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Lookback time as a test for $f(R)$ gravity in the Palatini approach*

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Abstract We use the recently released data of lookback time (LT)-redshift relation, the cosmic microwave background shift parameter and the baryon acoustic oscillation measurements to constrain cosmological parameters of $f(R)$ gravity in the Palatini formalism by considering the $f(R)$ form of type (a) $f(R) = R - \beta/R^n$ and (b) $f(R) = R + \alpha \ln R - \beta$. Under the assumption of a Friedmann-Robertson-Walker universe, we achieved the best fitting results of the free parameters (Ω_{m0}, n) for (a) and (Ω_{m0}, α) for (b). We find that current LT data can provide interesting and effective constraints on gravity models. Compared with other data, the LT constraints favor a smaller value of the non-relativistic matter energy density.

Key words: cosmology: cosmic microwave background

1 INTRODUCTION

The accelerating expansion of the universe is one of the most important discoveries in cosmology in the last few years. This surprising phenomenon is supported by a variety of cosmological observations, including measurements of luminosity-distance of type Ia supernovae (SNe Ia) (Riess et al. 1998; Hicken et al. 2009), the large scale structure (LSS) (Wang & Tegmark 2004), the mapping of the cosmic microwave background (CMB) anisotropy (Spergel et al. 2007; Komatsu et al. 2011) and measurements of the baryon acoustic oscillation (BAO) (Eisenstein et al. 2005; Percival et al. 2010), etc. However the underlying mechanism which causes this cosmic acceleration is still not clear. In principle, the explanations of this phenomenon can always be classified into two categories. One is to introduce exotic matter sources. This route is most commonly used and gives rise to the idea of a dark energy component (Santos et al. 2008), for example the cosmological constant. The other is related to the introduction of changes to the gravitational part of general relativity, i.e. modifying the geometric part of the gravitational theory. Among the latter, several approaches are proposed to solve the problem of cosmic acceleration, for example the $f(R)$ gravity, which examines the possibility of modifying Einstein's general relativity by adding terms proportional to powers of the Ricci scalar to the Einstein-Hilbert Lagrangian (Buchdahl 1970; Starobinsky 1980; Kerner 1982; Barrow & Cotsakis 1988; Li & Barrow 2007). Besides that, the $f(R)$ gravity also attracts us because this theory can describe the early inflation as well as the late time acceleration of the universe without introducing dark energy (DE).

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Another important aspect should also be noticed, that in the $f(R)$ gravity theory, there are two different variational approaches, namely, the metric and Palatini formalism (Sotiriou & Liberati 2007). In the metric formalism, the connections are assumed to be the Christoffel symbols defined in terms of the metric. The variation of the action is taken with respect to the metric. While in the Palatini variational approach, the affine connections and the metric are treated as independent fields and the variation is taken with respect to both of them. For a general $f(R)$ term in the action, these approaches give different equations of motion.

In the metric approach, the field equations are fourth order and this makes them difficult to deal with in practice. Furthermore, the simplest $f(R)$ model of type $f(R) = R - \beta/R^n$ has difficulties passing the solar system tests (Amendola & Tsujikawa 2008) and gaining the correct Newtonian limit (Sotiriou 2006b,a). In addition, such theories also suffer gravitational instability (Dolgov & Kawasaki 2003). On the other hand, variations using the Palatini approach derive the second order field equations which are free of instabilities (Meng & Wang 2003, 2004b). This theory can also satisfy the solar system tests and reach the correct Newtonian limit (Sotiriou 2006a). Remarkably, such a theory accounts for the present cosmic acceleration without introducing DE.

Thus in our analysis, we will consider the Palatini formalism for gravitation and will focus on its application to a flat Friedmann-Robertson-Walker (FRW) cosmological model. Under the assumption of homogeneity and isotropy, we will study the $f(R)$ gravity with the models of type (a) $f(R) = R - \beta/R^n$ and (b) $f(R) = R + \alpha \ln R - \beta$. Unlike the metric formalism, these models can produce the sequence of radiation-dominated, matter-dominated and de Sitter periods.

In order to test the theory, we will consider the observational data. In this paper, we use two data sets of lookback time (LT) versus redshift measurements, for galaxy-clusters (Capozziello et al. 2004) and for passively evolving galaxies (Simon et al. 2005). These data have been used to constrain the DE models (Samushia et al. 2010) and the results show a present accelerating expansion of our universe. So it is natural to test if the application of these data to the modified gravity can give similar results. Following this direction, we apply these data to the $f(R)$ models listed above and constrain the free parameters. The calculation can also be compared with the previous works which studied the SNe Ia, CMB and LSS data (Amarzguioui et al. 2006; Fay et al. 2007; Koivisto 2007; Fairbairn & Rydbeck 2007). In order to better constrain the free parameters of the $f(R)$ models, we combined the LT data with the BAO and CMB shift parameter data.

Our paper is organized as follows. In Section 2 we will give the formalisms of the $f(R)$ gravity in the Palatini approach. In Section 3 we describe the data used in the calculation. In Section 4, we show the models used in the calculation and present the constraint results with our analysis. Finally, we will give some discussion and conclusions.

2 $f(R)$ FORMALISM IN THE PALATINI APPROACH

The modified Einstein-Hilbert action in the Palatini $f(R)$ gravity is given as

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa}f(R) + \mathfrak{L}_m\right], \quad (1)$$

where f is a differentiable function of the Ricci scalar R, $\mathfrak{L}_m$ is the Lagrangian for the matter fields, and $\kappa = 8\pi G$. As was mentioned above, the variation of this action gives the second order field equation

$$f' R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f(R) = \kappa T_{\mu\nu}, \quad (2)$$

where f' denotes $f' = df/dR$, and $T_{\mu\nu}$ is the energy-momentum tensor. For a perfect-fluid system, we have $T_{\mu\nu} = (\rho_m + p_m)u_\mu u_\nu + p_m g_{\mu\nu}$; here ρ_m and p_m are the energy density and fluid pressure respectively, and u_μ is the fluid four-velocity. In the Palatini approach, the Ricci scalar is $R =$

$g^{\mu\nu}R_{\mu\nu}(\bar{\Gamma})$ with $R_{\mu\nu}(\bar{\Gamma})$ being defined as $R_{\mu\nu}(\bar{\Gamma}) = \bar{\Gamma}^{\alpha}_{\mu\nu,\alpha} - \bar{\Gamma}^{\alpha}_{\mu\alpha,\nu} + \bar{\Gamma}^{\alpha}_{\alpha\lambda}\bar{\Gamma}^{\lambda}_{\mu\nu} - \bar{\Gamma}^{\alpha}_{\mu\lambda}\bar{\Gamma}^{\lambda}_{\alpha\nu}$; here the connection $\bar{\Gamma}$ will be treated as an independent field separately from the metric.

Motivated by recent cosmological observations, we shall consider the spatially flat FRW universe with the metric

$$ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j, \tag{3}$$

where $a(t)$ is the cosmological scale factor. Considering the density $\Omega_{m0} = \kappa\rho_{m0}/3(H_0^2)$ and the redshift parameter $z = a_0/a - 1$, one can get the generalized Friedmann equation (Fay et al. 2007; Carvalho et al. 2008)

$$\frac{H^2}{H_0^2} = \frac{3\Omega_{m0}(1+z)^3 + f/H_0^2}{6f'\left[1 + \frac{9}{2}\frac{f''}{f'}\frac{H_0^2\Omega_{m0}(1+z)^3}{Rf''-f'}\right]^2}, \tag{4}$$

where ρ_{m0} is the present matter density. Additionally, the trace of the field equation gives another useful relation

$$Rf' - 2f = -3H_0^2\Omega_{m0}(1+z)^3. \tag{5}$$

One can easily find that the Friedmann equation will return to the Einstein-Hilbert one under the condition $f(R) = R$.

3 THE OBSERVATIONAL DATA

3.1 The Lookback Time Data

As one of the time-based cosmological tests, the LT observation is different from other widely-used distance-based cosmological tests (Samushia et al. 2010). Because this is a time-based method, the ages of distant objects are independent of each other. This feature makes it avoid the biases existing in techniques that use the distance of primary or secondary indicators in the cosmic distance ladder method. Such time-based methods contain the measurements of the absolute age of objects, differential age of objects and LT of objects (Samushia et al. 2010).

Since the seminal work of Sandage (Sandage 1988), the LT-redshift relation has been used to constrain cosmological models in several works (Samushia et al. 2010; Xu & Wang 2010; Pires et al. 2006). The LT is defined as the difference between the present age of the universe (t_0) and its age (t_z) when a particular light ray at redshift z was emitted

$$\begin{aligned} t_{\rm L}(z,p) \ &= \ t_0(p) - t(z) = \frac{1}{H_0}\left[\int_0^\infty \frac{dz'}{(1+z')E(z',p)} - \int_z^\infty \frac{dz'}{(1+z')E(z',p)}\right] \\ &= \ \frac{1}{H_0}\int_0^z \frac{dz'}{(1+z')E(z',p)}, \end{aligned} \tag{6}$$

where p are the parameters of the cosmological model under consideration (here is the particular $f(R)$ gravity models), $E(z,p) = H(z,p)/H_0$, $H(z,p)$ is the Hubble parameter at redshift z, and the Hubble constant $H_0 = 100\,h$ km s^{-1} Mpc^{-1}.

Following Capozziello et al. (2004), one can define the age $t(z_i)$ of an object (a galaxy, a quasar and so on) at redshift z_i as the difference between the age of the universe at z_i and the age z_F when the object was born,

$$t(z_i,p) = \int_{z_i}^\infty \frac{dz'}{(1+z')E(z',p)} - \int_{z_F}^\infty \frac{dz'}{(1+z')E(z',p)} = t_{\rm L}(z_F,p) - t_{\rm L}(z_i,p) \tag{7}$$

(The relations between these cosmological times can be clearly seen in fig. 1 in Pires et al. 2006).

Then the observed LT to an object at z_i can be defined as

$$t_{\rm L}^{\rm obs}(z_i) = t_{\rm L}(z_F) - t(z_i) = [t_0^{\rm obs} - t(z_i)] - [t_0^{\rm obs} - t_{\rm L}(z_F)] = t_0^{\rm obs} - t(z_i) - t_{\rm inc}. \tag{8}$$

Here $t_0^{\rm obs}$ is the measured current age of the universe, and $t_{\rm inc}$ stands for the delay factor or incubation time which accounts for our ignorance about the absolute age of the universe when the object is formed at $t(z_F)$.

In order to constrain the free parameters of the particular $f(R)$ gravity models, we use two age data sets. One is the ages of 32 passively evolving galaxies (Capozziello et al. 2004) with a redshift interval $0.117 \leq z \leq 1.845$. As suggested in Samushia et al. (2010), we consider a 12% one standard deviation uncertainty on the age measurements. The other is the ages of six galaxy clusters in the redshift range $0.10 \leq z \leq 1.27$ and their one standard deviation uncertainty is 1 Gyr. Thus we have 38 measurements of $t_{\rm L}^{\rm obs}$ with uncorrelated uncertainties σ_i to constrain the model parameters.

We apply the χ^2 statistic to constrain the parameters of each model

$$\chi^2_{\rm LT}(p, H_0, t_{\rm inc}, t_0^{\rm obs}) = \sum_{i=1}^{38} \frac{\left[t_{\rm L}(z_i, p, H_0) - t_{\rm L}^{\rm obs}(z_i, t_{\rm inc}, t_0^{\rm obs})\right]^2}{\sigma_i^2 + \sigma^2_{t_0^{\rm obs}}} + \frac{\left[t_0(p, H_0) - t_0^{\rm obs}\right]^2}{\sigma^2_{t_0^{\rm obs}}} \tag{9}$$

where $\sigma_{t_0}^{\rm obs}$ is the uncertainty in the estimate of t_0 and $t_{\rm L}(z_i, p)$ and $t_0(p)$ are the predicted values in the model under consideration. In order to get the constraint results of the parameter sets, we should calculate the likelihood function $L'(p, H_0, t_{\rm inc}, t_0^{\rm obs}) \propto \exp(-\chi^2/2)$. It can be easily seen that the likelihood function L is based on the total age of the universe $t_0^{\rm obs}$, the delay factor $t_{\rm inc}$ and the Hubble constant H_0. Similarly, we also analyze the DE constraints (Samushia et al. 2010). We will treat $t_{\rm inc}$ as a nuisance parameter and marginalize L' over it in an interval [0,20] Gyr. For $t_0^{\rm obs}$ we apply a Bayesian prior as a Gaussian function with central values and variance based on the WMAP estimate of the total age of the universe $t_0^{\rm obs} = (13.75 \pm 0.13)\,{\rm Gyr}$ in each model constraint. Furthermore, we treat H_0 as another nuisance parameter and marginalize over it with a Gaussian prior of $h = 0.742 \pm 0.036$ which is not the same but is consistent with the previous values of $h = 0.68 \pm 0.04$ (Chen et al. 2003) and $h = 0.72 \pm 0.08$ (Freedman et al. 2001). Thus the resulting LT likelihood function depends only on the parameter sets p. The best fit values of p can be achieved through minimizing $\chi^2_{\rm LT}$.

3.2 The CMB Data

The CMB shift parameter $\mathcal{R}$ is arguably one of the most model-independent parameters among those which can be inferred from CMB data. It is directly proportional to the ratio of the angular diameter distance to the decoupling epoch divided by the Hubble horizon size at that special epoch. That is

$$\mathcal{R} = \sqrt{\Omega_{m0} H_0^2} \int_0^{z_{\rm s}} \frac{dz}{H(z)}, \tag{10}$$

where $z_{\rm s} = 1089$ is the redshift of recombination. The value of $\mathcal{R}$ obtained from acoustic oscillations in the CMB temperature anisotropy power spectrum is $\mathcal{R} = 1.715 \pm 0.021$ (Hinshaw et al. 2009; Komatsu et al. 2009). One important aspect worth emphasizing is that the CMB shift parameter provides the information at a high redshift level. The calculation has to be integrated up to the matter/radiation decoupling, i.e. the contribution of radiation can be no longer be neglected and should be taken into account. So in our analysis, as Carvalho et al. (2008); Santos et al. (2008) suggest, a radiation component of $\Omega_r = 5 \times 10^{-5}$ has been included. For the CMB shift parameter data, the corresponding χ^2 is

$$\chi^2_{\rm CMB} = \left(\frac{\mathcal{R} - 1.715}{0.021}\right)^2. \tag{11}$$

3.3 The BAO Data

Similar to the case of shift parameter $\mathcal{R}$, the BAO peak detected in the Sloan Digital Sky Survey luminous red galaxies is another tool to test the model against observational data. BAO can be described by the dimensionless $\mathcal{A}$-parameter (Tegmark et al. 2004)

$$\mathcal{A} = \Omega_{m0}^{\frac{1}{2}} E(z_b)^{-\frac{1}{3}} \left[\frac{1}{z_b} \int_0^{z_b} \frac{dz'}{E(z')} \right]^{\frac{2}{3}}, \tag{12}$$

where $z_b = 0.35$ is the redshift at which the acoustic scale has been measured. In Eisenstein et al. (2005), the value of $\mathcal{A}$ has been determined to be $\mathcal{A}_{\rm obs} = 0.469(n_{\rm s}/0.98)^{-0.35} \pm 0.017$. In our calculation, the scalar spectral index $n_{\rm s}$ is taken to be 0.957 from Liu & Li (2009). Thus the χ^2 of the BAO data is

$$\chi^2_{\rm BAO} = \left(\frac{\mathcal{A} - A_{\rm obs}}{\sigma_{\mathcal{A}}} \right)^2. \tag{13}$$

4 THE $f(R)$ GRAVITY MODELS AND CONSTRAINTS

4.1 The $f(R)$ Gravity Models

We apply the data listed in Section 3 to two $f(R)$ gravity models

(a)

$$f(R) = R - \beta/R^n, \tag{14}$$

(b)

$$f(R) = R + \alpha \ln R - \beta. \tag{15}$$

Recently, the model of the type (a) case has been tested by several kinds of data, including SNe Ia, CMB, BAO and observational H(z) data (OHD) (Carvalho et al. 2008; Santos et al. 2008). Their constraints do not exactly show the same results, but they are consistent. Their best fit results of the parameters show a present accelerating expansion. Here we extended this by considering the LT data and combined them with the CMB and BAO data to test this model.

The type (b) model with a $\ln R$ term has also been discussed recently (Nojiri & Odintsov 2004; Meng & Wang 2004a; Fay et al. 2007). It is shown that in the Palatini formalism, the $\ln R$ gravity can drive a current exponential accelerated expansion and it reduces to the standard Friedmann evolution for the high redshift region. Although this model may have problems in the electron-electron scattering experiment, it has a well-defined Newtonian limit and may eliminate the need of dark energy to provide the current cosmic acceleration. So in this paper, we still consider this gravity model and compare the constraint results with the type (a) case.

It is should be mentioned that in both of the two $f(R)$ gravity models, there are three undetermined parameters. For (a), they are Ω_{m0}, n and β while for (b), they are Ω_{m0}, α and β. However, at $z = 0$, the evaluation of Equation (5) imposes a relation among these three parameters, so there are only two free parameters for each model. In our calculation, we choose (Ω_{m0}, n) for (a) and (Ω_{m0}, α) for (b) as the free parameters to work with.

4.2 Cosmological Constraints

Figure 1 shows the constraints of the LT and combined data results for each gravity model. The best fit values of (a) are $(\Omega_{m0}, n, \beta) = (0.01, -0.97, 1.03)$ for LT only and $(\Omega_{m0}, n, \beta) = (0.27, -0.04, 4.04)$ for LT+CMB+BAO. For (b), the results are $(\Omega_{m0}, \alpha, \beta) = (0.09, -4.7, -3.46)$ for LT only and $(\Omega_{m0}, \alpha, \beta) = (0.27, -0.2, 3.97)$ for LT+CMB+BAO. We can see that the data combination gives the same fitting value of the matter density parameter Ω_{m0} for both models. The 3σ confidence interval of Ω_{m0} in both constraints is contained in (0.2, 0.4). This is consistent with current observational results (Spergel et al. 2007, 2003), which show the universe is made of a large amount of energy density in the form of non-relativistic matter with a proportion up to 70%. However the LT constraints of both models are different from the combination ones; both Ω_{m0}

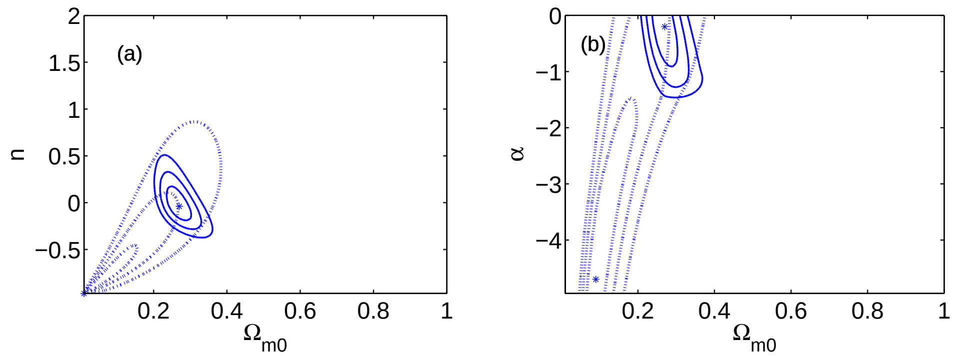

Fig. 1 (a) Confidence regions in the $\Omega_{m0} - n$ plane for the $f(R) = R - \beta/R^n$ gravity model. The dashed lines and solid lines for the constraint results from LT and LT+CMB+BAO respectively. (b) Confidence regions in the $\Omega_{m0} - \alpha$ plane for the $f(R) = R + \alpha \ln R - \beta$ gravity model. The dashed lines and solid lines for the constraint results from LT and LT+CMB+BAO respectively. The confidence regions at the 68.3%, 95.4% and 99.7% levels from inner to outer are presented respectively, and the stars in the centers stand for the best fit results.

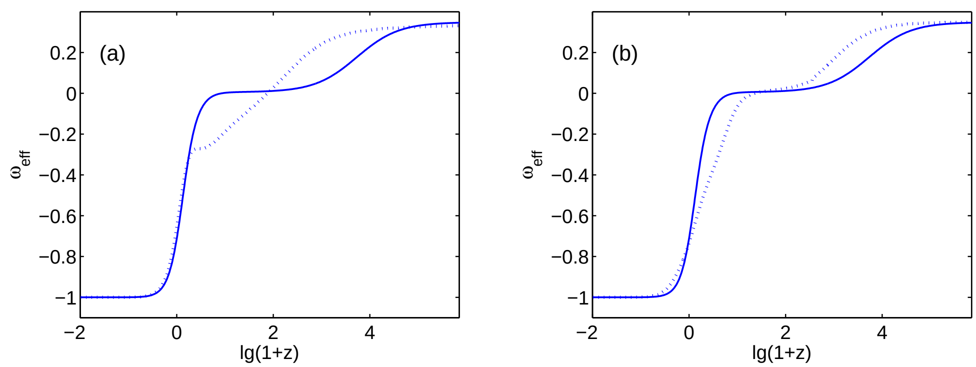

Fig. 2 Effective equation of state (EOS) as a function of redshift for the best fit results from the constraints of LT only (*dotted line*) and LT+CMB+BAO combined (*solid line*). (a) results for type (a), and (b) results for type (b).

values are smaller than other observation estimates and this result is consistent with the DE tests (Samushia et al. 2010). Moreover, from both the constraints of (b), we can see that the best fit value of β is nonzero which shows that the $\ln R$ term cannot derive the cosmic acceleration only without introducing dark energy. This is consistent with the result of Fay et al. (2007).

In a word, we can see that the LT data can give a comparable constraint on the $f(R)$ gravity models with other data. In order to verify if these two $f(R)$ gravity models in Palatini formalism can produce a standard matter-dominated era followed by an accelerating expansion, it is useful to calculate the effective equation of state (EOS) which is given by Santos et al. (2008)

$$\omega_{\rm eff} = -1 + \frac{2}{3}\frac{1+z}{H(z)}\frac{dH}{dz}. \tag{16}$$

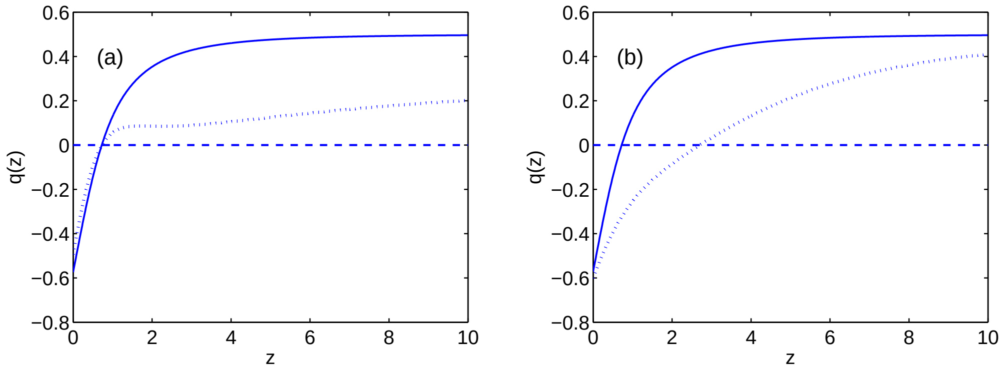

Fig. 3 Behavior of the deceleration parameter $q(z)$ as a function of redshift z. The dotted and solid lines are the results from LT only and LT+CMB+BAO jointly. *Left* panel: results for type (a) and *Right* panel: results for type (b).

This EOS curve is shown in Figure 2. It should be mentioned that a component of $\Omega_r = 5 \times 10^{-5}$ is included. We can see that the best fit values from constraints of LT only and LT+CMB+BAO combined give different evolutions of EOS. Both combinations of constraints in (a) and (b) show that the universe goes through the last three phases of cosmological evolutions: radiation-era ($\omega = 1/3$), matter-era ($\omega = 0$) and late time acceleration ($\omega = -1$). However, the constraints of LT only for both models show that there is no apparent matter-dominated era followed by an accelerated expansion. The evolution curve of LT only for (a) is consistent with the results achieved from OHD only in Carvalho et al. (2008) which behaves similarly as in the metric formalism (Amendola et al. 2007b,a).

Focusing on the behavior of the universe in the late time, one can calculate the deceleration parameter $q(z) \equiv -\frac{\ddot{a}}{a\dot{a}^2}$ which can be rewritten as (Nesseris & Shafieloo 2010)

$$q(z) = -1 + (1+z)\frac{d\ln(H(z))}{dz}. \tag{17}$$

Figure 3 shows $q(z)$ as a function of redshift z with the best fitting results in both constraints. All the constraints show that the universe is now undergoing an accelerated phase as widely suggested. As we go back, the universe will enter a decelerating phase. The moment that the expansion of the universe changes from decelerated to accelerated can also be calculated. Especially for (a), we can see that the LT constraint and the LT+CMB+BAO constraint give the same time of deceleration-acceleration transition in the $q(z)$ curve.

5 DISCUSSION AND CONCLUSIONS

As an alternative way to solve the problem of the accelerated expansion of the universe, the issue of modified gravity has been studied from different aspects of DE. The combination with observational data is maybe one of the most important steps to test theories. In this paper, we analyze the $f(R)$ gravity of type (a) $f(R) = R - \beta/R^n$ and (b) $f(R) = R + \alpha \ln R - \beta$ in the Palatini approach by assuming a spatially flat FRW cosmology. By use of the lookback time-redshift relation, we check the cosmological behavior of these gravity models.

As the previous works show, the LT can give an efficient contribution in constraining the cosmological parameters in DE models (Samushia et al. 2010; Xu & Wang 2010; Pires et al. 2006), so the role that LT plays in modified gravity should also be focused on in future work. Because the time-based observation is different from the ones achieved from the distance-based method, whether this test can give researchers some new information is worth noticing. Following this direction, we use the LT to constrain the free parameters of the particular modified gravity models. The results using LT only show that the universe is now undergoing an accelerating expansion phase. However this conclusion is not perfectly satisfactory because it cannot derive an apparent matter-dominated era right after the radiation-dominated era. So the combination with other data becomes necessary. When CMB and BAO are being considered, the combined constraints show that the universe goes through the last three phases of cosmological evolution: radiation era, matter era and a late time cosmic acceleration. This is consistent with previous works which use SNe Ia and OHD (Fay et al. 2007; Carvalho et al. 2008; Santos et al. 2008).

From the above analysis and constraint, we find that our results from LT are believable. Although the constraint of LT only is not perfectly obeyed, the combination with other data can derive a well-behaved gravity model. With more and better data being collected in the future, we can imagine that the LT may give more efficient constraints in both $f(R)$ gravity models and DE models. This can also provide more information about the evolution of our universe.

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