

Steady state equilibrium condition of npe^\pm gas and its application to astrophysics *

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Abstract The steady equilibrium conditions for a mixed gas of neutrons, protons, electrons, positrons and radiation fields (abbreviated as npe^\pm gas) with or without external neutrino flux are investigated, and a general chemical potential equilibrium equation $\mu_n = \mu_p + C\mu_e$ is obtained to describe the steady equilibrium at high temperatures ($T > 10^9$ K). An analytic fitting formula of coefficient C is presented for the sake of simplicity, when neutrinos and antineutrinos are transparent. It is a simple method to estimate the electron fraction for the steady equilibrium npe^\pm gas that adopts the corresponding equilibrium condition. As an example, we apply this method to the GRB accretion disk and confirm that the composition in the inner region is approximately in equilibrium when the accretion rate is low. For the case with external neutrino flux, we calculate the initial electron fraction of neutrino-driven wind from the proto-neutron star model M15-11-r1. The results show that the improved equilibrium condition makes the electron fraction decrease significantly more than the case $\mu_n = \mu_p + \mu_e$ when the time is less than 5 s post bounce, which may be useful for r-process nucleosynthesis models.

Key words: nuclear reactions — nucleosynthesis, weak-interaction GRB accretion disk — neutrino-driven wind

1 INTRODUCTION

It is a classic and simple approximation for the practical application in many astrophysical situations that matter compositions can be considered as a mixture of the neutrons, protons and electrons, the so called npe^- system. If the temperature is very high ($T > 10^9$ K), lots of photons, positrons, and even neutrinos and antineutrinos will appear in the system, i.e., the system becomes a mixture of electrons, positrons, nucleons and radiation fields (abbreviated as npe^\pm gas). Many astrophysical situations can be regarded as an npe^\pm gas, such as (i) the hot fireball jetted from the central engine of a Gamma Ray Burst (GRB) (Pruet & Dalal 2002), (ii) the matter produced after the core-collapse of a supernova due to the photodisintegration of the iron nuclei (Marek & Janka 2009), (iii) the neutrino-driven wind emanating from a proto-neutron star (PNS) where $T > 10^9$ K (Martínez-Pinedo 2008), (iv) the outer core of a young neutron star (Yakovlev et al. 2008; Baldo & Ducoin

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2009), (v) the accretion disk of a GRB (Liu et al. 2007; Janiuk & Yuan 2010) and (vi) the early universe before the decoupling of neutrinos (Dutta et al. 2004; Harwit 2006). In summary, npe^- and npe^\pm gas is applied widely in present studies. The equilibrium steady state of npe^- or npe^\pm gas is an important stage for many cases. Many authors have addressed this issue for several decades. A typical approach to the steady state equilibrium of the npe^- system was proposed by Shapiro & Teukolsky (1983). They gave an important result that $\mu_n = \mu_p + \mu_e$ for a steady equilibrium npe^- system, where μ 's are the chemical potentials for neutrons, protons and electrons respectively. This result has been accepted by most authors. However, because Shapiro & Teukolsky (1983) only considered the electron capture and its reverse interaction at 'low temperature,' they ignored the appearance of positrons when the temperature of the system is high enough for this to be a factor. Recently, Yuan (2005) argued that many positrons can exist at high temperatures, which leads to a great increase of the positrons' capture rate. The positron capture significantly affects the condition of steady state equilibrium. If the neutrinos can escape freely from the system with plenty of e^\pm pairs, the equilibrium condition should be $\mu_n = \mu_p + 2\mu_e$ instead. However, for a more general condition when the temperature is moderate, the equilibrium condition has not been researched. Liu et al. (2007) have previously proposed a method in which they assume that the coefficient of μ_e varies exponentially from $\mu_n = \mu_p + \mu_e$ to $\mu_n = \mu_p + 2\mu_e$ in the accretion disk of GRBs, but it is not a rigorous method. Therefore, a detailed and reliable database or fitting function to describe the steady state equilibrium of the npe^\pm gas at any temperature is necessary. Furthermore, the above discussions are limited to an isolated system, ignoring the external neutrino flux. In this paper, we investigate the chemical equilibrium condition for npe^\pm gas at any temperature from 10^9 to 10^{11} K, and give a concrete application to the GRB accretion disk. We also calculate the initial electron fraction of the neutrino-driven wind in a PNS, in which the external strong neutrino flux cannot be ignored. This paper is organized as follows. In Section 2, we present the equilibrium conditions when neutrinos are transparent or opaque for an isolated system. Section 3 contains a detailed discussion of the initial electron fraction of a neutrino-driven wind for the PNS model M15-I1-r1 (Arcones et al. 2007, 2008). Finally, we analyze the results and draw our conclusions.

2 EQUILIBRIUM CONDITION OF npe^\pm GAS WITHOUT THE EXTERNAL NEUTRINO FLUX

For a mixed gas of npe^\pm and a radiation field in different physical conditions, we divide it into two cases: neutrino transparency and opacity. To guarantee self-consistency, we give a simple estimate for the opaque critical density of the npe^\pm gas. The mean free path of the neutrino is

$$l_\nu = \frac{1}{n\sigma_\nu^{\text{sac}} + n_n\sigma_\nu^{\text{abs}}},$$

where n and n_n are the number density of baryons and neutrons respectively. Here

$$n = \rho N_A, \quad n_n = \rho(1 - Y_e)N_A,$$

where ρ is the mass density, Y_e is the electron fraction, N_A is Avogadro's constant, σ_ν^{sac} and σ_ν^{abs} are the scattering cross section of baryons and absorption cross section of neutrons respectively. Also,

$$\sigma_\nu^{\text{sac}} \approx \left(\frac{E_\nu}{m_e c^2}\right)^2 10^{-44},$$

(Kippenhahn & Weigert 1990), where E_ν is the energy of the neutrino, $m_e c^2$ is the mass energy of an electron, and c is the speed of light. In addition,

$$\sigma_\nu^{\text{abs}} \approx \frac{A}{\pi^2} E_e p_e \approx \frac{A}{\pi^2} E_e^2$$

(Qian & Woosley 1996; Lai & Qian 1998), where

$$A = \pi G_F^2 \cos^2 \theta_c (C_V^2 + 3C_A^2),$$

$$G_F = 1.436 \times 10^{-49} \text{ erg cm}^3$$

is the Fermi weak interaction constant, and $\cos^2 \theta_c = 0.95$ refers to the Cabbibo angle. $C_V = 1$, $C_A = 1.26$, E_e and p_e are the energy and momentum for the electron, respectively. Due to the energy conservation of the nuclear reaction,

$$E_e = E_\nu + Q, \quad Q = (m_n - m_p)c^2 = 1.29 \text{ MeV},$$

where m_n and m_p are the mass of a neutron and proton respectively. At high density, the electrons are strongly degenerate and relativistic, so $E_e \approx E_F = [(3\pi^2 \bar{\lambda}_e^3 n_e)^{2/3} + 1]^{1/2}$ (in units of $m_e c^2$) and $\bar{\lambda}_e = \frac{\hbar}{m_e c}$ is the reduced electron Compton wavelength. Substituting $\rho Y_e N_A$ for n_e , we obtain

$$E_e \approx (3\pi^2 \bar{\lambda}_e^3 \rho Y_e N_A)^{1/3}.$$

Therefore, the mean free path of the neutrino is

$$l_\nu = \frac{1}{\rho N_A [(3\pi^2 \bar{\lambda}_e^3 \rho Y_e N_A - Q)^{2/3} \times 10^{-44}] + [\frac{A}{\pi^2} (3\pi^2 \bar{\lambda}_e^3 \rho Y_e N_A)^{2/3}] \rho (1 - Y_e) N_A}. \quad (1)$$

If we assume $l_\nu = 10 \text{ km}$ is the criterion for neutrino opacity, $\rho_{\text{cri}}^\nu = 5.58, 4.50, 4.10, 3.96$, and $3.96 \times 10^{10} \text{ g cm}^{-3}$ for $Y_e = 0.1, 0.2, 0.3, 0.4$, and 0.5 respectively. Rigorously speaking, here we overestimate the absorption cross section because we ignore a block factor $(1 - f_e)$, so ρ_{cri}^ν is the minimum critical density. When $\rho < \rho_{\text{cri}}^\nu$, the neutrino is transparent, otherwise it is opaque. By a similar method, one only needs to replace ν , $E_e = E_\nu + Q$, and n_n with $\bar{\nu}$, $E_e = E_{\bar{\nu}} - Q$, and n_p respectively to obtain the mean free path of an antineutrino. The critical density for an antineutrino $\rho_{\text{cri}}^{\bar{\nu}} = 1.43, 0.86, 0.62, 0.48$, and $0.40 \times 10^{11} \text{ g cm}^{-3}$ for $Y_e = 0.1, 0.2, 0.3, 0.4$, and 0.5 , respectively.

Another more precise way to judge the transparency of a neutrino is by defining a parameter, neutrino optical depth τ , which is closely related to the object's composition and structure. Following Arcones et al. (2008), we obtain

$$\tau = \int_r^\infty \langle \kappa_{\text{eff}} \rangle dr,$$

where r is the neutrino transport distance,

$$\langle \kappa_{\text{eff}} \rangle = \sqrt{\langle \kappa_{\text{abs}} \rangle (\langle \kappa_{\text{abs}} \rangle + \langle \kappa_{\text{sac}} \rangle)},$$

where κ_{abs} and κ_{sac} are the absorption opacity and scatter opacity respectively, $\kappa_{\text{sac}} = n \sigma_{\text{sac}}$, $\kappa_{\text{abs}} = \sum_i n_i \sigma_{\text{abs}(i)}$, and $\sigma_{\text{abs}(i)}$ and n_i are the neutrino absorption cross section and number density of the target particle respectively. Usually authors define $\tau < \frac{2}{3}$ or 1 as the criterion for neutrino transparency (Cheng et al. 2009; Janka 2001). In the following, we investigate the chemical equilibrium condition for the two different cases.

2.1 Case 1. Neutrinos are Transparent

When the npe^\pm gas is in equilibrium and transparent to neutrinos and antineutrinos ($\mu_\nu = \mu_{\bar{\nu}} = 0$, that is, we have taken their number densities to be zero), the beta reactions are the most important physical processes (Yuan 2005). The condition of steady state equilibrium is achieved via the following beta reactions,



Reactions (2)–(4) denote the electron capture (EC), positron capture (PC) and beta decay (BD) respectively. Since the system is transparent to neutrinos and antineutrinos, neutrinos and antineutrinos produced by reactions (2)–(4) can immediately freely escape, inducing much energy loss, so the reverse reactions, neutrino capture and antineutrino capture are negligible. If the system is in its equilibrium state, its composition is fixed and the electron fraction Y_e stays constant. EC decreases the Y_e , while PC and BD increase Y_e , so a general steady state equilibrium condition is

$$\lambda_{e-p} = \lambda_{e+n} + \lambda_n, \quad (5)$$

where λ 's are the respective reaction rates, and subscript symbols denote reaction particles (with the same convention in the following sections). Other reactions, such as $\gamma + \gamma \leftrightarrow e^- + e^+ \leftrightarrow \nu + \bar{\nu}$ also exist, but they do not directly influence the electron fraction. These beta reaction rates can be obtained from previous studies, and we list them (We employ the natural system of units here with $m_e = \hbar = c = 1$. In normal units, they would be multiplied by $\frac{(m_e c^2)^5 c}{(\hbar c)^7}$) (Yuan 2005; Langanke & Martínez-Pinedo 2000)

$$\lambda_{e-p} \simeq \frac{A}{2\pi^4} n_p \int_Q^\infty dE_e E_e p_e (E_e - Q)^2 F(Z, E_e) f_e, \quad (6)$$

$$\lambda_{e+n} \simeq \frac{A}{2\pi^4} n_n \int_{m_e}^\infty dE_e E_e p_e (E_e + Q)^2 F(-Z, E_e) f_{e+}, \quad (7)$$

$$\lambda_n \simeq \frac{A}{2\pi^4} n_n \int_{m_e}^Q dE_e E_e p_e (Q - E_e)^2 F(Z + 1, E_e) (1 - f_e). \quad (8)$$

Considering charge neutrality, $Y_e = Y_p$, and conservation of baryon number, $Y_n + Y_p = 1$, so n_p and n_n in Equations (6)–(8) are equal to $\rho Y_e N_A$ and $\rho(1 - Y_e) N_A$, respectively. $F(\pm Z, E_e)$ is the Fermi function, which corrects the phase space integral for the Coulomb distortion of the electron or positron wave function near the nucleus. It can be approximated by

$$F(\pm Z, E_e) \approx 2(1 + s)(2p_e R)^{2(s-1)} e^{\pi\eta} \left| \frac{\Gamma(s + i\eta)}{\Gamma(2s + 1)} \right|^2, \quad (9)$$

where Z is the nuclear charge of the parent nucleus, $Z = 1/0$ is for proton/neutron, $s = (1 - \alpha^2 Z^2)^{1/2}$, α is the fine structure constant, R is the nuclear radius, $\eta = \pm \alpha Z E_e / p_e$, and $\Gamma(x)$ is the Gamma function. We do not adopt any limiting form for the Fermi function. Compared to the derivation of the rates in Yuan (2005), we additionally consider the Coulomb screening of the nuclei. Here f_e and f_{e+} are the Fermi-Dirac functions for electrons and positrons. We have

$$f_e = \left[1 + \exp\left(\frac{E_e - \mu_e}{kT}\right) \right]^{-1},$$

and

$$f_{e+} = \left[1 + \exp\left(\frac{E_e + \mu_e}{kT}\right) \right]^{-1},$$

where k is Boltzmann's constant; electron chemical potential μ_e can be calculated as follows (energy is in units of $m_e c^2$ and momentum in units of $m_e c$):

$$\rho N_A Y_e = \frac{8\pi}{\lambda_e^3} \int_0^\infty (f_e - f_{e+}) p^2 dp, \quad (10)$$

where $\lambda_e = \frac{\hbar}{m_e c}$ is the electron Compton wavelength. Note that the calculation method of the chemical potential of an electron (including the chemical potentials of the proton and neutron in Equations (11)–(12)) also differs from the method in Yuan (2005). For one system with given

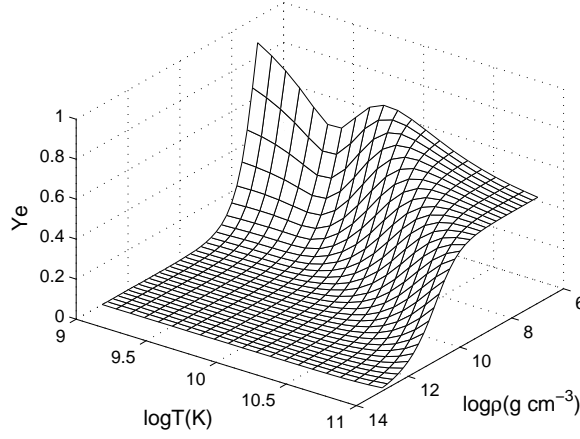


Fig. 1 Electron fraction Y_e as a function of T and ρ for npe^\pm gas in the equilibrium state.

temperature T and density ρ , electron fraction Y_e can be determined by the iterative technique of Equation (5).

Figure 1 shows T , Y_e and ρ that satisfy the equilibrium condition. It can be found that the Y_e decreases with density. As $\rho > 10^{11} \text{ g cm}^{-3}$, Y_e tends to zero, especially for lower temperatures. This is consistent with the results in figure 5 of Reddy et al. (1998). At high density, the β decay is almost forbidden and the positron capture rate is smaller than that of the electron. In order to sustain the equilibrium, electron number density n_e must be very low, which causes the Y_e to obviously decrease. Note that it is quite different from the direct Urca process for strongly degenerate baryons (Shapiro & Teukolsky 1983), in which $n_p/n_n > 1/8$. The baryons here are nondegenerate since their chemical potentials (minus their rest mass) are very low, even negative. After ρ , T and Y_e are found, chemical potentials μ_n and μ_p can be calculated as below (energy is in units of $m_e c^2$ and momentum in units of $m_e c$),

$$\rho N_A Y_p = \frac{8\pi}{\lambda_e^3} \int_0^\infty p^2 \left[1 + \exp\left(\frac{E_p - \mu_p}{kT}\right) \right]^{-1} dp, \quad (11)$$

$$\rho N_A (1 - Y_e) = \frac{8\pi}{\lambda_e^3} \int_0^\infty p^2 \left[1 + \exp\left(\frac{E_n - \mu_n}{kT}\right) \right]^{-1} dp, \quad (12)$$

where conservation of baryon number and charge density are also included.

In order to describe the numerical relationship of μ_e , μ_n and μ_p , we define a factor C : $\mu_n = \mu_p + C\mu_e$. Tables 1 and 2 are the results at $T = 10^9 \text{ K}$ and $5 \times 10^9 \text{ K}$ (μ_p' , μ_e' and μ_n' are chemical potentials without rest mass), respectively. It can be seen from Table 1 that $\lambda_{e-p} = \lambda_n \gg \lambda_{e+n}$, i.e., the positron capture rate at this time can be ignored. $C \approx 1$ means that $\mu_n = \mu_p + \mu_e$ is valid, while from Table 2 one can find $\lambda_{e-p} \approx \lambda_{e+n} \gg \lambda_n$, i.e., beta decay becomes negligible because many positrons take part in the reactions at high temperature. Correspondingly, the equilibrium condition becomes $\mu_n = \mu_p + C\mu_e$, with $C \approx 2$. It is quite different from the well known result $\mu_n = \mu_p + \mu_e$. This result was first observed by Yuan (2005), and a detailed explanation can be found in Yuan (2005). Here we give a simple explanation that,

$$\lambda_{e-p} \propto n_e n_p \propto f_e f_p, \quad \lambda_{e+n} \propto n_e n_n \propto f_n f_{e+},$$

so

$$\lambda_{e-p} - \lambda_{e+n} \propto f_e f_p - f_n f_{e+} = f_p f_{e+} \left(\frac{f_e}{f_{e+}} - \frac{f_n}{f_p} \right).$$

Considering

$$f_e \approx \exp\left(\frac{E_e - \mu_e}{kT}\right), \quad f_{e+} \approx \exp\left(\frac{E_e + \mu_e}{kT}\right), \quad f_p \approx \exp\left(\frac{E_p - \mu_p}{kT}\right)$$

and

$$f_n \approx \exp\left(\frac{E_n - \mu_n}{kT}\right),$$

we find $\mu_n = \mu_p + 2\mu_e$ is valid when $\lambda_{e-p} = \lambda_{e+n}$. For a more universal case, none of the terms λ_{e-p} , λ_{e+n} or λ_n can be ignored, and the coefficient C will vary with the physical conditions.

Table 1 Steady state chemical equilibrium condition when the neutrino is transparent for $T = 10^9$ K.

Y_e	ρ (g cm ⁻³)	λ_{e-p}	λ_{e+n}	λ_n	$\mu_{e'}$	$\mu_{n'}$	$\mu_{p'}$	C
		(cm ⁻³ s ⁻¹)			(MeV)			
0.10	1.50E+08	8.56E+26	3.52E+19	8.56E+26	0.84	-0.43	-0.62	1.09
0.20	6.83E+07	5.21E+26	2.22E+19	5.21E+26	0.80	-0.51	-0.63	1.07
0.30	4.27E+07	3.72E+26	1.63E+19	3.72E+26	0.78	-0.56	-0.63	1.06
0.40	3.02E+07	2.79E+26	1.26E+19	2.79E+26	0.76	-0.60	-0.64	1.05
0.50	2.29E+07	2.14E+26	9.98E+18	2.14E+26	0.74	-0.64	-0.64	1.03

Table 2 Steady state chemical equilibrium condition when the neutrino is transparent for $T = 5 \times 10^9$ K.

Y_e	ρ (g cm ⁻³)	λ_{e-p}	λ_{e+n}	λ_n	$\mu_{e'}$	$\mu_{n'}$	$\mu_{p'}$	C
		(cm ⁻³ s ⁻¹)			(MeV)			
0.10	2.32E+08	2.00E+29	1.57E+29	4.28E+28	0.64	-3.00	-3.94	1.95
0.20	8.38E+07	9.51E+28	7.73E+28	1.79E+28	0.45	-3.49	-4.08	1.96
0.30	4.43E+07	5.71E+28	4.75E+28	9.61E+27	0.33	-3.82	-4.18	1.97
0.40	2.71E+07	3.71E+28	3.14E+28	5.64E+27	0.23	-4.10	-4.27	1.98
0.50	1.78E+07	2.47E+28	2.13E+28	3.38E+27	0.14	-4.35	-4.35	1.99

Figure 2 shows the coefficient C at different T and Y_e values. It can be found that C mainly depends on temperature T . When $T < 10^9$ K, $C \approx 1$; when T increases from 10^9 K to 5×10^9 K, C increases significantly from 1 to 2; when $T > 5 \times 10^9$ K, $C \approx 2$. However, when $T > 3 \times 10^{10}$ K and $Y_e > 0.4$, C is obviously larger than 2. The reason is that the fiducial analysis in Yuan (2005) ignores the Fermi function. If we set the Fermi function equal to 1, $C \approx 2$ is still valid. For convenience in practical application, we give a practical analytic fitting formula,

$$C = 2 - \left[1 + \exp\left(\frac{T_9 - A_i}{B_i}\right) \right]^{-1}, \quad (13)$$

where $A=[2.8643, 2.9249, 2.9785, 2.9902, 3.0094]$ and $B=[0.79138, 0.72181, 0.66331, 0.61813, 0.57999]$ corresponds to $Y_e=[0.1, 0.2, 0.3, 0.4, 0.5]$. T_9 is the temperature in units of 10^9 K ($T_9 \in [1 - 6]$). The accuracy of the fitting is generally better than 1%.

As an example, we introduce an application to analyze the electron fraction of a GRB accretion disk. A GRB is one of the most violent events in the universe, but its explosion mechanism is still

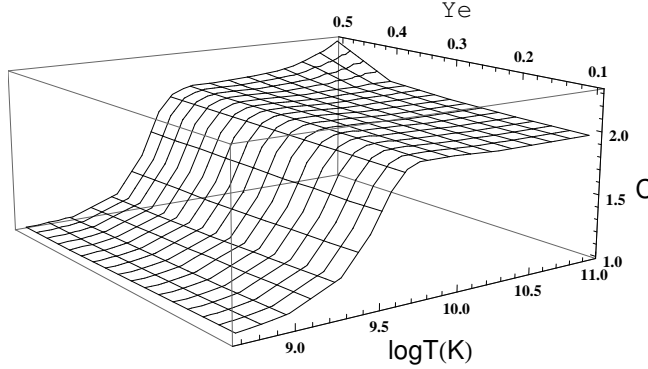


Fig. 2 Coefficient C of chemical potential equilibrium condition $\mu_n = \mu_p + C\mu_e$ as a function of T and Y_e .

not clear. Many authors support the view that a GRB originates from the accretion disk of a stellar mass black hole. Various accretion rates (from $0.01 M_\odot s^{-1}$ to $10 M_\odot s^{-1}$) cause quite a significant difference in the disk structure and composition. Since the temperature of the accretion disk is generally larger than $10^{10} K$, all nuclei are dissociated to become free nucleons, so the npe^\pm gas can describe the composition well. For lower accretion rates ($\dot{M} \leq 0.1 M_\odot s^{-1}$), the disk is transparent to neutrinos and antineutrinos, and neutrino and antineutrino absorption are not important (Surman & McLaughlin 2004). Adopting the steady equilibrium condition, Y_e of the disk model PWF99 (Popham et al. 1999) (accretion rate $\dot{M} = 0.1 M_\odot s^{-1}$, alpha viscosity $\alpha = 0.1$, and black hole spin parameter $a = 0.95$) are obtained in Figure 3. The dashed line and solid line are the results from the steady equilibrium condition and the full calculation by Surman & McLaughlin (2004), respectively. It shows that in the inner region of the disk (from 20 km to 120 km), the electron fraction calculated from different methods are principally confirmed, which indicates that the composition in the disk is approximately in an equilibrium state, but our result is generally smaller than that of Surman & McLaughlin (2004). This is caused by the Fermi function correction in our calculation. Although in the outer region of the disk Y_e deviates from equilibrium, this deviation increases with the accretion disk's radius.

Surman & McLaughlin (2004) did not bother with specifying the radial profile of the temperature and the density of the accretion disk when calculating the electron fraction as a function of the radius for the model introduced by Popham et al. (1999). Here we rewrite the temperature and the density formulae of Popham et al. (1999)'s analytical model as

$$T = 1.3 \times 10^{11} \alpha^{0.2} M_1^{-0.2} R^{-0.3} K, \quad (14)$$

$$\rho = 1.2 \times 10^{14} \alpha^{-1.3} M_1^{-1.7} R^{-2.55} \dot{M}_1 \text{g cm}^{-3}, \quad (15)$$

where M_1 is the mass of the accreting black hole in M_\odot , and R is the radius given in terms of gravitational radius r_g ($r_g \equiv GM_1/c^2$, which is equal to 1.4767 km for $M_1 = 1 M_\odot$). Since the explicit formulae are given, we adopt the equilibrium condition of npe^\pm gas to obtain some representative values of Y_e in Figure 4 at a radius larger than the inner edge (six gravitational radii) of the accretion disk. One can find from Figure 4 that Y_e values have a rapid increase with radius because both densities and temperatures decrease rapidly when radius increases and the variation of Y_e is very sensitive to density and temperature as shown in Figure 1. Regarding the different accretion rates, the accretion rate is larger, and the Y_e value is larger. This means the distribution of Y_e values along the radius strongly depends on the structure equations of the disk.

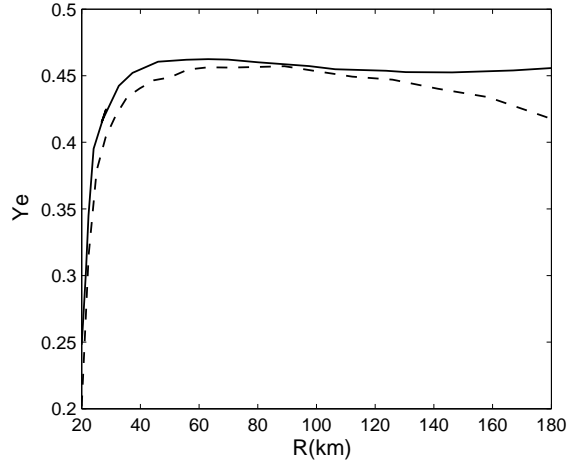


Fig. 3 Y_e as a function of accretion disk radius for model $\dot{M} = 0.1$, alpha viscosity $\alpha = 0.1$, and black hole spin parameter $a = 0.95$. The dashed line shows Y_e from the steady state equilibrium condition, while the solid line is the full calculation by Surman & McLaughlin (2004).

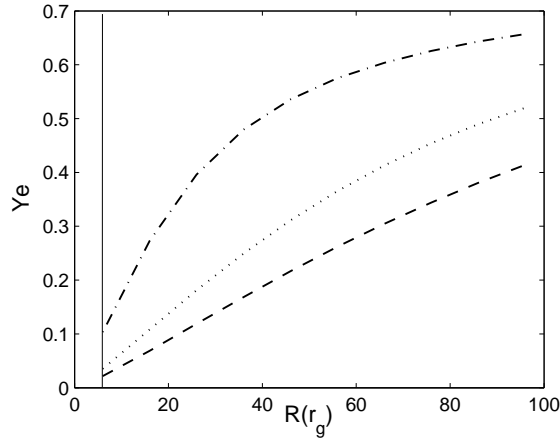


Fig. 4 Y_e as a function of accretion disk radius for the thin disk analytical model ($\alpha = 0.1$, $a = 0$, $M_1 = 3$). The long-dashed line, dotted line and dot-dashed line show Y_e as the accretion rate $\dot{M} = 0.01$, 0.05 , and 0.1 , respectively. The vertical solid line denotes the inner boundary of the accretion disk (six gravitational radii).

2.2 Case 2. Neutrinos are Opaque

In neutrino-opaque and antineutrino-opaque matter, neutrinos and antineutrinos will be absorbed by protons and neutrons, except for the reactions (2)–(4) as follows,

$$\nu_e + n \rightarrow e^- + p, \quad (16)$$

$$\bar{\nu}_e + p \rightarrow e^+ + n. \quad (17)$$

By using cross sections

$$\sigma_{\nu_{e\text{n}}}^{\text{abs}} = \frac{A}{\pi^2} (E_{\nu_e} + Q) [(E_{\nu_e} + Q)^2 - 1]^{1/2} (1 - f_e),$$

and

$$\sigma_{\bar{\nu}_{e\text{p}}}^{\text{abs}} = \frac{A}{\pi^2} (E_{\bar{\nu}_e} - Q) [(E_{\bar{\nu}_e} - Q)^2 - 1]^{1/2} (1 - f_{e^+}),$$

we obtain their rates (in the system of natural units)

$$\lambda_{\nu_{e\text{n}}} = \frac{A}{2\pi^4} n_n \int_0^\infty (E_{\nu_e} + Q) [(E_{\nu_e} + Q)^2 - 1]^{1/2} F(Z + 1, E_{\nu_e} + Q) (1 - f_e) E_{\nu_e}^2 f_{\nu_e} dE_{\nu_e}, \quad (18)$$

$$\lambda_{\bar{\nu}_{e\text{p}}} = \frac{A}{2\pi^4} n_p \int_0^\infty (E_{\bar{\nu}_e} - Q) [(E_{\bar{\nu}_e} - Q)^2 - 1]^{1/2} F(-Z + 1, E_{\bar{\nu}_e} - Q) (1 - f_{e^+}) E_{\bar{\nu}_e}^2 f_{\bar{\nu}_e} dE_{\bar{\nu}_e}, \quad (19)$$

where f_{ν_e} and $f_{\bar{\nu}_e}$ are the Fermi-Dirac distribution function of neutrinos and antineutrinos,

$$f_{\nu_e} = \left[1 + \exp\left(\frac{E_{\nu_e} - \mu_{\nu_e}}{kT}\right) \right]^{-1},$$

and

$$f_{\bar{\nu}_e} = \left[1 + \exp\left(\frac{E_{\bar{\nu}_e} - \mu_{\bar{\nu}_e}}{kT}\right) \right]^{-1}.$$

The number densities of neutrinos and antineutrinos are

$$n_{\nu_e} - n_{\bar{\nu}_e} = \frac{4\pi}{h^3} \int p^2 dp \frac{1}{1 + \exp\left(\frac{E_{\nu_e} - \mu_{\nu_e}}{kT}\right)} - \frac{4\pi}{h^3} \int p^2 dp \frac{1}{1 + \exp\left(\frac{E_{\bar{\nu}_e} + \mu_{\nu_e}}{kT}\right)}. \quad (20)$$

When $n_{\nu_e} = n_{\bar{\nu}_e}$, i.e., number density of neutrinos is equal to that of antineutrinos, $\mu_{\nu_e} = \mu_{\bar{\nu}_e} = 0$. In this case, the equilibrium condition of Equation (5) becomes

$$\lambda_{e^-p} - \lambda_{\nu_{e\text{n}}} = \lambda_{e^+n} - \lambda_{\bar{\nu}_{e\text{p}}} + \lambda_n. \quad (21)$$

One can find from Table 3 that even at $T = 5 \times 10^{10}$ K (μ_p' , μ_e' and μ_n' are chemical potentials without rest mass), C is still approximately equal to 1. In other words, when systems with neutrinos and antineutrinos are opaque and their chemical potentials are zero, $\mu_n = \mu_p + \mu_e$ is always effective no matter what the temperature is, just as expected.

Table 3 Steady state chemical equilibrium condition when neutrinos are opaque for $T = 5 \times 10^{10}$ K.

Y_e	ρ (g cm^{-3})	λ_{e^-p}	$\lambda_{\nu_{e\text{n}}}$	λ_{e^+n}	$\lambda_{\bar{\nu}_{e\text{p}}}$	λ_n	μ_e'	μ_n'	μ_p'	C
		(cm $^{-3}$ s $^{-1}$)					(MeV)			
0.10	2.32E+11	7.92E+37	7.89E+37	7.88E+36	7.56E+36	1.23E+31	10.18	-15.14	-24.65	1.01
0.20	6.17E+10	2.03E+37	2.01E+37	4.17E+36	4.01E+36	5.87E+30	6.69	-21.40	-27.38	1.01
0.30	2.46E+10	7.35E+36	7.26E+36	2.48E+36	2.39E+36	3.10E+30	4.37	-25.95	-29.60	1.01
0.40	1.05E+10	2.75E+36	2.71E+36	1.40E+36	1.35E+36	1.52E+30	2.48	-30.29	-32.03	1.01
0.50	3.40E+09	7.56E+35	7.40E+35	5.59E+35	5.43E+35	5.17E+29	0.75	-35.94	-35.93	1.02

3 EQUILIBRIUM CONDITION OF npe^\pm GAS WITH EXTERNAL NEUTRINO FLUX

As discussed in Section 2, we only consider that npe^\pm gas is isolated, but for many astrophysical environments, the external strong neutrino and antineutrino fluxes cannot be ignored. These processes involve some complex and difficult problems that concern both the neutrino transport and the interactions with nucleons. Here we discuss the neutrino-driven wind (NDW) from a proto-neutron star (PNS) as a typical example. NDW is regarded as a major site for r-process nucleosynthesis according to the observations of metal-poor stars in recent years (see e.g., Qian 2008, 2000; Martínez-Pinedo 2008). Since the NDW was first proposed by Duncan et al. (1986), many detailed analyses of this process have been done by many authors, including Newtonian and general relativity hydrodynamics and other physical inputs: rotation, magnetic field, termination shock and so on (Qian & Woosley 1996; Thompson 2003; Metzger et al. 2007; Kuroda et al. 2008; Thompson et al. 2001; Fischer et al. 2009). A basic scenario of r-process nucleosynthesis in the NDW can be simply described as (see Martínez-Pinedo 2008): soon after the birth of the PNS, many neutrinos are emitted from its surface; because of the photodisintegration of the shock wave, the main composition at the surface of a PNS consists of protons, neutrons, electrons and positrons (i.e., npe^\pm gas); in the circumambience of the PNS, the main reactions are the neutrino or antineutrino's absorption and emission by nucleons (the so called 'neutrino heat region'); in a further region, the electron fraction Y_e stays constant and α particles are formed; above this region, other particles, such as ^{12}C and ^9Be , are produced until the seed nuclei are formed; abundant neutrinos are successively captured by seed nuclei. The previous researches show that steady state is a good approximation to the NDW in the first 20 s (Thompson et al. 2001; Thompson 2003; Qian & Woosley 1996; Fischer et al. 2009).

Usually, neutron-to-seed ratio, electron fraction, entropy and expansion timescale are four essential parameters for a successful r-element pattern. It is very difficult to fulfill all those conditions self-consistently. Electron fraction Y_e is one of the most important parameters. Recent research by Wanajo et al. (2009) shows that the puzzle of the excess of the r-element of $A = 90$ may be solved if Y_e can increase 1%–2% (Wanajo et al. 2009). The evolution of Y_e is usually obtained by solving the group of differential equations which is related to the Equation of State (EOS), neutrino reaction rates and hydrodynamic condition (Thompson et al. 2001). The initial value of Y_e at the start of the wind is an important boundary condition. Considering that the neutrinos are emitted from a neutrino sphere, Y_e at the neutrino sphere can be regarded as the initial value of Y_e for the wind. For a given model, the initial value of Y_e can be determined by making the assumption that the matter in the neutrino sphere is in beta equilibrium (Arcones et al. 2008). To compare the results with the previous work of Arcones et al. (2007, 2008), we employ the same PNS model M15-11-r1. The model has a baryonic mass of $1.4 M_\odot$, which is obtained in a spherically symmetric simulation of the parameterized $15 M_\odot$ supernova explosion model. Detailed research shows that there are a few α particles which will appear in the neutrino sphere, but the number density of the α particles is much smaller than that of the protons or neutrons, so it is reasonable to ignore the α particle effect on the electron fraction, i.e., the matter is regarded as an npe^\pm gas. Simultaneously, although many neutrinos and antineutrinos are emitted from the PNS, their number densities are equal, which means $\mu_{\nu_e} = \mu_{\bar{\nu}_e} = 0$. Since the neutrinos and antineutrinos are transparent to the matter in the neutrino sphere, neutrinos produced by reactions (2)–(3) cannot interact with nucleons, but for the neutrinos and antineutrinos coming from the core region of the PNS, absorption reactions (16) and (17) are permitted. Their rates are

$$\lambda_{\nu_{en}} = \frac{L_{n,\nu_e}}{4\pi R_\nu^2} \sigma_{\nu_{en}}^{\text{abs}} \rho (1 - Y_e) N_A, \quad (22)$$

$$\lambda_{\bar{\nu}_{eP}} = \frac{L_{n,\bar{\nu}_e}}{4\pi R_\nu^2} \sigma_{\bar{\nu}_{eP}}^{\text{abs}} \rho Y_e N_A, \quad (23)$$

where $L_{n,\nu}$ and $L_{n,\bar{\nu}_e}$ are the number luminosity of the neutrino and antineutrino respectively, and R_ν is the neutrino sphere's radius. Considering too many physical factors (EOS, transport equation and so on) will influence the number luminosity and the neutrino energy, so we simply assume the number luminosity and the energy of neutrinos and antineutrinos are the same as those in the wind. First, we obtain the electron fraction by using a general equilibrium condition

$$\lambda_{e-p} - \lambda_{\nu en} = \lambda_{e+n} - \lambda_{\bar{\nu}_e p} + \lambda_n.$$

In other words, if the density and temperature are fixed for the equilibrium system, the electron fraction is unique. Then the coefficient C in the chemical potential equilibrium condition is determined (rightmost column in Table 4). The results for model M15-11-r1 are shown in Table 4.

In Table 4, t is the time post bounce, R_ν is the neutrino sphere's radius, L_n is the number luminosity for neutrino and antineutrino, and $\langle E_{\nu_e} \rangle$ and $\langle E_{\bar{\nu}_e} \rangle$ are the average energy of the neutrino and antineutrino respectively. All parameters above refer to Ref. (Arcones et al. 2008). Y_e^a is the electron fraction for an extreme case, $C = 1$, which is adopted in reference (Arcones et al. 2008); Y_e^b is the result in which the steady equilibrium condition is valid and the external neutrino flux is also considered. We can find Y_e^b is universally smaller than Y_e^a , which means the external neutrino flux strongly influences the composition of the equilibrium system. Comparing Y_e^a with Y_e^b , one can find that the improved equilibrium condition makes the electron fraction significantly decrease when the time is less than 5 s post bounce. After 5 s the electron fractions are similar to the case $C = 1$. Note that it is just a conclusion for the model M15-11-r1. Due to the huge differences between the different models, the results may be quite different for the other models. More detailed consideration will be done in our further work. Initial electron fraction is an important boundary condition to determine the electron fraction of the wind. Since r-process nucleosynthesis is strongly dependent on the electron fraction, an accurate electron fraction is useful for the final r-process nucleosynthesis.

Table 4 Evolution of the initial electron fraction in chemical equilibrium conditions of different steady states.

t (s)	R_ν (km)	T (MeV)	L_n (10^{56} s^{-1})	$\langle E_{\nu_e} \rangle$ (MeV)	$\langle E_{\bar{\nu}_e} \rangle$ (MeV)	ρ (g cm^{-3})	Y_e^a	Y_e^b	C
2	10.55	6.34	6.05	20.71	25.64	5.50E+11	0.113	0.084	1.39
5	9.82	5.14	3.55	17.1	22.6	1.30E+12	0.050	0.039	1.22
7	9.68	4.73	3.03	15.9	21.69	1.40E+12	0.042	0.035	1.15
10	9.59	4.37	3.06	15.05	21.86	2.00E+12	0.029	0.028	1.03

4 CONCLUSIONS

In this work, we derive the chemical potential equilibrium conditions $\mu_n = \mu_p + C\mu_e$ for npe^\pm gas in two cases (with and without external neutrino flux). Especially for neutrino-transparent matter, employing the fitting Equation (13) for the transition from low temperature to high temperature is a more convenient method than the usual method for calculation of interaction rates. Since chemical potentials are dependent on three parameters: density, electron fraction and temperature, any one of these three parameters can be determined if the other two parameters are given. Although the variation of factor C is complicated when the external neutrino flux cannot be ignored, one can obtain the extremum of those parameters assuming $C = 1$ or 2. Furthermore, our results can be regarded as the reference value for non-equilibrium states. Considering the simplicity and far-ranging aspects of the astrophysical environment, the results in this paper are expected to be used widely in the further related works.

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