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The frequencies of high frequency quasi-periodic oscillations in two different frames in black hole LMXBs

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Abstract We discuss the modes of the Alfvén waves in the accretion disk with a toroidal magnetic field in black hole low mass X-ray binaries in a rotating frame. By solving the perturbed general relativistic magnetohydrodynamic equations in the rotating frame, we find two stable modes of the Alfvén wave which are the same as those in the fiducial observer frame. This gives a feasible way to transform between the two different frames, which validates the possible Alfvén wave modes in the accreting celestial bodies with a toroidal magnetic field.

Key words: accretion, accretion disks — magnetohydrodynamics — relativity

1 INTRODUCTION

High frequency quasi-periodic oscillations (HFQPOs) in the observed X-ray fluxes are often shown in low mass X-ray binaries (LMXBs). Characteristic frequencies of these QPOs range from 50 to 1300 Hz. In about 20 neutron star LMXBs (NS-LMXBs), two peak kHz QPOs (twin kHz QPOs) have been detected and it has been discovered that the centroid frequency separation roughly equals either the spin frequency of the neutron star or half of its value. In these NS-LMXBs, the centroid frequency separation decreases typically by a few tens of Hz when the QPO frequencies increase by hundreds of Hz (Boutloukos et al. 2006; Shi & Li 2009). In some black hole LMXBs (BH-LMXBs), HFQPOs have been seen at nearly constant frequencies from a given source and the frequency ratio of the HFQPOs in pairs observed in BH-LMXBs is usually roughly consistent with a fixed 3:2 ratio (McClintock & Remillard 2006; Strohmayer 2001), such as these observed frequencies (450, 300 Hz in GRO J1655+40, Strohmayer 2001; 67, 41 Hz and 168, 113 Hz in GRS 1915+105, Remillard 2004; 276, 184 Hz in XTE J1550–564, Miller et al. 2001; 240, 165 Hz in H1743–322, Homan et al. 2005). Being different from the BH systems, the frequency ratio of the twin kHz QPOs in the NS systems is concentrated around 3:2 (Abramowicz et al. 2005; Török et al. 2005; Török & Stuchlík 2005, Török et al. 2008a,b; Boutelier et al. 2010) but other ratios, such as 5:4 and 4:3, have also been found (Török 2009; Stuchlík et al. 2011) and it is controversial whether there is an intrinsically preferred ratio in NS-LMXBs (Belloni et al. 2007). At present, a lot of QPO theories are put forward but no one can account for the different phenomena in BH- and NS-LMXBs. Zhang (2004), Li & Zhang (2005) and Rezania & Samson (2005) discussed the twin kHz QPOs in NS-LMXBs by using magnetohydrodynamic (MHD) oscillations.

In addition to the models of kHz QPOs, a lot of researchers suggested many models to account for the HFQPOs in BH-LMXBs. In the Kerr metric, the four frequencies: the Keplerian frequency, the radial epicyclic frequency, the vertical epicyclic frequency and the Lense-Thirring frame-dragging frequency have been listed (e.g., Perez et al. 1997) and these frequencies have been used to explain the observed 3/2 commensurability in various combinations for several models. A series of papers discussed the 3:2 internal epicyclic or Keplerian resonance among the first three frequencies such as Aschenbach (2004); Török & Stuchlík (2005); Török et al. (2005); Stuchlík et al. (2007, 2008). Cui et al. (1998) suggested that the 300 Hz QPOs in GRO J1655–40 corresponded to the Lense-Thirring nodal precession frequency near the inner stable circular orbit radius. Stella & Vietri (1998, 1999); Stella et al. (1999) considered the periastron precession frequency and the Keplerian frequency of the hot-blob at various radii r in the inner parts of the accretion disk as the lower and upper frequencies of the twin HFQPOs respectively for BH and NS sources. In this case, a relativistic precession model and a massive ($\sim 2M_{\odot}$) NS is usually required to match the observations in this model. Čadež et al. (2008); Kostić et al. (2009) suggested that the QPOs were generated by a "tidal disruption" due to the large accreting inhomogeneities and the related characteristic frequency is far lower than the observed frequencies (Török et al. 2011). Wagoner et al. (2001) selected the g-modes and c-modes of the diskoseismic wave as the measured frequencies of the HFQPOs in BH-LMXBs and then they estimated the masses and angular momenta of some BHs. Ortega-Rodríguez et al. (2002); Šrámková et al. (2007); Fu & Lai (2009) discussed the diskoseismic modes (the inertial oscillations, acoustic oscillations & corrugation modes) further and the possible sources of HFQPOs were suggested. Rezzolla et al. (2003) also estimated the BH spin by the inertial-acoustic modes which came from the centrifugal and pressure gradients in a small-size torus and they found that the BH spin should be close to the maximal value to produce the 3:2 ratio, i.e. a group of extreme Kerr BHs should exist in the BH-LMXBs with twin HFQPOs. Abramowicz and Kluźniak (Abramowicz & Kluźniak 2001; Kluzniak & Abramowicz 2001; Kluźniak et al. 2004) discussed their model containing a non-linear parametric resonance in accretion disk global oscillations that could lead to the twin-peak HFQPOs in LMXBs. Kato (2001, 2004, 2005, 2008) discussed the inertial-acoustic mode and g-mode oscillations in the warped disk. Tassev & Bertschinger (2008) discussed the kinematic density waves in the accretion disks and several modes in pairs close to the ratio (3:2) could be obtained but the correct frequencies could not be reproduced. Shi & Li (2009, 2010) also considered the MHD oscillations and they suggested the MHD model for NS-LMXBs and the general relativistic magnetohydrodynamics (GRMHD) model for BH-LMXBs; then the spins of some neutron stars were estimated (Shi 2010).

Shi & Li (2009) suggested an explanation that the coupling of the two resonant MHD modes based on MHD oscillation modes in neutron star magnetospheres might lead to the twin kHz QPOs in NS-LMXBs. Including the spin of a neutron star, this model naturally related the upper and lower kHz QPO frequencies. Shi & Li (2010) suggested that the two modes of the Alfvén wave produced in the transition region between the inner advection-dominated accretion flows (ADAFs) and the outer thin disk might lead to the double HFQPOs in BH-LMXBs. The accretion disks with toroidal magnetic fields were considered and the 3:2 relation for the upper and lower frequencies of the QPOs was shown in the result. From that it could be estimated that the HFQPOs might come from the place inside 100 gravitational radii and there is strong evidence supporting the origin of the twin HFQPOs (van der Klis 2006). Considering the similarities in terms of general relativity for the accretion disks in BH-LMXBs and in NS-LMXBs, it should be discussed urgently why there is no identical observation in the HFQPOs in NS-LMXBs; an example is the ratio 3:2 of the twin HFQPOs in BH-LMXBs, which is not prominent in NS-LMXBs. The differences in the configuration of the magnetic fields and the structure of the accretion disks in BH- and NS-LMXBs may be the main reasons (Shi & Li 2010) and we should thoroughly discuss those cases.

van der Klis (2006) suggested that the kHz QPOs in NS-LMXBs and the HFQPOs in BH-LMXBs could be interpreted by a unified model. The two above models of Shi & Li (2009, 2010) have been discussed in two different frames of reference respectively and now we should transform them into the same frame of reference to unify the two models. Here we will only discuss the GRMHD modes in BH-LMXBs in the rotating frame because those in the fiducial observer (FIDO) frame have already been discussed (Shi & Li 2010).

This paper is organized as follows. In the next section, we give the the two modes of the Alfvén wave in the rotating frame from the GRMHD equations. Lastly, we give the discussion and conclusions.

2 THE TWO MODES OF THE GRMHD WAVE

There are two kinds of configurations of the magnetic field in BH-LMXBs; one is similar to the dipolar magnetic field and the other is the toroidal magnetic field. Now many investigations (Tout & Pringle 1992; Ruediger et al. 1995; Hawley 2000; Hirose et al. 2004; Moss & Shukurov 2004) show that the toroidal component of the magnetic field may be predominant in the accretion disk around a BH.

Here we discuss the Alfvén wave modes by GRMHD in an ideal adiabatic magnetofluid in the rotating frame. Koide (2003) discussed the frame in four types of reference frames in Kerr spacetime with GRMHD. The rotating frame, which is a non-inertial frame, was also discussed as a "locally nonrotating frame" by Bardeen et al. (1972). Shi & Li (2010) discussed the progress of how to produce the GRMHD waves in the Boyer-Lindquist coordinates $(ct', r, \theta, \varphi)$ in the FIDO frame. The FIDO frame is a locally inertial frame and we can define the line element as $(ds)^2 = -(cdt)^2 + \sum_{i=1}^{3}(dx^i)^2$. Here c is the speed of light in vacuum, r, θ, and φ are the coordinates in the spherical coordinate system; the Roman indices (i) run from 1 to 3, and (ct, x^1, x^2, x^3) are the coordinates of the FIDO frame.

In the accretion disk of BH-LMXBs, i.e. $\theta = \pi/2$, the oscillation for the plasma rotating around the BH often takes place because of the slim perturbation and the Alfvén wave can also develop. Now we begin with the form of the 3+1 split of the GRMHD equations about the perturbed quantities to a first-order approximation in the FIDO frame for the perturbed plasma as in Shi & Li (2010),

$$\frac{\partial(\gamma\rho_{\rm s})}{\partial t} = -\nabla\cdot[\alpha\gamma(\rho_0\boldsymbol{v}_s + \rho_{\rm s}\boldsymbol{v} + \rho_{\rm s}c\boldsymbol{\beta})], \tag{1}$$

$$\frac{\partial\varepsilon_{\rm s}}{\partial t} = -\nabla\cdot[\alpha(c^2\boldsymbol{P}_s - \gamma c^2\rho_0\boldsymbol{v}_s - \gamma c^2\rho_{\rm s}\boldsymbol{v}_0 + \varepsilon_{\rm s}c\boldsymbol{\beta})] - (\nabla\alpha)\cdot c^2\boldsymbol{P}_s - \widetilde{\boldsymbol{T}}_{\rm s} : \widetilde{\boldsymbol{\sigma}}, \tag{2}$$

$$\frac{\partial\boldsymbol{P}_s}{\partial t} = -\nabla\cdot[\alpha(\widetilde{\boldsymbol{T}}_s + c\boldsymbol{\beta}\boldsymbol{P}_{\rm s})] - (\varepsilon_{\rm s} + \gamma\rho_{\rm s}c^2)\nabla\alpha + \alpha\boldsymbol{f}_{{\rm curv},s} - \boldsymbol{P}_s\cdot\widetilde{\boldsymbol{\sigma}}, \tag{3}$$

$$\frac{\partial\boldsymbol{B}_s}{\partial t} = -\nabla\times[\alpha(\boldsymbol{E}_s - c\boldsymbol{\beta}\times\boldsymbol{B}_s)], \tag{4}$$

$$\boldsymbol{E}_s + \boldsymbol{v}_0\times\boldsymbol{B}_s + \boldsymbol{v}_s\times\boldsymbol{B}_0 = 0, \tag{5}$$

$$\nabla\cdot\boldsymbol{B}_s = 0, \tag{6}$$

$$p_{\rm s} = \frac{\Gamma p_0}{\rho_0}\rho_{\rm s}. \tag{7}$$

Here ρ is the plasma density, p the barometric pressure, $\boldsymbol{v}$ the velocity of the plasma in the FIDO frame, γ the Lorentz factor, Γ the adiabatic index, $\boldsymbol{E} = \boldsymbol{E}'/\sqrt{\mu_0}$ and $\boldsymbol{B} = \boldsymbol{B}'/\sqrt{\mu_0}$ (here $\boldsymbol{B}'$ is the magnetic field, $\boldsymbol{E}'$ is the electric field and μ_0 is the magnetic permeability in the vacuum). The bold characters denote vectors, the superscript $\sim$ corresponds to tensors, the subscript 0 corresponds to physical variables when the accretion is in a steady state and the subscript 's' corresponds to perturbed quantities. In Equation (2), $\widetilde{\boldsymbol{T}}_s{:}\widetilde{\boldsymbol{\sigma}}$ expresses the inner product of the two tensors, and $\boldsymbol{f}_{\rm curv,s}$ is a vector.

In addition, the lapse function (α) and the shift velocity ($\boldsymbol{\beta}$) can be expressed as,

$$\alpha = \sqrt{\frac{r^4 - 2r^3 r_g + a^2 r^2 r_g^2}{r^4 + a^2 r^2 r_g^2 + 2a^2 r r_g^3}}, \tag{8}$$

and

$$\beta^i = (\beta_1, \beta_2, \beta_3) = (0, 0, \frac{2ar_g^2}{r\sqrt{r^2 - 2rr_g + a^2 r_g^2}}), \tag{9}$$

where r is the distance of the plasma from the BH and $r_g = GM/c^2$, $a = Jc/GM^2$ (M and J are the mass and the angular momentum of the BH, G is the gravitational constant, and c is the speed of light in a vacuum); here $\boldsymbol{\beta}$ is a vector parallel to the toroidal velocity of the plasma. The perturbed quantities of momentum density ($\boldsymbol{P}_{\rm s}$), energy density ($\boldsymbol{\varepsilon}_{\rm s}$) and the energy-momentum tensor ($\widetilde{\boldsymbol{T}}_{\rm s}$) can be written as,

$$\boldsymbol{P}_{\rm s} = \frac{\gamma^2}{c^2}(\psi_0 \boldsymbol{v}_{\rm s} + \psi_{\rm s} \boldsymbol{v}_0) + \frac{1}{c^2}\boldsymbol{E}_{\rm s} \times \boldsymbol{B}_0 + \frac{1}{c^2}\boldsymbol{E}_0 \times \boldsymbol{B}_{\rm s}, \tag{10}$$

$$\varepsilon_s = \psi_s \gamma^2 - p_s - \gamma \rho_s c^2 + \boldsymbol{B}_0 \cdot \boldsymbol{B}_s + \frac{\boldsymbol{E}_0 \cdot \boldsymbol{E}_s}{c^2}, \tag{11}$$

$$\begin{aligned}\widetilde{\boldsymbol{T}}_s = (p_s + \boldsymbol{B}_0 \cdot \boldsymbol{B}_s + \frac{1}{c^2}\boldsymbol{E}_0 \cdot \boldsymbol{E}_s)\widetilde{\boldsymbol{I}} + \frac{\psi_s}{c^2}\gamma^2 \boldsymbol{V}_0 \boldsymbol{V}_0 + \frac{\psi_0}{c^2}\gamma^2(\boldsymbol{V}_0 \boldsymbol{V}_s + \boldsymbol{V}_s \boldsymbol{V}_0) \\ -(\boldsymbol{B}_0 \boldsymbol{B}_s + \boldsymbol{B}_s \boldsymbol{B}_0) - \frac{1}{c^2}(\boldsymbol{E}_0 \boldsymbol{E}_s + \boldsymbol{E}_s \boldsymbol{E}_0),\end{aligned} \tag{12}$$

respectively. Here $\psi_0 = \rho_0 c^2 + \frac{\Gamma p_0}{\Gamma - 1}$, $\psi_s = \rho_s c^2 + \frac{\Gamma p_s}{\Gamma - 1}$, which correspond to the relativistic enthalpy density in steady state and the perturbed relativistic enthalpy density, respectively.

The other physical quantities in the accretion disk in the Kerr space-time can be simplified as,

$$\boldsymbol{f}_{\rm curv} \equiv \sum\nolimits_j (G_{ij} T^{ij} - G_{ji} T^{jj}), \tag{13}$$

$$\sigma_{ij} = \left\{\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{2acr_g^2(3r^2 + a^2 r_g^2)\sqrt{r^2 + a^2 r_g^2 + \frac{2a^2 r_g^3}{r}}}{\sqrt{\frac{r^2}{r^2 + a^2 r_g^2 - 2rr_g}}(r^3 + ra^2 r_g^2 + 2a^2 r_g^3)^2} & 0 & 0 \end{matrix}\right\}, \tag{14}$$

and

$$G_{ij} = -\left\{\begin{matrix} \frac{r_g(a^2 r_g - r)}{r^2\sqrt{r^2 - 2rr_g + a^2 r_g^2}} & 0 & 0 \\ \frac{\sqrt{r^2 - 2rr_g + a^2 r_g^2}}{r^2} & 0 & 0 \\ \frac{(r^3 - a^2 r_g^3)\sqrt{r^2 - 2rr_g + a^2 r_g^2}}{(r^3 + ra^2 r_g^2 + 2a^2 r_g^3)r^2} & 0 & 0 \end{matrix}\right\}. \tag{15}$$

Now the physical quantities in the FIDO frame can be converted into the quantities in the rotating frame as follows,

$$\frac{d\boldsymbol{r}_s}{dt} = \boldsymbol{v}_s = \boldsymbol{v}_s{}' + \boldsymbol{\Omega} \times \boldsymbol{r}_s,$$

$$\frac{d\boldsymbol{r}_0}{dt} = \boldsymbol{v}_0 = \boldsymbol{v}_0{}' + \boldsymbol{\Omega} \times \boldsymbol{r}_0 = \boldsymbol{\Omega} \times \boldsymbol{r}_0,$$

$$\frac{\partial \boldsymbol{r}_s}{\partial t} = \frac{\partial \boldsymbol{r}_s}{\partial t'} = \boldsymbol{v}_s{}',$$

$$\frac{\partial (\boldsymbol{\Omega} \times \boldsymbol{r}_s)}{\partial t} = \frac{\partial (\boldsymbol{\Omega} \times \boldsymbol{r}_s)}{\partial t'} = \boldsymbol{\Omega} \times \boldsymbol{v}_s{}',$$

where the quantities with the superscript $'$ denote the variables in the rotating frame and $\boldsymbol{\Omega}$ is the angular velocity of locally non-rotating frames. We differentiate Equations (1)–(4) while Equations (5), (7) and the above expressions are substituted, and we can derive the following

$$\frac{\partial^2 (\gamma \rho_s)}{\partial t^2} = -\nabla \cdot \left[\alpha \gamma (\rho_0 \frac{\partial}{\partial t} \boldsymbol{v}_s{}' + \rho_0 \boldsymbol{\Omega} \times \boldsymbol{v}_s{}' + \frac{\partial}{\partial t} \rho_s \boldsymbol{v}_0 + \frac{\partial}{\partial t} \rho_s c \boldsymbol{\beta}) \right], \tag{16}$$

$$\begin{aligned} \frac{\partial^2 \varepsilon_s}{\partial t^2} \; = \; & -\nabla \cdot \left[\alpha \left(c^2 \frac{\partial}{\partial t} \boldsymbol{P}_s - \gamma c^2 \rho_0 \Big(\frac{\partial}{\partial t} \boldsymbol{v}_s{}' + \boldsymbol{\Omega} \times \boldsymbol{v}_s{}' \Big) - \gamma c^2 \frac{\partial}{\partial t} \rho_s \boldsymbol{v}_0 + \frac{\partial}{\partial t} \varepsilon_s c \boldsymbol{\beta} \right) \right] \\ & -(\nabla \alpha) \cdot c^2 \frac{\partial}{\partial t} \boldsymbol{P}_s - \frac{\partial}{\partial t} \widetilde{\boldsymbol{T}}_s : \widetilde{\boldsymbol{\sigma}}, \end{aligned} \tag{17}$$

$$\begin{aligned} \frac{\partial^2 \boldsymbol{P}_s}{\partial t^2} \; = \; & -\nabla \cdot \left[\alpha \left(\frac{\partial}{\partial t} \widetilde{\boldsymbol{T}}_s + c \boldsymbol{\beta} \frac{\partial}{\partial t} \boldsymbol{P}_s \right) \right] - \left(\frac{\partial}{\partial t} \varepsilon_s + \gamma \frac{\partial}{\partial t} \rho_s c^2 \right) \nabla \alpha \\ & + \alpha \frac{\partial}{\partial t} \boldsymbol{f}_{\mathrm{curv,s}} - \frac{\partial}{\partial t} \boldsymbol{P}_s \cdot \widetilde{\boldsymbol{\sigma}}, \end{aligned} \tag{18}$$

$$\frac{\partial^2 \boldsymbol{B}_s}{\partial t^2} = \nabla \times \left[\alpha \Big(\boldsymbol{v}_0 \times \frac{\partial}{\partial t} \boldsymbol{B}_s + c \boldsymbol{\beta} \times \frac{\partial}{\partial t} \boldsymbol{B}_s + \Big(\frac{\partial}{\partial t} \boldsymbol{v}_s{}' + \boldsymbol{\Omega} \times \boldsymbol{v}_s{}' \Big) \times \boldsymbol{B}_0 \Big) \right]. \tag{19}$$

Equations (10)–(12) can be transformed as follows

$$\begin{aligned} \widetilde{\boldsymbol{T}}_s = \widetilde{\boldsymbol{T}}_s{}' + \tfrac{1}{c^2} (\boldsymbol{v}_0 \times \boldsymbol{B}_0) \cdot [(\boldsymbol{\Omega} \times \boldsymbol{r}_s) \times \boldsymbol{B}_0] \widetilde{\boldsymbol{I}} + \tfrac{1}{c^2} \psi_0 \gamma^2 (\boldsymbol{v}_0 (\boldsymbol{\Omega} \times \boldsymbol{r}_s) + (\boldsymbol{\Omega} \times \boldsymbol{r}_s) \boldsymbol{v}_0) \\ - \tfrac{1}{c^2} [(\boldsymbol{v}_0 \times \boldsymbol{B}_0)((\boldsymbol{\Omega} \times \boldsymbol{r}_s) \times \boldsymbol{B}_0) + ((\boldsymbol{v}_s{}' + \boldsymbol{\Omega} \times \boldsymbol{r}_s) \times \boldsymbol{B}_0)(\boldsymbol{v}_0 \times \boldsymbol{B}_0)], \end{aligned} \tag{20}$$

$$\boldsymbol{P}_s = \boldsymbol{P}_s{}' + \tfrac{1}{c^2} \gamma^2 \psi_0 (\boldsymbol{\Omega} \times \boldsymbol{r}_s) - \tfrac{1}{c^2} ((\boldsymbol{\Omega} \times \boldsymbol{r}_s) \times \boldsymbol{B}_0) \times \boldsymbol{B}_0, \tag{21}$$

$$\varepsilon_s = \varepsilon_s{}' + \tfrac{1}{c^2} (\boldsymbol{v}_0 \times \boldsymbol{B}_0) \cdot ((\boldsymbol{\Omega} \times \boldsymbol{r}_s) \times \boldsymbol{B}_0), \tag{22}$$

where

$$\begin{aligned} \widetilde{\boldsymbol{T}}_s{}' \; = \; & \Big[\frac{\Gamma p_0}{\rho_0} \rho_s + \boldsymbol{B}_0 \cdot \boldsymbol{B}_s + \tfrac{1}{c^2} (\boldsymbol{v}_0 \times \boldsymbol{B}_0) \cdot (\boldsymbol{v}_0 \times \boldsymbol{B}_s) \Big] \widetilde{\boldsymbol{I}} + \tfrac{1}{c^2} \psi_s \gamma^2 \boldsymbol{v}_0 \boldsymbol{v}_0 - (\boldsymbol{B}_0 \boldsymbol{B}_s + \boldsymbol{B}_s \boldsymbol{B}_0) \\ & - \tfrac{1}{c^2} [(\boldsymbol{v}_0 \times \boldsymbol{B}_0)(\boldsymbol{v}_0 \times \boldsymbol{B}_s) + (\boldsymbol{v}_0 \times \boldsymbol{B}_s)(\boldsymbol{v}_0 \times \boldsymbol{B}_0)] + \tfrac{1}{c^2} (\boldsymbol{v}_0 \times \boldsymbol{B}_0) \cdot (\boldsymbol{v}_s{}' \times \boldsymbol{B}_0) \widetilde{\boldsymbol{I}} \\ & + \tfrac{1}{c^2} \psi_0 \gamma^2 (\boldsymbol{v}_0 \boldsymbol{v}_s{}' + \boldsymbol{v}_s{}' \boldsymbol{v}_0) - \tfrac{1}{c^2} [(\boldsymbol{v}_0 \times \boldsymbol{B}_0)(\boldsymbol{v}_s{}' \times \boldsymbol{B}_0) + (\boldsymbol{v}_s{}' \times \boldsymbol{B}_0)(\boldsymbol{v}_0 \times \boldsymbol{B}_0)], \end{aligned} \tag{23}$$

$$\boldsymbol{P}_s^{'} = \tfrac{1}{c^2} \gamma^2 \psi_0 \boldsymbol{v}_s^{'} - \tfrac{1}{c^2} (\boldsymbol{v}_s^{'} \times \boldsymbol{B}_0) \times \boldsymbol{B}_0 - \tfrac{1}{c^2} (\boldsymbol{v}_0 \times \boldsymbol{B}_s) \times \boldsymbol{B}_0 + \tfrac{1}{c^2} \gamma^2 \psi_s \boldsymbol{v}_0 - \tfrac{1}{c^2} (\boldsymbol{v}_0 \times \boldsymbol{B}_{\mathbf{0}}) \times \boldsymbol{B}_s, \tag{24}$$

$$\varepsilon_s^{'} = \psi_s \gamma^2 - \frac{\Gamma p_0}{\rho_0} \rho_s - \gamma \rho_s c^2 + \boldsymbol{B}_0 \cdot \boldsymbol{B}_s + \tfrac{1}{c^2} (\boldsymbol{v}_0 \times \boldsymbol{B}_0) \cdot (\boldsymbol{v}_0 \times \boldsymbol{B}_s) + \tfrac{1}{c^2} (\boldsymbol{v}_0 \times \boldsymbol{B}_0) \cdot (\boldsymbol{v}_s^{'} \times \boldsymbol{B}_0). \tag{25}$$

Here $\widetilde{\boldsymbol{I}}$ is the unit tensor. Substituting Equations (20)–(22) into Equations (16)–(19) and carrying out Fourier transformation ($e^{i\boldsymbol{k}\cdot\boldsymbol{r}-i\omega t}$) for Equations (16)–(19), we then get the dispersion equations,

$$\omega^2\gamma\rho_s = \alpha\gamma\omega\boldsymbol{k}\cdot(\rho_0\boldsymbol{v}_s^{'} + \rho_s\boldsymbol{v}_0 + \rho_s c\boldsymbol{\beta}) + i\rho_0\boldsymbol{k}\cdot(\boldsymbol{\Omega}\times\boldsymbol{v}_s^{'}), \tag{26}$$

$$\begin{aligned} -\omega^2\varepsilon_s \;=\; & -\omega\boldsymbol{k}\cdot[\alpha c^2\boldsymbol{P}_s - \gamma c^2\rho_0\boldsymbol{v}_s^{'} - \gamma c^2\rho_s\boldsymbol{v}_0 + \varepsilon_s c\boldsymbol{\beta})] + i\gamma c^2\rho_0\boldsymbol{k}\cdot(\boldsymbol{\Omega}\times\boldsymbol{v}_s^{'}) \\ & -\alpha c^2\omega\boldsymbol{k}\cdot\boldsymbol{P}_s + i\omega\widetilde{\boldsymbol{T}}_s : \widetilde{\boldsymbol{\sigma}}, \end{aligned} \tag{27}$$

$$\omega\boldsymbol{P}_s^{'} = \boldsymbol{k}\cdot[\alpha(\widetilde{\boldsymbol{T}}_s^{'} + c\boldsymbol{\beta}\boldsymbol{P}_s^{'})] + (\varepsilon_s + \gamma\rho_s c^2)\boldsymbol{k}\alpha + i(\alpha\boldsymbol{f}_{\mathrm{curv,s}}^{'} - \boldsymbol{P}_s^{'}\cdot\widetilde{\boldsymbol{\sigma}}), \tag{28}$$

$$-\omega^2\boldsymbol{B}_s = \alpha\omega\boldsymbol{k}\times(\boldsymbol{v}_0\times\boldsymbol{B}_s + c\boldsymbol{\beta}\times\boldsymbol{B}_s + \boldsymbol{v}_s^{'}\times\boldsymbol{B}_0) + i\boldsymbol{k}\times[(\boldsymbol{\Omega}\times\boldsymbol{v}_s^{'})\times\boldsymbol{B}_0], \tag{29}$$

where $\boldsymbol{k}$ is the wave vector and ω is the the oscillation frequency.

Shi & Li (2010) found that only the Alfvén modes in GRMHD in the accretion disk with a toroidal magnetic field are stable, so now we discuss the Alfvén modes in the following text. We can then calculate $\boldsymbol{k}\|\boldsymbol{v}_0\|\boldsymbol{B}_0\|\boldsymbol{\beta}$, $\boldsymbol{k}\perp\boldsymbol{v}_s^{'}$ and derive $\boldsymbol{k}\perp\boldsymbol{B}_s$ from the dispersion equation $\boldsymbol{k}\cdot\boldsymbol{B}_s = 0$ which is obtained from Equation (6) by the Fourier transformation, where $\|$ denotes parallel. The perturbed density of the plasma ρ_s and the perturbed equivalent energy density ε_s should be real numbers, so we can get $\boldsymbol{k}\cdot(\boldsymbol{\Omega}\times\boldsymbol{v}_s^{'}) = 0$ from Equations (26) and (27). According to those conditions, the result $\rho_s = 0$, $\varepsilon_s = 0$ of those two equations is suitable for the Alfvén wave. Now Equations (23), (24), (28) and (29) can be simplified as follows

$$\widetilde{\boldsymbol{T}}_s^{'} = \tfrac{1}{c^2}\psi_0\gamma^2(\boldsymbol{v}_0\boldsymbol{v}_s^{'} + \boldsymbol{v}_s^{'}\boldsymbol{v}_0) - (\boldsymbol{B}_0\boldsymbol{B}_s + \boldsymbol{B}_s\boldsymbol{B}_0), \tag{30}$$

$$\boldsymbol{P}_s^{'} = \frac{\gamma^2}{c^2}\psi_0\boldsymbol{v}_s^{'} + \frac{v_0 B_0}{c^2}\boldsymbol{B}_s + \frac{B_0^2}{c^2}\boldsymbol{v}_s^{'}, \tag{31}$$

$$\begin{aligned} & \omega(\tfrac{1}{c^2}\gamma^2\psi_0\boldsymbol{v}_s^{'} + \tfrac{B_0^2}{c^2}\boldsymbol{v}_s^{'} - \tfrac{v_0B_0}{c^2}\boldsymbol{B}_s) - \alpha\boldsymbol{k}\cdot[-(\boldsymbol{B}_0\boldsymbol{B}_s + \boldsymbol{B}_s\boldsymbol{B}_0) \\ & + \tfrac{1}{c^2}\psi_0\gamma^2(\boldsymbol{v}_0\boldsymbol{v}_s^{'} + \boldsymbol{v}_s^{'}\boldsymbol{v}_0)] - [\alpha c(\boldsymbol{k}\cdot\boldsymbol{\beta})]\boldsymbol{P}_s^{'} = 0, \end{aligned} \tag{32}$$

$$\boldsymbol{B}_s = \frac{-\alpha(\boldsymbol{k}\cdot\boldsymbol{B}_0)\boldsymbol{v}_s^{'}}{\omega - \alpha(\boldsymbol{k}\cdot\boldsymbol{v}_0) - \alpha c(\boldsymbol{k}\cdot\boldsymbol{\beta})}. \tag{33}$$

Here Equation (33) can be obtained because the solution of $\omega - \alpha(\boldsymbol{k}\cdot\boldsymbol{v}_0) - \alpha c(\boldsymbol{k}\cdot\boldsymbol{\beta}) = 0$ is not a physical one. When Equations (30), (31) and (33) are substituted into Equation (32) and the nonzero oscillation velocity is considered, Equation (32) is simplified as

$$(\gamma^2\psi_0 + B_0^2)[\omega - \alpha c(\boldsymbol{k}\cdot\boldsymbol{\beta})]^2 - 2\gamma^2\psi_0\alpha k v_0[\omega - \alpha c(\boldsymbol{k}\cdot\boldsymbol{\beta})] - k^2\alpha^2B_0^2c^2 + \alpha^2k^2v_0^2\gamma^2\psi_0 = 0. \tag{34}$$

The modes of the Alfvén waves are solved the same as in Shi & Li (2010),

$$\omega = k\alpha[\beta_3 c + \frac{\gamma^2\psi_0 v_0 \pm B_0\sqrt{B_0^2c^2 + (c^2 - v_0^2)\gamma^2\psi_0}}{\gamma^2\psi_0 + B_0^2}]. \tag{35}$$

The group velocities of the Alfvén waves and the phase velocities of these Alfvén waves in special relativity are the same as those obtained by De Villiers & Hawley (2003),

$$v_A = \frac{v_0 \pm \eta\sqrt{\frac{1}{\gamma^2} + \eta^2}c}{\eta^2 + 1},$$

where $\gamma^2\eta^2 = B_0^2/\psi_0$.

3 DISCUSSION AND CONCLUSIONS

According to the conclusions of many researchers, we suggest several basic hypotheses:

(1) The toroidal magnetic field in accretion disks is generated by a dynamo mechanism (Hawley 2000; Moss & Shukurov 2004; Ruediger et al. 1995; Ruzmaikin et al. 1979; Tout & Pringle 1992) and that accretion is driven by the magnetic stress (e.g., Brandenburg et al. 1995; Matsumoto & Tajima 1995; Stone et al. 1996).
(2) The thickness of the accretion disks with strong toroidal magnetic fields can be estimated by Begelman & Pringle's accretion disk theory (Begelman & Pringle 2007) which agrees with observations (Robinson et al. 1999; Shafter & Misselt 2006).
(3) The HFQPOs in BH-LMXBs are generated from the truncated accretion disk because HFQPOs in BH-LMXBs are generally observed in the steep power-law (SPL) state, i.e., very high state (VHS); the accretion disk might contain an inner ADAF surrounded by an outer thin disk in VHS (see Yuan 2001).
(4) The frequencies of the two Alfvén waves correspond to the frequencies of the two peak HFQPOs in BH-LMXBs.

We can estimate the toroidal velocity of the accretion plasma from the velocity of the circular orbit relative to Bardeen observers as in Camenzind (2007). From the conclusion of that paper, we can simplify Equation (35) and the detailed process was listed by Shi & Li (2010),

$$\omega \;=\; 1.2756\times 10^{6}\Big(\frac{M}{M_{\odot}}\Big)^{-1}\frac{1}{i}\sqrt{\frac{i^4-2i^3+a^2i^2}{i^4+a^2i^2+2a^2i}}\left[\frac{2a}{i\sqrt{i^2-2i+a^2}}\right. \\ \left. +\frac{\gamma^2\pm\sqrt{0.048+0.0023v_{\varphi}^2/c^2}}{\gamma^2+0.048v_{\varphi}^2/c^2}\frac{v_{\varphi}}{c}\right], \tag{36}$$

where $i=r/r_g$, $M_{\odot}$ is the mass of the Sun and v_{φ} is the toroidal component of the velocity of the plasma. If we substitute the known mass and spin of some BHs for one of the two peak HFQPOs into Equation (36), then we can get the truncated radius of the accretion disk, the frequency of the other HFQPO and the ratio of the two peak HFQPOs. For example, we can find the ratio (1.55) of the two peak HFQPOs in GRO J1655−40 after substituting the mass ($6.3\,M_{\odot}$), the spin ($a\,=\,0.7$) of the system and the frequency (450 Hz) into Equation (36) (Shi & Li 2010). According to the two modes, the approximate ratio 3:2 of the twin HFQPOs in BH-LMXBs can be derived and those QPOs might be the origin of the Alfvén waves in the accretion disk of the BH-LMXBs (Shi & Li 2010).

Now there are three main kinds of mechanisms of the HFQPOs: oscillations, waves and spin with and without general relativity effects. We have considered the modes including the three kinds of mechanisms in the GRMHD model (Shi & Li 2010) but the effect from the spin is not prominent. In LMXBs the high energy X-ray radiation could mainly originate from the interaction of the plasma with the magnetic field and so the change of the magnetic field could modulate the X-ray flux. A natural result is that the Alfvén waves may lead to the HFQPOs in LMXBs. In the border between the thin accretion disk and the thick accretion disk, such as the ADAF, some slight perturbation can cause the production of the Alfvén waves. The frequency ratio of the two Alfvén modes is very close to 3:2 and these modes may be the two frequencies of the twin peak HFQPOs.

In this paper we have adopted the conventional assumption that a star is well described by the Kerr geometry. Urbanec et al. (2010) drew a conclusion that the combination of disk-oscillation modes, which differs from the geodesic radial and vertical epicyclic modes, or a modulation mechanism that differs from the Paczyński modulation, should be involved in the resonance model for NS kHz QPOs; so the MHD modes, which are different from epicyclic modes, may be a possible explanation for the HFQPOs. When the Kerr metric is assumed, the various combinations of epicyclic modes can be used to fit the HFQPOs. Lin et al. (2011) fit the data by some HFQPO models with some input values such as the the mass or the spin of the NS. Török et al. (2010) have presented a detailed analysis that demonstrated properties of the Kerr geometry imply a relation between the mass and spin of the NS through fitting the data with some HFQPO models (e.g. the relativistic precession one). Similar to the resonance model, there are also several parameters in our model (Shi & Li 2010) and there might be a relation between the mass and spin of the BH. This aspect could be studied subsequently when the observational data are much more abundant.

In the rotating frame we get the same results as Shi & Li (2010), i.e. the modes of the Alfvén waves in the accretion disk with the toroidal magnetic field in BH-LMXBs have the same form in the two different reference frames. This effect opens up a road for exploring the MHD waves from the accretion disk in the LMXBs in different frames of reference. According to the transformation process in the text, we will continue to explore the Alfvén modes using GRMHD in NS-LMXBs which might be the source of the twin kHz QPOs.

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