensity reaches a minimum, the intensity of the two images will be approximately equal to each other. As a result, the value of the induced phase will be nearly the actual phase error that needs to be measured. The optimization algorithm should minimize the following equation:

$$\min \ \sum_{x=1}^{M}\sum_{y=1}^{N} |I_{\mathrm{r}}(x, y) - I_{\mathrm{t}}(x, y)| \,, \quad \mathrm{subject\ \ to} - \lambda/2 \le \Psi \le \lambda/2 \,, \tag{8}$$

assuming the CCD has $M \times N$ pixels.

Substituting Equations (6) and (7) into Equation (8), the problem has become to minimize:

$$\min \ \sum_{x=1}^{M}\sum_{y=1}^{N} ||\boldsymbol{F}[A_{\mathrm{t}}(u, v)\mathrm{e}^{-i\Psi(u,v)}]|^2 - |\boldsymbol{F}[A(u, v)\mathrm{e}^{-i\phi(u,v)}]|^2| \,. \tag{9}$$

Here in this paper, we use the Zernike polynomial to represent both the original phase error and the induced phase

$$\phi = \sum_{n=1}^{N} a_n Z_n, \qquad \Psi = \sum_{n=1}^{N} a_n' Z_n \,, \tag{10}$$

where a_n and a_n' are the Zernike coefficients; N is the order of the Zernike polynomial of Z_n; because each order of the Zernike polynomial is uncorrelated with every other, this can guarantee the existence of a solution for Equation (9).

In the procedure of optimization, we provide an initial phase (for instance, $\Psi = 0$), as the start of the iterative algorithm (the first step). In each iterative step, the induced phase Ψ will change towards the direction that makes the residual intensity become smaller. With a reasonable initial phase, the wavefront can be reconstructed very quickly in several steps. Once the wavefront error of the optics system can be precisely measured, it may be corrected by utilizing an appropriately-shaped DM and the speckle noise surrounding the bright starlight image will be expected to be effectively eliminated. A numerical simulation based on the iterative algorithm will be presented in detail in the next section.

3 NUMERICAL SIMULATION

The feasibility of the wavefront sensing algorithm discussed above can be verified by the following numerical simulation. For the purpose of demonstration, we create a distorted optics system by introducing a random phase error to an ideal optics system with a circular pupil. Here the root mean square (RMS) of the phase error is ~ 0.2114 rad, where the phase error needs to be measured. The aberrated image on the focal plane will be used as the reference image that will be compared with the reconstructed image. In this paper, we use a 36-order Zernike polynomial to represent the phase and the 36 Zernike coefficients are created randomly, to represent a general situation. Since each order of the Zernike polynomial is uncorrelated with every other, this can guarantee the existence
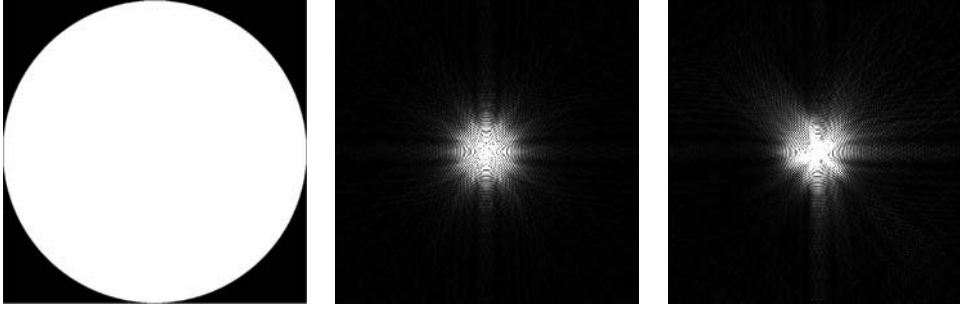
Fig. 1 *Left*: The amplitude pattern of the pupil function; *Middle*: The theoretical PSF of the starlight image; *Right*: The aberrated image with a phase error to be measured.

of a solution for the iterative optimization algorithm. The amplitude pattern of the pupil function (image taken on the pupil), and the theoretical and aberrated PSF (the starlight image on the focal plane) of the system are shown in Figure 1.

In the procedure of iterative optimization, a dynamic phase, as the only variable of the optimization problem, will be introduced and added to the theoretical pupil function and the intensity of the associated PSF can be calculated directly by using the Fourier transformation (here in this simulation, we use a 2-D fast Fourier transformation). In each iterative step, such a calculated intensity that changes with the induced phase will be subtracted from the intensity of the aberrated image (the reference image) with its original phase error. In the next step, the phase will change towards the direction that makes the residual intensity become smaller. Once the residual intensity reaches a minimum, the iterative procedure will stop and the phase in the last step will be the optimum phase that best approximates the original one.

Such a problem has been transformed to a 2-D constrained nonlinear minimization problem. Here the numerical simulation to test the optimization algorithm is based on the GUI of the Optimization Tool in Matlab, which needs an *objective function* to describe the problem, a *constraint function* to limit the variable and a *start point* as the first step to begin the search for a minimum. The objective function of such an optimization problem needs to find a minimum of the residual intensity between the reference image and the reconstructed image, as defined in Equation (9). Supposing the wavefront error of the system is within a wavelength, and the constraint function of the problem can be represented as $-\lambda/2 \le \Psi(u, v) \le \lambda/2$, where $\Psi(u, v)$ is the phase that will be induced on the pupil plane. With a reasonably initialized phase that is the start point for the optimization problem, the algorithm will only take several steps and the minimization solver converges very quickly. Here we initialize the start point to be $\Psi(u, v) = 0$ for a general purpose situation.

A trade-off is needed between the accuracy of the wavefront sensing process and the time consuming task of the iterative optimization procedure. For example, during the iterative procedure, the convergence rate will be very fast for the first 50 steps, with a remaining phase error of RMS on the order of 10^{-2} rad. However, the convergence rate will greatly decrease when the remaining phase error is very small. That means it may take a long time if an extremely high precision for the wavefront sensing process is needed. Here we manually stop the optimization procedure after 86 steps with an acceptable accuracy. The reconstructed wavefront has an RMS ~ 0.211485 rad, with a remaining wavefront error on the order of 10^{-3} rad (RMS). Comparing with the original wavefront of ~ 0.2114 rad, the relative error for the iterative wavefront sensing algorithm is 0.004%.

Figure 2 shows the original phase map to be measured and the reconstructed wavefront map by using the algorithm. It clearly indicates that the reconstructed wavefront is very consistent with the

original one based on the iterative optimization algorithm. Figure 3 shows the remaining wavefront error and the residual intensity between the reconstructed image and the aberrated starlight image of the actual optics system.
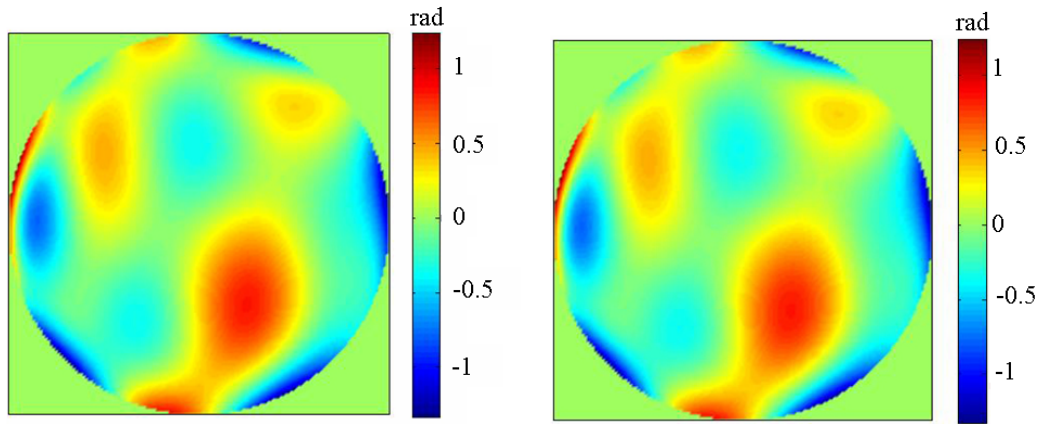

Fig. 2 Original wavefront map that will be measured (*left*) and the reconstructed wavefront map based on the iterative optimization algorithm (*right*).
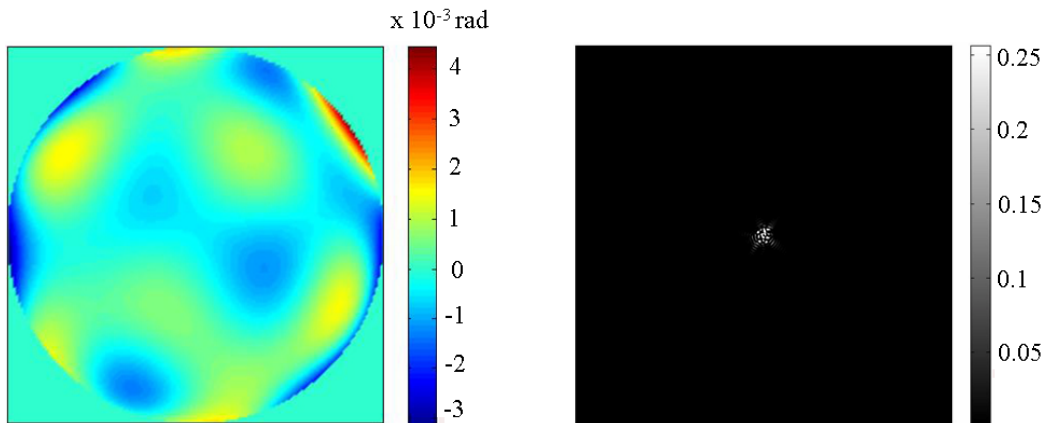

Fig. 3 Remaining wavefront error (*left*) and the residual intensity between two starlight images (*right*).

Since the original wavefront error was created randomly, the algorithm should be suitable for a general situation. To test this, we also used other wavefront maps to replace the one that has been used above, and achieved the same result, which gives us confidence that the performance of the iterative optimization algorithm is reliable and promising for wavefront measurement in high-contrast imaging systems.

4 SUMMARY AND CONCLUSIONS

Based on the iterative optimization algorithm, the wavefront error of an optics system can be precisely measured directly from one focal plane image, which has been demonstrated in our numerical simulation. The iterative optimization algorithm we have proposed here is completely different from other multiple image focal plane wavefront sensing algorithms. Since no approximation is performed on the original wavefront error or on the DM induced phase during the whole procedure, such an algorithm can be used for the measurement of large wavefront errors, which is impossible for the previously proposed 3-image focal plane wavefront sensing algorithm. Although in this paper we have only considered the case of a monochromatic wavelength, for a general coronagraph system, wavefronts at other wavelengths can be simply scaled according to the actual wavelength. At present, a laboratory test system has been set up for such a wavefront sensing experiment. We will discuss the later results in a future publication.

References

Borde, P. J., & Traub, W. A. 2006, ApJ, 638, 488.
Brown, R. A., & Burrows, C. J. 1990, ICARUS, 87, 484.
Dou, J. P., Ren, D. Q., Zhu, Y. T., et al. 2009, Science in China G: Physics and Astronomy, 52, 1284
Dou, J. P., Ren, D. Q., & Zhu, Y. T. 2010, RAA (Research Astron. Astrophys.), 10, 189
Give'on, A., Beliko, R., Shklan S., et al. 2007, Opt. Express, 15, 12338
Guyon, O., Pluzhnik, E. A., Kuchner, M. J., et al. 2006, ApJS, 167, 81
Kasdin, N. J., Vanderbei, R. J., Littman, M. J., et al. 2004, Proc. SPIE, 5487, 1312
Malbet, F., Yu, J. W., & Shao, M. 1995, PASP, 107, 386
Ren, D. Q., & Serabyn, E. 2005, Appl. Opt., 44, 7070
Ren, D. Q., & Wang, H. M. 2006, ApJ, 640, 530
Ren, D. Q., & Zhu, Y. T. 2007, PASP, 119, 1063
Ren, D. Q., Dou, J. P., & Zhu, Y. T. 2010, PASP, 122, 590
Serabyn, E., Mawet, D., & Burruss, R. 2010, Nature, 464, 1018